CHAPTER 3

BILINGUAL STUDENTS USING TWO LANGUAGES DURING PEER MATHEMATICS DISCUSSIONS

¿Qué Significa? Estudiantes Bilingües Usando Dos Idiomas en sus Discusiones Matemáticas: What Does it Mean?

William Zahner
Judit Moschkovich
Department of Education
University of California, Santa Cruz

INTRODUCTION

On a typical Wednesday morning in Ms. B’s sixth-grade mathematics class in a dual-immersion bilingual school located in California, a group of six students were sitting at their table and working on percentage problems from a worksheet. Claudia, Amber, and Francisco disagreed over their answer to the exercise (see Figure 3.1), and they asked Ms. B for help. Claudia
thought the answer should be 75%, while Amber and Francisco thought it was 33%. Claudia ultimately prevailed in the debate when the teacher verified her answer. Francisco was still not sure how his 'group-mates had obtained 75% from the fraction \( \frac{3}{4} \), and he asked for more help. The group's discussion is presented in Excerpt 3.1.

Excerpt 3.1. (Wednesday class, session 1, 25:20–25:55)

1 Francisco: "Ah—why six over eight—seventy five percent?"

2 Claudia: [(bobbing head slowly)] "Divide it."

3 Amber: "OK watch. Six goes inside divide by eight [\( \frac{3}{4} \)] =

4 Claudia: 

5 Amber: = "And [that]."

6 Claudia: ""[\( \text{Por eso 'era' } \)"

7 Amber: = ""[Uh-huh."

8 Claudia: = "\( \text{Ocho por siete, put a seven here, it's fifty-six.} \)"

(\"eight times seven , put a seven here, it's fifty-six.\")

9 Dennis: "\( \text{Four.} \)"

10 Francisco: = "\( \text{And that's) four.} \)"

11 Claudia: = "\( \text{Four.} \)"

12 Claudia: = "\( \text{Five, it's ten is four.} \)"

13 Francisco: = "\( \text{Oh yeah, seventy five percent.} \)"

14 Claudia: = "\( \text{Y luego [zero]} \)"

15 Dennis: = "\( \text{Zero, then five.} \)"

16 Claudia: = "\( \text{Y luego es seventy-five percent.} \)"

(\"And then it is seventy-five percent.\")

These students attended a dual immersion bilingual school where classes were taught in both Spanish and English, and bilingualism was both encouraged and valued. All of the students in this group were bilingual, although some reported a preference for speaking Spanish and others preferred speaking English both inside and outside of school. Given that all of the students in the group were able to talk about this exercise in both Spanish and English, why did Claudia mix both Spanish and English in her explanations? What social and mathematical functions were served by her use of two languages? The purpose of this chapter is to explore these questions in more depth using examples from bilingual students' mathematical discussions to understand explanations for why bilingual students use multiple languages during mathematical discussions.

We think that a chapter about using multiple languages during mathematical discussions is especially relevant in the current U.S. context, where bi/multilingualism is increasingly becoming the norm. According to the U.S. Census Bureau (2008), a language other than English is spoken in 40% of all California households. In some areas of the state (such as Los Angeles and the San Francisco Bay area), the concentration of bilingual households is significantly higher. At the same time that demographic trends show an increase in societal bilingualism, restrictive language policies in schools have led to mandated English-only schooling in several states, including California (Gándara & Contreras, 2009). In the midst of a social and political shift, few mathematics educators have looked carefully at the function of using multiple languages in mathematical discussions.

In this chapter we follow up on Moschkovich's (2007b) analysis of bilingual students' use of two languages while doing mathematics. In our analysis, we confirm and corroborate Moschkovich's previous findings about one of the functions of using two languages during mathematical discussions. We extend Moschkovich's work by showing how the use of two (or more) languages by bilingual (or multilingual) students provides a set of linguistic resources for managing the social and cognitive demands of a group mathematics discussion. Finally, we consider the possible functions of using two languages during mathematical discussions in light of alternative theories about language in general and bilingualism in particular. In the discussion, we argue that future research relating language(s) to students' mathematical reasoning must carefully align theory, data, methods, and conclusions.

