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William Zahner[a]; Judit Moschkovich[a]
[a] University of California, Santa Cruz

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Talking while Computing in Groups: The Not-So-Private Functions of Computational Private Speech in Mathematical Discussions

William Zahner and Judit Moschkovich
University of California, Santa Cruz

Students often voice computations during group discussions of mathematics problems. Yet, this type of private speech has received little attention from mathematics educators or researchers. In this article, we use excerpts from middle school students' group mathematical discussions to illustrate and describe "computational private speech." We analyze four examples of computational private speech using lenses from neo-Vygotskian psychology, sociolinguistics, and distributed cognition. Our analyses of computational private speech challenge the individualistic developmental assumptions of some neo-Vygotskian theories of private speech, and we show how this form of private speech can serve socio-cognitive functions during group mathematical discussions.

Observing that young children and even adults occasionally talk to themselves is not new. Both Piaget (1959/2002) and Vygotsky (1986) developed theories about this phenomenon, which they called *egocentric speech*. Likewise, Goffman (e.g., 1978) and other sociolinguists have analyzed the form and function of what they call "response cries" by both adults and children. In this article we describe a related phenomenon that we call "computational private speech" (CPS)—people talking to themselves when doing computations while working on mathematics problems with others. We focus on middle school students verbalizing computations during group work in mathematics classrooms, and we use three theoretical lenses to analyze this type of talk: neo-Vygotskian psychology, sociolinguistics, and an activity theory/distributed cognition perspective. We conclude by proposing that CPS may serve a regulatory function for the "distributed mind" of a group of students collaboratively doing calculations.

Our goal in this article is to advance the conceptualization of private speech (PS) by documenting and describing a type of PS that has received little attention in its own right. We investigate the function of this talk in classroom mathematical discussions and we use four examples of CPS from our studies of students' in-class mathematics discussions to define CPS and to illustrate how three different theoretical perspectives illuminate this type of PS.

Correspondence should be sent to William Zahner, Education Department, UC Santa Cruz, 1156 High Street, Santa Cruz, CA 95064. E-mail: wzahner@ucsc.edu

AN INTRODUCTORY EXAMPLE

We start with an example of CPS to ground our subsequent discussion in a real example of the phenomenon. The following excerpt of a mathematical discussion is from a recording of students in a sixth-grade classroom in a bilingual school in California. Six students were seated around a table and completing a worksheet with 27 exercises involving computing percentages (e.g., "write the following fractions as percents"; see Figure 1).

Each student had an individual worksheet, but the teacher encouraged the students to work together and to help their groupmates complete the exercises. In the midst of the students' group work we recorded the following discussion.

Excerpt 1[1]

Participants: Claudia, Dennis, Amber, Francisco, Joaquin, and Louis (did not speak).

Physical arrangement: The six students were sitting around six desks pushed together to form a table. Each student had a worksheet.

1.	Claudia:	It's twenty four [percent ((looks up at Amber))
2.	Joaquin:	[xxxx ((looking off camera))
3.	Claudia:	It's twenty four percent. ((erases paper)) Amber, it's twenty four percent.
4.	Dennis:	What?
5.	Amber:	Qué? (("What?"))
6.	Claudia:	The number eight's twenty four percent.
7.	Francisco:	Twenty four?
8.	Amber:	My bad. Twenty fi:ve six that's three no ((Amber writes on her paper while talking and Claudia walks away from the table)) times four is t- twenty four that's (one that's) [zero=
9.	Joaquin:	[xxx
10.	Amber:	= that goes in (.) [two
11.	Francisco:	[Oh yeah, twenty four percent
12.	Amber:	How is it twenty four?
13.	Francisco:	OK cause look OK xxx when you put the decimal there then with that it will be two ten that'll be fifty then you have to bring down then you have- wait what the heck?

The underlined portions of lines 8 and 10 are examples of what we call CPS. In line 8, Amber began her turn by responding directly to Claudia ("my bad") and acknowledging that her answer to question number 8 on the worksheet did not match Claudia's answer. The first part of line 8 is *not* an example of PS, because it formed the second part of an adjacency pair initiated by Claudia in line 6. However, we would label the second half of Amber's utterance in line 8 (underlined for emphasis) as CPS. Although we do not have a copy of Amber's written work, the video shows her writing a long division and her talk corresponds with doing several steps in the long

Convert the following into a percent.

$$\frac{6}{25} = \_\_\_\_\_\%$$

FIGURE 1 The worksheet exercise discussed in Excerpt 1.

[1]In these transcripts we use Jefferson's transcription conventions as described in Schiffrin (1994). Utterances in Spanish are translated in comments enclosed by double parenthesis and quotation marks: (("this is translated text")). We use underlining to mark text for discussion.

What Amber Said	Our reconstruction of what Amber wrote
Twenty fi:ve six	$6\overline{)25}$
that's three no times four	4 $6\overline{)25}$
is t- twenty four	4 $6\overline{)25}$ 24
that's (one that's) zero that goes in (.) [two	4 $6\overline{)25.0}$ $\underline{24}$ 10

FIGURE 2 Our reconstruction of what Amber wrote during Excerpt 1 based on her talk and the video recording.

division algorithm for 25 ÷ 6. Figure 2 shows our reconstruction of what Amber wrote while she was talking.

Based on her talk it appears that Amber had not reached a terminating or repeating decimal in the long division algorithm when she said, "How is it twenty four" in line 12 (note that 25 ÷ 6 = 4.1666 . . .).

There are several indicators that Amber was not addressing Claudia or any other specific group member in the underlined parts of lines 8 and 10. First, Amber's gaze was fixed on her paper. Second, Claudia walked away from the table in the middle of Amber's utterance, but Amber continued talking while writing on her paper. Finally, in contrast to her "private" utterances in lines 8 and 10, in line 12 Amber directly responded to Francisco's initiation in line 11 while nodding her head toward him. We analyze this discussion in more detail later in the article. For now, we use this example to illustrate our definition of CPS. We also note that we have documented talk similar to Amber's CPS in several small group mathematical discussions in other settings.