Our analysis is organized in a format that reflects our own path of inquiry as mathematics education researchers working with a research group. We begin with an overall conceptual framework for the analysis of students' mathematical discussions. Next, we provide a brief overview of the data that we used in Excerpt 3.1 and in Excerpt 3.2 (see Appendix B). Then we consider several competing hypotheses to explain students' use of two languages in mathematical discussions. As we consider various hypotheses...
about students' use of two or more languages in mathematical discussions, we use theory, previous research findings, and examples from the two excerpts to consider the merits of each of these hypotheses in turn. Finally, we conclude with a discussion of themes that emerge from this work.

CONCEPTUAL FRAMEWORK

Our perspective on learning mathematics and learning languages draws on assumptions from the sociocultural tradition. Learning is signaled by changing patterns of participation in a community (Newman, Griffin, & Cole, 1989; Rogoff, 2003; Vygotsky, 1978), and we specifically assume that learning mathematics is mediated by the use of multiple semiotic tools such as spoken and written words, specialized mathematical symbols, graphs, gestures, and mathematical discourse practices (Forman, 1996; Lemke, 2005; Moschkovich, 2007c; Radford, 2001, 2003). In mathematics classrooms, learning mathematics is inextricably bound to participating in mathematical discourse practices that are related to both the discipline of mathematics as well as the unique forms of mathematics found in schools (Forman, 1996; Moschkovich, 2002, 2004). In our analysis of language, we draw on theories and methods of sociolinguistics, focusing on the social uses of language rather than grammatical structures and prescriptive rules for usage (Drew & Heritage, 1992; Sacks, Schegloff, & Jefferson, 1974; Zentella, 1997).

Our approach to research is also grounded in K. D. Gutiérrez and Rogoff's (2003) recommendations for describing learners and communities. We treat culture as a set of practices, as something that people do, rather than as a static attribute of individuals or groups of people. Following K. D. Gutiérrez and Rogoff, we describe children's bilingualism in terms of how they participated in bilingual discussions rather than treating bilingualism as an individual attribute, as something they have acquired, or a school program category. K. D. Gutiérrez and Rogoff also emphasized that individuals participate in "constellations" of multiple cultural practices and that people's practices are constantly changing, so we use the past tense to describe what these students did in this setting at the time of data collection, rather than writing as if our observations generalize to all bilingual students across time and settings. Despite our local orientation, we believe that insights from our analysis will be useful for mathematics education research and teacher education in other settings.

Defining language is beyond the scope of this chapter, but we use our understanding of Spanish and English as speakers of these two languages to attend to when an utterance is all in Spanish (e.g., Excerpt 3.2, line 21: "No porque Claudia quatro por seis es veinte cuatro"), all in English (e.g., Excerpt 3.1, line 1: "Ah—why six over eight, seventy five percent?"), or a mixture of both (e.g., Excerpt 3.1, line 6: "Goes in the oserita, you take out eight, you put a decimal, porque no se puede put a diecim—put a zero"). Previous researchers have used a wide variety of terms to describe the practice of using two or more languages during one unit of discourse, and linguists make a number of subtle distinctions in regard to this practice, including whether switches involve single words, using loan words, and whether switches occur at the end of an utterance and at the end of a unit of discourse (Moschkovich, 2007b; Stache, 1994). These disparate practices have been called, among many terms, code mixing, borrowing, or code switching. While we acknowledge that many of these distinctions are important for linguists, for mathematics education, we think the most relevant issue is how the use of two languages facilitates (or not) students' participation in mathematical discourse practices. In this chapter, we follow Moschkovich (2007b) and refer to the use of two languages during a continuous mathematical discussion as code switching.