A NOTE ON TERMINOLOGY AND DEFINITION OF COMPUTATIONAL PRIVATE SPEECH

Both Piaget and Vygotsky used the term "egocentric speech" to describe the phenomenon of children talking with no apparent addressee. Goffman (1978) and sociolinguists use the term "response cries" for a similar class of utterances. Subsequent investigators have developed a plethora of neologisms for this type of talk including "private speech," "private verbal thinking," "verbalized inner speech," talking on one's "private channel," "self-focused talk," and still several other terms. In this article we use the term private speech (PS) to describe the general phenomenon of a speaker voicing utterances with no immediately apparent addressee and computational private speech (CPS) to specify when private speech utterances are part of computational activity. Also, we restrict our focus to CPS in small-group discussions rather than during individual computational activity. The utterances that we call CPS meet the following criteria: (a) The speaker is engaging in computation; (b) the utterance is part of a "broken" adjacency pair—it does not respond to an initiation by another speaker and it does not constitute an initiation; and (c) the speaker does not use extralinguistic cues of addressivity such as gaze, gesture, and posture oriented toward others. A shorthand method we have developed to identify CPS in a transcript of a group mathematical discussion is by the criterion that, when CPS utterances are removed from the transcript, the dialogue remains intact although some *meaning* may be lost (analogous to how prepositional phrases can be removed from a sentence).

DEVELOPMENTAL PSYCHOLOGY PERSPECTIVES

Piaget observed that young children "think aloud" and he viewed engaging in "egocentric" speech as part of a child's normal developmental trajectory (Piaget, 1959/2002). However, he used the term *egocentric* to emphasize that talking to oneself is one of the stages a child passes through en route to becoming a well-adjusted, social user of language. Piaget observed that as children develop, they start producing little egocentric speech, then the child's production of egocentric speech peaks, and eventually they use less egocentric speech. Vygotsky (1986) observed the same phenomenon, but he disputed Piaget's negative evaluation of egocentric speech. Vygotsky interpreted the developmental pattern in children's production of egocentric speech as evidence that children internalize social speech. He viewed egocentric speech as an intermediate stage that children pass through as they internalize social speech and transform it into inner speech—or verbal thought. To support this claim, Vygotsky documented changes in the relative order of egocentric speech and action, the relationship between egocentric speech and task difficulty, and the relative frequency of egocentric speech when children are alone or in the presence of others.

More recent psychological studies of PS in the neo-Vygotskian tradition have typically arranged situations where children or adults solve problems and researchers manipulate and/or measure the amount of PS that participants produce. A typical example of a neo-Vygotskian experiment recorded 46 five- and six-year-old children solving a puzzle in the presence of an experimenter (Fernyhough & Fradley, 2005). We use this study as a typical example, but there are many other similar studies in this area. Fernyhough and Fradley found that the participants' production of PS peaked on medium difficulty tasks, which they interpreted as evidence that a child verbalizes PS most often while working on tasks that are in the child's zone of proximal development.

Most neo-Vygotskian analyses of PS with a developmental perspective focus on how PS mediates individual participants' self-regulation and executive functioning. Some of these studies have explored whether environmental conditions, including the presence of others in the room, alter children's production of PS. Yet, in most of these studies, the *social* focus is limited to exploring how verbal thought (or "inner speech") is internalized social speech, and little attention is paid to exploring possible interpersonal communicative functions of PS or recipient design of PS utterances—a stance that makes sense if we assume that PS is completely private. In the next section of this article we describe how sociolinguistic studies of "response cries" (or

PS) help uncover more information about possible interpersonal communicative functions and recipient design of PS utterances. We also revisit this theme in the section on distributed cognition/activity theory perspectives on CPS.

Neo-Vygotskian Analysis of Excerpt 1

An analysis of Excerpt 1 from a neo-Vygotskian perspective might start by focusing on the difficulty of the computation that Amber verbalized in lines 8 and 10. Based on the problem she was doing, her talk, and her writing visible on the video, Amber appeared to be talking through the steps for computing 25 ÷ 6 using long division (note that if her intention was to find the percentage by long division, then the computation should have been 6 ÷ 25). One hypothesis might be that Amber voiced her computations because this long division problem was relatively difficult for her and, therefore, her PS served self-regulatory functions. Just as prior research found that the amount of PS peaks on medium difficulty tasks (Ferynhough & Fradley, 2005), Amber may have been regulating her activity with PS. This interpretation is plausible, but it raises questions regarding *what part of the problem* was difficult for Amber. Given that she appeared to be doing the division "backward," perhaps Amber's difficulty was not with carrying out the steps of the long division algorithm but rather with *setting up the computation*. This ambiguity highlights one of the limitations of using theories developed in clinical interview settings to analyze CPS occurring in a classroom, because it may not be possible to tell which aspects of a task are difficult or responsible for increasing the occurrence of PS in a classroom.

Amber's CPS in Excerpt 1, together with other recordings from this classroom, do show that she was using a shared semiotic tool in her computational talk. One of Vygotsky's primary interests in PS was that PS offers evidence of children's internalization of language, that is, their appropriation of socially shared semiotic tools for thinking. We have some evidence that Amber's CPS was connected to a socially shared semiotic tool—the long division algorithm accompanied by the inscription and verbal description of that algorithm. To illustrate this connection, we next show how Amber's teacher and classmates used long division to solve a similar problem during a whole-class discussion 27 min prior to the discussion in Excerpt 1.

Excerpt 2

Participants: Teacher and 27 students in the class. The students from the group featured in Excerpt 1 are named and other student voices are labeled as "Student."

Problem: The task is to find the percent that is shaded in a 3 x 5 rectangle with 6 shaded squares (similar to the task in Figure 4). At the start of this excerpt the teacher had already written the fraction $\frac{6}{15}$ *on the board.*

1.	Teacher:	So how do I make a per- a fraction into a percent. Where are all the different ways to convert it? There are posters all over this room. In what area xxx that say how to convert xxx from one thing to another? Just point where they are. They're all over there at the door. Right and one of them is falling off. ((referring to a poster that is falling off the wall in the back of the classroom)).
2.	Claudia:	Divide it
3.	Teacher:	We divide right. What goes in the house, the six or the fifteen?
4.	Student:	xxx
5.	Teacher:	Six ((writes 6 in division sign)) fifteen ((writes 15 outside of division sign)). Fifteen goes into six. Yes
6.	Claudia:	Um, you put a decimal

7. Student: No
8. Teacher: Decimal, [OK=
9. Claudia: [And then a zero
10. Teacher: =go straight up and then there ((writes decimal in the quotient))
11. Student: five fifteen goes into oh four four times
12. Teacher: Four times ((writes 60 below 6.0 in the dividend)) OK I'm done? xxx lets add a zero. Let's add a zero so it's going to be forty percent or forty point forty or its going to be forty percent. OK?

The teacher and the students' talk detail the steps for setting up and completing the following long division that the teacher wrote on the board during Excerpt 2 (see Figure 3).

Whereas the whole-class discussion in Excerpt 2 did not include CPS, there are several parallels between the teacher and the students' talk in Excerpt 2 and Amber's CPS in Excerpt 1. We propose that these parallels are evidence that Amber was using a shared semiotic tool in her CPS during Excerpt 1. First, both the teacher in Excerpt 2 and Amber in Excerpt 1 used long division to find a percentage from a fraction. Second, they both described setting up the computation in similar ways while writing the first number under the long division sign and the second number outside (Compare line 5 of Excerpt 2 "six fifteen" with line 8 of Excerpt 1 "twenty fi:ve six"). Third, further evidence that the long division algorithm, together with its inscription and verbal description, was a socially shared tool in this classroom is that Amber's talk while putting numbers in the quotient in Excerpt 1 was similar to the talk by other students who contributed to the whole-class use of the division algorithm in Excerpt 2. Notice the parallel between the student contribution "five fifteen goes into oh four four times" (line 11 of Excerpt 2) and Amber's description of the algorithm, "that's three no times four is twenty four" (line 8 of Excerpt 1). Taken together, these parallels between the whole-class talk in Excerpt 2 and Amber's CPS in Excerpt 1 support the neo-Vygoskian claim that PS (and by extension CPS) is internalized social speech.

What Was Said	**Our reconstruction of the Teacher's writing on the board**
Teacher: Six fifteen. Fifteen goes into six. Yes	$15\overline{)6}$
Claudia: Um you put a decimal Teacher: Decimal OK Claudia: And then a zero	$\begin{array}{r} . \\ 15\overline{)6.0} \end{array}$
Student: five fifteen goes into oh four four times. Teacher: Four times	$\begin{array}{r} .4 \\ 15\overline{)6.0} \\ \underline{60} \\ 0 \end{array}$
Teacher: Lets add a zero so it's going to be forty percent	$\begin{array}{r} .40 \\ 15\overline{)6.00} \\ \underline{60} \\ 0 \end{array}$

FIGURE 3 Our reconstruction of what the teacher wrote during Excerpt 2.

Challenges of Using a Neo-Vygotskian Perspective

One of the advantages of neo-Vygotskian studies of PS done in clinical settings is that by carefully designing tasks and controlling settings, researchers have shown how PS varies with task difficulty, the participant's level of development, and changes in the social arrangement. On the other hand, this control also raises the issue of ecological validity: It is not clear how most insights about PS from clinical studies generalize beyond laboratory settings. In this article we present examples of PS recorded during classroom discussions, and thus we are documenting talk related to mathematical thinking in a naturalistic setting. The advantage of recording children's talk in a naturalistic setting is that we can see the emergence of types of PS, like CPS in group discussions, that have not previously been studied. Yet a liability of studying children's talk in a naturalistic setting is that we cannot determine factors such as the difficulty of a task a priori (e.g., Ferynhough & Fradley, 2005) or control the social arrangement of the group doing mathematics. Given the data in Excerpt 1, it is not possible to attribute Amber's CPS to the task difficulty, the type of task, or the presence of others.

A second challenge for applying a neo-Vygotskian psychological lens to our examples of CPS is that defining PS in naturalistic settings is more challenging than in laboratory-based studies because in mathematical discussions children appear to rapidly shift between *social* and *private* speech. For example, in line 8 of Excerpt 1 we see evidence that Amber's single turn contains both social and private utterances with little or no pause between the social utterance ("my bad") and CPS.

A final drawback of applying neo-Vygotskian approaches to PS to CPS is that neo-Vygoskian studies have frequently included a developmental hierarchy for types of PS. One prominent example is Kohlberg's seven-level hierarchy of types of PS ranging from low-level PS such as *word play* to the highest stage of PS development, *silent inner speech* (Kohlberg, Yaeger, & Hjertholm, 1968). Numerous psychological researchers have used this hierarchical framework in their subsequent studies of PS. In contrast with neo-Vygotskian theories of PS, we do not think there is necessarily a strong relationship between an individual's level of development and her use of CPS. In our research and teaching we observe CPS not only among young children but also among groups of mathematically educated people as they solved mathematics problems. Thus, we conjecture that using CPS is not directly tied to development, mathematical expertise, or one's level of education in mathematics. One possibility that has been raised in the literature is that there are many different types of PS and that the use of these types of PS depends on the task rather than on the individual's level of development. For instance, one study treated verbal rehearsal of a list of items to be memorized as a distinct form of PS rather than a type of PS that occurs along a developmental trajectory (Flavell, Beach, & Chinsky, 1966). We are inclined to agree with Flavell's theory, and in the following sections we show how a combination of social and cognitive aspects of the problem-solving situation influence children's use of CPS.

SOCIOLINGUISITIC PERSPECTIVES

Sociolinguists have also documented that people sometimes "talk to themselves." These researchers have focused on the *social* functions of this type of talk rather than the self-regulatory functions of PS. One class of utterances, which Goffman termed “response cries,” appears similar to what psychologists call PS. Response cries can be defined as utterances that cannot neatly be classified as part of an adjacency pair, and Goffman’s rule about response cries is simple: Talking to one’s self is not *generally* permitted (Goffman, 1978). However, he noted that the exceptions to this rule shed light on the functions of response cries. Goffman argued that most of the occasions where response cries are used pertain to issues of self-presentation.

Goffman’s perspective reveals one possible *social* function of CPS. In Excerpt 1, Amber’s CPS made her computations present for her groupmates while she worked on the problem. By voicing computations in lines 8 and 10, Amber provided evidence for Claudia that she was acting on Claudia’s suggested answer in a socially acceptable manner for school mathematics group work. If Amber were only to have said “my bad” while writing “24%” in the answer space of her worksheet, then Claudia might have inferred that Amber was copying her answer—a violation of typical school norms as enacted in this classroom. In this regard Amber’s CPS served a social function by showing Claudia that she was not copying her answer.

Another social/linguistic approach that may help shed light on CPS is the theory of *speech genres*. “Each separate utterance is individual, of course, but each sphere in which language is used develops its own relatively stable types of these utterances. These we may call *speech genres*” (Bakhtin, 1986, p. 60). In this case, if we consider a peer-group (school) mathematical discussion as a speech genre, then perhaps voicing computations is one common type of utterance in this speech genre. This might explain why Amber’s groupmates did not mark her PS in Excerpt 1 as unusual.

The idea that verbalizing computations is relatively common in the genre of mathematical discussions is bolstered by the fact that other researchers have also documented this type of talk. Berk and Garvin (1984) had one example of a child working alone and voicing computations in their study of Appalachian children’s PS at school. In the group setting, other researchers have observed children voicing utterances not addressed to their partners during group mathematical discussions (e.g., Barron 2003). We would not categorize what these researchers documented as *computational private speech,* because the talk was not explicitly describing the carrying out of calculations during a group discussion. Nonetheless, these observations may be indirect evidence that verbalized PS is relatively common in the speech genre of small group mathematical discussions.

Supposing that verbalizing CPS is part of the genre of mathematical discussions, this does not tell us the *function* of these utterances. Another component of Bakhtin’s theory that may help when considering the functions of CPS is *addressivity*, the notion that every utterance responds to prior utterances and anticipates responses (Bakhtin, 1986). Addressivity is not limited to the immediate audience, and it does not depend on the speaker’s intention. Talking about the *addressivity* of a *private* utterance may seem absurd, but Goffman (1978) observed that despite their appearance of lacking an addressee, response cries (i.e., PS utterances) do serve social functions. But who (or what) is the addressee of CPS? Although addressivity may be impossible to ascribe, we can use a functional approach to language and change the question to what *social functions* does CPS accomplish for the speaker and her groupmates?

In addition to managing self-presentation, we see two additional, possible functions of CPS. One of our hypotheses is that CPS signals and reflexively creates a position of authority in peer mathematical discussions. A second hypothesis is that CPS promotes intersubjectivity or joint focus of attention. We use Excerpt 3 to elaborate these possibilities.

One way to exert authority in a peer discussion is to “take the floor” and keep it. In middle school students’ peer mathematical discussions, the conversational floor frequently appears contested (Barron, 2003), and we have observed that voicing computations is one move that helps a student maintain control of the floor. Excerpt 3 involves the same group of sixth graders from Excerpt 1. It illustrates how one group member, Claudia, used CPS to hold the floor after she had finished answering a peer’s question. The problem that the students were discussing is shown in Figure 4.

Excerpt 3
Participants: Francisco, Claudia, Amber, Dennis.
Physical Arrangement: Same as in Excerpt 1.

1.	Francisco:	ah- why six over eight seventy five percent?
2.	Claudia:	((bobbing head slowly)) Divide it.
3.	Amber:	OK watch. Six goes inside divide by eight [xxx=
4.	Claudia:	[Por eso ‘ira ((“that’s why, look”))
5.	Amber:	=and [that
6.	Claudia:	[goes in the casita ((“little house”)), you take out eight you put a decimal porque no se puede ((“because you cannot”)) [put a decim- you put a zero=
7.	Amber:	[Uh-huh
8.	Claudia:	=ocho por siete ((“eight times seven”)) put a seven here it’s fifty-six
9.	Dennis:	four
10.	Francisco:	(and that’s) four
11.	Dennis:	[[four
12.	Claudia:	[[five it’s ten is four
13.	Francisco:	Oh yeah, seventy five percent
14.	Claudia:	y (luego) ((“and then”)) [xxx zero
15.	Dennis:	[Zero then five
16.	Claudia:	y luego es ((“and then it is”)) seventy five percent

Our reconstruction of the written work that Claudia produced while she was talking in Excerpt 3 is in Figure 5.

Based on who contributed to the talk, this discussion appears to have three stages where the function of Claudia’s talk shifted. In the first stage from lines 1 to 7, Francisco posed a question, and Amber and Claudia appeared to dispute the right to take the floor and answer Francisco’s question. Claudia gave a terse answer in line 2. However, as Amber began to elaborate on Claudia’s response, Claudia interrupted her and retook the floor. These interruptions indicate that the right to explain the answer to Francisco was contested, so in addition to answering Francisco’s question, Claudia and Amber were also negotiating control of the conversational floor. The second stage of the discussion spans lines 8 through 11, and we do not consider Claudia’s voicing of calculations in these lines CPS, because she was responding to Francisco’s request for information in line 1. In this case, the function of her talk was to give

Find the percent that is shaded in the following figure.

FIGURE 4 The problem discussed in Excerpt 3.

What Claudia Said	Our reconstruction of what Claudia wrote
Goes in the casita ((“little house”)), you take out eight you put a decimal porque no se puede ((“because you cannot”)) [put a decim- you put a zero ocho por siete ((“eight times seven”)) put a seven here it’s fifty-six five it’s ten is four y (luego) ((“and then”)) [xxx zero y luego es ((“and then it is”)) seventy five percent	0.75 8) $^{5}\not{6}.^{10}\not{0}0$ –56↓ 40 –40 0

FIGURE 5 Our reconstruction of Claudia’s computation.

information in response to a request. However, the function of Claudia’s talk appeared to shift again during a third stage starting in line 12 and continuing through 16 when she used CPS. Again we have underlined the CPS for emphasis.

Why is Claudia’s talk on lines 12, 14, and 16 CPS? When Claudia reached the step 60 – 56 in the algorithm (line 8), both Francisco and Dennis interjected the answer of four (lines 9, 10, and 11). This did not stop Claudia from continuing her description of the algorithm steps; moreover, in line 12 she explicitly voiced the “carry step” that the boys had carried out implicitly in order to arrive at an answer of 4. The function of Claudia’s speech shifted away from “giving information” as she continued voicing computations even after Francisco said, “Oh yeah, seventy-five percent” in line 13.

In Excerpt 3 we see evidence for one social function of CPS: establishing and maintaining authority in a group mathematical discussion. As with Amber’s CPS, the parallel between Claudia’s talk in Excerpt 3 and the teacher’s talk in Excerpt 2 are evidence that a socially shared semiotic tool—the long division algorithm and the talk describing it—served as a basis for Claudia’s CPS. Furthermore, Claudia’s ventriloquism of the teacher’s talk in Excerpt 2 is striking, down to the shared metaphor of the house (“casita”) to refer to the long division sign. Imitating the teacher is another way for a student to construct herself as an authority in the classroom. Although on the surface Claudia’s CPS appears to not have an addressee, one function of Claudia’s CPS may have been to establish herself as an authority and maintain her position in that role.

The second social function we have identified for CPS is maintaining or establishing intersubjectivity and focusing the group discussion. Goodwin (1996) argued that response cries are “organized as social phenomena that provide very powerful resources for shaping the perception and action of others” (p. 393), and his analysis of talk-in-interaction among airline employees shows how response cries function in this way. In Goodwin’s case, the response cry of one employee focused a problem-solving discussion on the visual manifestation of a reported problem (a focus that ultimately proved unproductive for resolving the issue).

Claudia’s CPS in Excerpt 3 and Amber’s CPS in Excerpt 1 shaped the group interactions and suggested a very specific interpretation of these percentage problems. Both girls’ CPS focused on one particular solution method to such problems—doing long division. Their groupmates also adopted this solution method and subsequent discussions focused on the correct implementation of the algorithm. For example, after Excerpt 1, the group discussed how to do 6 ÷ 25 using long division for several minutes. Although long division is one way to convert a fraction to a percentage, it is not the only possible way to solve this type of problem. We can imagine a different classroom where the norms for discussions include giving not only procedural descriptions of how one solved a problem but also explanations of why a particular procedure was chosen, justifications for choosing this procedure, or other arguments to support the validity of the chosen solution. In this classroom, doing these fraction-to-percentage conversions by long division was an endorsed solution method (see Excerpt 2), and Amber and Claudia's CPS further reinforced that method and focused the discussion by assuming that these problems should be solved using long division.

When we (appear to) speak to ourselves, our talk still serves social functions. Response cries and CPS may well have an addressee, despite appearances to the contrary. However, a sociolinguistic interpretation of CPS does not necessarily illuminate how CPS helps a group of students solve problems—that is, think—together. Why do students (and adults) sometimes voice computations in a group that they might not voice while working alone? We think that answering this question is best accomplished by invoking the notion of distributed cognition.

ACTIVITY THEORY AND DISTRIBUTED COGNITION

Thus far we have used analytical lenses that focus on the psychological and sociolinguistic functions of PS and we have used those lenses to examine examples of CPS from students' discussions of calculations. Does CPS also facilitate students' collaborative problem solving? In the following section we propose an affirmative answer to this question and use examples from a group discussion in a different mathematics classroom to illustrate how CPS might help a group of students reason together as they work on nonalgorithmic mathematical tasks.

In our overall analysis we treat peer mathematical discussions as activity systems using an approach rooted in cultural historical activity theory as well as distributed cognition. A mathematical discussion among students is an activity system that includes subject(s), object(s), mediating artifacts, rules, a larger community context, and a division of labor. For example, in our analysis of the construction of authority in the group, we have examined the reflexive relationship between *division of labor (or roles)* in the group and emergent *rules* for verbal interaction between group members.

The notion of distributed cognition frames reasoning not as an individual performance but as distributed across people, time, artifacts, and cultural norms and activities. For example, Hutchins (1993) used distributed cognition to frame his analysis of navigating naval ships, and he argued that no single person does the task of navigation or does all of the computations needed to solve that problem. Much of the computational work is simplified by using navigational charts and tools; the creators of navigational charts and other specialized tools for navigation did much of the computational work of navigation long before any specific navigator or ship's Navigation Team locates and pilots a ship.

One key step in locating a ship near shore as described by Hutchins (1993) involves sighting three landmarks from the ship and then "fixing" the ship's location on the navigational chart by drawing the observed bearings to each landmark on the chart. When one person does this task, he or she goes outside to the wings of the bridge and sights each landmark, memorizes the sighted bearings (or writes them down), and then goes inside the chart room and records those bearings in a log and draws lines on the chart to triangulate the ship's location. However, in restricted waters this task must be done rapidly, so three to six people coordinate their actions to measure bearings simultaneously and transfer them to the chart. Each member of the navigation team is on the same telephone circuit and can hear all the others. In this case, the "memory" of bearings is not held inside one person's head (or on paper), but rather it is communicated via telephone from the wings of the bridge where the observations are made to the plotter at the navigation chart in the chart room. Hutchins argues that this is an example of *memory* being transformed into *communication*. Furthermore, this shared channel of communication allows for self-correction in the system. Inconsistent information transmitted by one member of the team is recognizable, and it is either corrected or ignored.

Combining distributed cognition and Vygotsky's (1986) theory of inner speech yields another interpretation of CPS: CPS allows group members to *reason together* while doing computations. Just as naval navigators use interpersonal language as a form of memory, CPS allows multiple students in a mathematical discussion to collectively "think about" a computation—an activity that is usually done silently and individually. We use Excerpts 4 and 5 to illustrate how CPS can function this way during a peer mathematical discussion in a school setting.

These excerpts are from a group discussion among four girls in an eighth-grade mathematics class where their tasks were to make a distance-time table based on written notes about a bicycle trip and then to create a graph based on their table (see Figure 6 for the problem and the girls' final table). In contrast with the students in Excerpts 1 and 3, whose discussions focused solely on calculations and who were working in a classroom where most discussions had a calculational orientation, these students had extensive experience working in groups on open-ended and conceptually focused problems. In addition to showing how CPS allows a group to think about a computation together, this excerpt also illustrates how CPS can occur during a group discussion in a classroom characterized by conceptually oriented discussions, especially when a calculation must be done to complete a task.

Excerpt 4 starts in the middle of an extended discussion when the group of students was making a distance-time table describing a bike trip. They had already made a table with 16 rows (representing each half hour of the 8-hour trip), and the girls were negotiating filling in the distances for each half hour. In line 13 Susan, who was recording numbers in the group's table, summed the numbers in the distance column to see whether they added to 80 (one of the given constraints).

Excerpt 4

Participants & Setting: Susan, Erin, Marta, and Kristin seated around a table with four chairs. Susan is writing in her notebook.

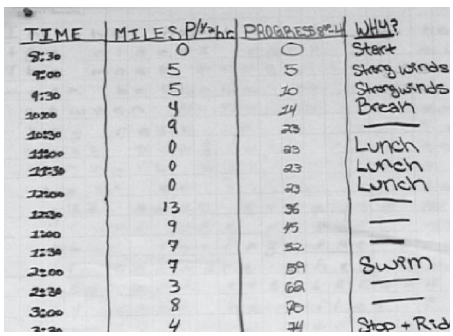
Prompt: Make a table of (time, distance) data that reasonably fits the information in Malcom and Sarah's Notes (*Figure 2*)

1. Susan: No this one should be fifteen
2. Erin: Yeah I was about to say that (1) that should be fifteen that way fifteen then point

On day 4, the group traveled from Chincoteague Island to Norfolk, Virginia. Norfolk is a major base for the United States Navy Atlantic Fleet. Malcom and Sarah rode in the van. They forgot to record the distance traveled for each half hour, but they did write some notes about the trip.

- We started at 8:30 A.M. and rode into a strong headwind until our midmorning break.
- About midmorning, the wind shifted to our backs.
- We stopped for lunch at a barbecue stand and rested for about an hour. By this time we had traveled about halfway to Norfolk.
- At around 2:00 P.M. we stopped for a brief swim in the ocean.
- At around 3:30 P.M. we had reached the north end of the Chesapeake Bay Bridge and Tunnel. We stopped for a few minutes to watch the ships passing by. Since bikes are prohibited on the bridge, the riders put their bikes in the van and we drove across the bridge.
- We took about 7 ½ hours to complete today's 80-mile trip.

A. Make a table of (time, distance) data that reasonably fits the information in Malcom and Sarah's notes.
B. Sketch a coordinate graph that shows the same information.



TIME	MILES P/½ hr	PROGRESS 8:30–4	WHY?
8:30	0	0	Start
9:00	5	5	Strong winds
9:30	5	10	Strongwinds
10:00	4	14	Break
10:30	9	23	—
11:00	0	23	Lunch
11:30	0	23	Lunch
12:00	0	23	Lunch
12:30	13	36	—
1:00	9	45	—
1:30	7	52	—
2:00	7	59	Swim
2:30	3	62	—
3:00	8	70	—
3:30	4	74	Stop + Rid

FIGURE 6 The problem for Excerpts 4 and 5 (Lappan et al., 1997, pp. 24–25) and the group's final table. *Note*. that the numbers in the final table differ from the numbers discussed in Excerpts 4 and 5.

3. Marta: (Like you had) a ten
4. Susan: Yeah do like ten and then nine and then seven
5. Marta: Yeah
6. Susan: And then three they get more energy because they stopped for a second. So they can go like um
7. Marta: six
8. Unknown: ten
9. Erin: Ok, now add it up
10. Kristin: Yeah, add all that up
11. Susan: OK
12. Kristin: It doesn't look like
13. Susan: Lets see ((begins writing)) <u>eighty take away five ten fifteen twenty six twenty six plus fifteen ((looks up to right)) is eleven ((looking down)) uh:: four forty one fifty one (1.5) sixty sixty seven sixty eight sixty nine seventy eighty four.</u> ((lifts head looks to Erin)) It's eighty four miles.
14. Erin: So we have to xxx subtract four
15. Susan: [We should take
16. Kristin: [We have to subtract a little bit more then four
17. Susan: [[So make this less
18. Erin: [[Why it when eighty miles
19. Susan: Yeah but we still have another little spot to fill in.

In lines 1 to 8, as well as in the minutes prior to the start of Excerpt 4, the girls collaboratively created their table, the object of activity, through discussing how each part of the notes should be translated into times and distances in their table (e.g., how to translate “midmorning” into a time for the group to take a break). In lines 9 and 10, the girls told Susan to “add it up,” which we interpret to mean check whether the numbers they have put in the “miles” column sum to 80. In line 13 Susan engaged in an extended period of CPS (underlined in Excerpt 4) lasting 27 seconds while her groupmates silently listened and watched her.

Although the discussions in Excerpt 1, 3, and 4 all include CPS, this group discussion differs markedly from the discussions in Excerpts 1 and 3 in one important way. The group discussion in Excerpt 4 (and in the minutes prior to the start of the excerpt) was not just about doing a calculation to solve a routine exercise. The girls were doing a nonalgorithmic task—converting words such as “midmorning” into numerical entries in the table. In this classroom, conceptually oriented discussions including the students and teacher were relatively common, and in the first half of Excerpt 4 the girls were having a conceptual discussion while collaboratively building their table. Thus, although the CPS in line 13 was in service of doing a calculation (i.e., verifying that the numbers in the distance column sum to 80), the group’s discussion as a whole did not have a calculational orientation, as we see in Excerpts 1, 2, and 3.

In the previous section we noted that one *social* purpose of CPS is to allow for groupmates to monitor each other’s activity. Adopting a distributed cognition perspective illuminates a *socio-cognitive* function for CPS: By voicing her steps in line 13 of Excerpt 4, Susan made her computations audible for other members of the group, and it allowed her groupmates to monitor her progress (and accuracy) during this step. Susan’s groupmates intently gazed at her and did not move while she spoke, implying that they were attending to her computations. In other words, Susan’s *computational private speech* in line 13 facilitated the regulation of the group’s activity, and it may have functioned as the *inner speech* of the distributed mind (or, in Hutchin’s, 1993, words, “functional system”) in the group discussion.

To support the contention that Susan’s CPS was like inner speech for the group, we turn back to one more part of Vygotsky’s theory—the grammatical structure of inner speech. Vygotsky (1986) noted that initially children speak to themselves in complete utterances, and then over time their egocentric speech becomes much less coherent to an eavesdropping researcher. Specifically, he noted that as children’s PS becomes inner speech, *sense,* or personal meaning, is emphasized over semantic or grammatical *meaning*, and full sentences are replaced with abbreviated utterances or whispers. Wertsch (1979) built on Vygotsky’s theory and he noted that in social speech new information is emphasized (through tone, volume, etc.) while “known” information is not. Wertsch argued that what is verbalized in PS is precisely what is “given-new” information for the child in the context of the individual problem solving activity.

Returning to the line 13 of Excerpt 4, Susan’s CPS was voicing “given-new” information for the *group* while she did the computation, and her words tracked her progress. In fact we can re-create many of Susan’s computations and the numbers she was adding by carefully interpreting her talk as calculations. Her CPS “five ten fifteen twenty six twenty six plus fifteen is eleven uh::: forty one” corresponds with $5 + 5 + 5 + 11 + 15 = 41$. But Susan did not speak in complete sentences or explain her steps, as we might expect a teacher would do while standing at the board and presenting the same computation to a class (assuming the goal of such a lesson is to teach the algorithm). Of interest, Susan’s CPS is even more abbreviated and not necessarily as algorithmic as the public computational speech used by a teacher and her students in Excerpt 2 (recall, however, that this teacher was not Susan's teacher). Rather than speaking in complete sentences, Susan emphasized sense over meaning. Yet the function of her CPS remains key: Susan shared new information with her groupmates as she did steps of the computation; she was communicating—thinking—together with her group.

In another instance of CPS by Susan about one minute after Excerpt 4, we see how Susan's groupmates actively monitored her computations by attending to her CPS. This excerpt illustrates how CPS can be both private and social. In Excerpt 4 Susan's groupmates did not evaluate her computation until it was finished, but in Excerpt 5 we see evidence of Susan's groupmates actively responding to voiced calculations. This excerpt starts after the group put new numbers into their table and Susan again attempted to compute the sum of the distances.

Excerpt 5
Participants & Setting: Same as Excerpt 4

1. Susan: Five ten fif- ehh this was twenty six
2. Erin: uh-huh
3. Susan: Thirty six thirty nine (3) forty eight ((starts counting on fingers)) forty nine fifty fifty one fifty two fifty three fifty four fifty f(h)ive ((laughs and looks up))
4. Erin: its supposed xxx
5. Susan: (I know) it was fifty six ((laughter))

After line 5 Susan started over and continued adding the numbers aloud a third time. In Excerpt 5 Erin was monitoring Susan's work and offered an affirmation in line 2 and a correction in line 4. Susan's CPS appears to be PS because she was computing a sum and using talk to regulate her activity. The truncated grammatical form of Susan's talk fits with predictions about PS (Wertsch, 1979). Yet Susan's talk also invited and made possible responses from her groupmates; at two points during Susan's CPS Erin contributed to Susan's computation. First, Erin interjected "uh-huh" in line 2 and she verified that the first block of numbers sum to 26 in line 2 even though Susan did not look at her. Then, after Susan began counting on her fingers, she looked to her groupmates and began to laugh (perhaps being self-conscious about using her fingers to add numbers). Like the laugh by a controller in Goodwin (1996), Susan's laugh may have functioned, in part, as a focusing response cry in the middle of her talk. At this point, Erin interjected a correction. It is important to note that, had Susan not been voicing computations, Erin would not have been able to correct her.

The interaction in Excerpt 5 highlights how, when framed using the notion of distributed cognition, we can see how CPS can serve both social and cognitive functions for the group simultaneously. The social functions of CPS in this group were to facilitate joint participation and promote intersubjectivity; the cognitive function was to monitor Susan's—and the group's—computational accuracy.

DISCUSSION

We are not the first to notice that PS is more common in some social settings than in others. Previous research found that children's production of PS increased in the presence of others who could understand them. Contemporary psychology researchers have replicated this finding. Vygotsky's (1986) interpretation of this finding was that the increase in PS in settings where there was a potential to be understood showed evidence that inner speech (verbal thought) originates in social speech.

In principle, our analysis is compatible with the major components of Vygotsky's findings (e.g., we show how CPS is related to a shared semiotic tool). However, our analysis extends neo-Vygotskian theories of PS by providing a preliminary explication of one context-dependent type of PS. The examples of CPS we present here are a compelling counterexample to the notion that there is a unitary developmental trajectory for the production of PS. Rather than treating all PS as part of the same developmental trajectory, we recognize that some types of PS serve specific sociocognitive functions and emerge during group problem-solving situations regardless of the problem difficulty or the participant's level of development. In this regard, we recognize that certain types of PS, such as verbal rehearsal for memorization, serve specific functions and are not dependent on a child's level of development. Adopting such a situated and functional view of PS is important, because it is a shift away from using culturally dependent school-like norms for universal developmental milestones.

We note previously that other researchers have documented PS in the context of students doing calculations alone and during peer mathematical discussions. In mathematics education research, the distinction between procedural and conceptual reasoning (or discourse) is a crucial one, both theoretically and in practice. Researchers have described whole-class discussions with calculational and conceptual orientations, how mathematical tasks can be implemented by teachers and approached by students as procedural or conceptual problems, how students engage in conceptual discussions, and how peer discusssions can support conceptual change. Overall, current policy documents and research in mathematics education reflect a growing and converging agreement that, in the path toward mathematical proficiency, procedures and concepts are both important, are mutually supportive, and should be developed simultaneously.

The excerpts presented here were selected because they included students voicing calculations while solving problems in a small group. Thus, perhaps by definition, the examples are of talk focused on calculations. However, we are not making any claims here about the nature of CPS as it relates to either a calculational or a conceptual orientation in a classroom. We provided excerpts from two classrooms, one where most discussions had a calculational orientation and one where discussions had a documented conceptual orientation (Moschkovich, 2008), in part to show that CPS can occur in both settings. We know from other classroom studies that computations will play a different role in each type of classroom. We also know that the norms for students' small-group discussions will largely be created and maintained during whole-class discussions. Thus, our observations about CPS are not intended to make further claims or arguments regarding the procedural or conceptual nature of these students' discussions overall.

Barron (2003) reported that one child's PS was not helpful for engendering understanding among his or her groupmates during conceptual mathematical discussions. We lack the data to make any generalization based on this observation, but one possible explanation for why PS could confound a conceptual discussion is that conceptual discussions may rely less on shared algorithms than calculations. Recall that in Excerpts 1, 2, and 3, the students and their teacher used a socially shared semiotic tool, the long division algorithm and its verbal description, while voicing computations. That shared tool made Amber and Claudia's CPS intelligible to others in the group and allowed us to reconstruct their calculations from their words. Exploring the relationship between computational and conceptual group discussions and PS/CPS may be a fruitful avenue for future research.

Our observations about the occurrence of CPS raise the question of whether certain group work configurations might promote it or hinder it. Some tasks, such as locating a ship in restricted waters, *require* the rapid coordination of multiple people and artifacts. As we previously noted, the communication between team members is how the functional system of the navigation team "remembers" while fixing the ship's location on a chart (Hutchins, 1993). Most multistep computational tasks such as adding a column of 16 numbers (Excerpts 4 and 5) could be done in sequence or parallel. However, organizing parallel computation distributed across a group of students requires planning the implementation of the algorithm and managing the data flow. For most group mathematics tasks used in schools, the overhead cost of setting up parallel computation is too high to justify, and students do computations sequentially. For instance, in Excerpts 4 and 5, Susan added a column of 16 numbers independently rather than giving each girl in her group four numbers to sum.

An interesting follow-up study would be to investigate whether the use of CPS is different when computational tasks are done in parallel. For example, we could modify the task and ask a group of four students to find the sum of 100 numbers arranged in a table in four columns. It is possible that group members might delegate the task of summing each column of 25 numbers to each group member, and then one person would sum the four results (the layout of the table may even suggest the use of partial sum method). If a group of four students do computations in parallel, we would hypothesize that each person would *not* voice their computations because verbalizing CPS while doing computations in parallel would serve no sociocognitive or regulatory function for the group, and it would probably hinder each group member's ability to find the correct sum independently. In the case where one person is doing the computation for the group, CPS serves the function of focusing the group's activity and allowing others to monitor her work.

CONCLUSION

What does it mean when students voice computations while working in a group on a mathematical task? In this article we considered three possible theoretical lenses for interpreting this type of talk. Psychologically, PS is often interpreted as a developmental stage that children pass through en route to internalizing speech as thought (inner speech). PS serves as a way for children to regulate their activity, and it can be manipulated by changing the task difficulty (Fernyhough & Fradley, 2005). Because PS apparently re-emerges in this group-work context, one hypothesis is that these computational tasks are difficult for these students and require PS for self-regulatory purposes. However, this hypothesis does not shed much light on the examples we have presented here, because tasks like adding a column of numbers were most likely not beyond these children's level of routine mathematical activity. (We acknowledge that this argument is stronger with Excerpts 4 and 5. The task in Excerpt 1 may have been a much more conceptually difficult task for Amber.)

Socially, CPS may signal one is behaving in a socially acceptable way, and it may enable group members to monitor each other's work. Amber's CPS in Excerpt 1 showed Claudia that she was "playing by the rules" of school and not just copying Claudia's answer. CPS may also be a common element of the *speech genre* of group mathematical discussions and other mathematics education researchers have observed a similar phenomenon in studies of group work (e.g., Barron, 2003). The relationship between voicing computations, promoting intersubjectivity, framing the discussion, and controlling the conversational floor (i.e. exerting authority) is an area where further research may add to our understanding of this type of talk.

Finally, if we use a distributed cognition perspective and treat the group of students working together as an activity system, then CPS may, in fact, be the inner speech—the memory and thought—of the functional system of the group doing shared problem solving (Hutchins, 1993). This hypothesis lends itself to an eclectic view of CPS. Not all PS is the same, and CPS may be a type of PS that occurs specifically when working on mathematics tasks during group discussions, just as reading aloud and reciting a list of items to be memorized are also task-dependent forms of PS.

Our observations about CPS are preliminary. We did not set out to study this phenomenon or design an experimental research study to intentionally manipulate students' production of CPS. We have used three theoretical lenses to illuminate this type of talk from different perspectives. We concluded by showing how an activity theory, distributed cognition perspective, shows how this talk serves both *social* and *cognitive* functions for the group. In this case, then, some forms of PS may not be so private after all.

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