DATA, SETTING, AND METHODS

The student discussions we use in this chapter were recorded during classroom observations by the primary author. These observations were part of a study of sixth-graders' peer interactions during mathematics group work. The overarching research questions for the project were the following:

1. In what mathematical discourse practices did students engage during peer mathematics discussions?
2. What were the rules of these discussions?
3. How did intellectual authority mediate these discussions?

In this chapter, we focus on part of the first and second questions.

Setting and Participants

The participating students were sixth graders (age 11–12) in a K–8 dual-immersion bilingual school in a rural town in central California. Over 90% of the students in this school identified as Latino/a, and all students were bilingual in Spanish and English. The sixth-grade students in this school were in two math classes, one taught in Spanish and the other class taught in English. The two sixth-grade teachers divided the sixth-grade mathematics curriculum; Ms. B taught her sections in English while the other sixth-grade teacher taught his sections in Spanish. All of the sixth graders had both teachers for mathematics, so they studied half of the sixth grade mathemat-
ics curriculum in English and the other half of the curriculum in Spanish. Both teachers used the same English-language mathematics text although, in earlier grades, students used Spanish-language mathematics texts. All observations for this study took place in the English-language mathematics class. Although Ms. B lectured and conducted whole-class discussions in English, she was bilingual.

**Data and Methods**

In collaboration with Ms. B, we selected one focal group of students from the classroom for this study. Ms. B’s class had a total of 28 students sitting in four tables made of seven desks each. Ms. B. selected students to sit at the focal group (near our video camera) by choosing typical students who she thought would be likely to engage in lively discussions. We videotaped all interactions of the focal group during the students’ mathematics class for one week. Adopting a naturalistic (Moschkovich & Brenner, 2000) and ethnomethodological (Garfinkel, 1967; Heritage, 1984; Mehan, 1979) approach, we chose to record the students’ peer mathematics discussions in the class; we allowed the teacher to structure the groups and decide on the mathematical activities on which the class would work. Between four and seven students sat at the table at which the focal group worked. All seven students were bilingual, and, in a language preference survey, four of them reported speaking primarily Spanish outside of school while two reported speaking primarily English outside of school (the seventh student did not complete the survey).

We videotaped a total of five hours of the focal students’ in-class interactions, and we selected excerpts from the data when the students engaged in sustained mathematical discussions for analysis. We used Pini and Schwarzenberger’s (1988) definition of a mathematical discussion: “purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction” (p. 461). Based on our selection criteria, we transcribed a total of 56 minutes of the students’ interactions using a modified version of Jefferson’s transcription conventions (Schiffrin, 1994). Our analysis included two phases. During the first phase, we focused on the students’ mathematical discourse practices and we attended to the content frame (Barnes & Todd, 1995) of the students’ discussions. We analyzed how the students used the peer discussion to accomplish their mathematical goals. During the second phase of the analysis, we used methods from conversation analysis (Drew & Heritage, 1992; Schiffrin, 1994) to understand the implicit rules of these students’ peer discussions, specifically attending to the rules for turn taking (Sacks et al., 1974), the function of questions as a signal of authority (Lindt, 1988), and details such as overlapping speech and ignored propositions (Barron, 2003). This chapter represents a third phase of this project in which we make connections between the interactional norms, the mathematical reasoning, and the two languages used in these discussions.

**FINDINGS AND NEW QUESTIONS RAISED BY THE STUDY**

There are two overarching findings from this study. First, these students primarily engaged in “calculational” discussions (Thompson, Philipp, Thompson, & Boyd, 1994, p. 79). Although their discussions involved hybrid discourse practices (K. D. Güírez, Baquedano-López, & Tejeda, 1999), including mixing everyday, school, and academic mathematical discourse practices, their discussions were principally oriented towards calculations rather than concepts. Second, intellectual (or mathematical) authority mediated the conversational rules of the students’ peer discussions. However, intellectual authority did not necessarily align with who students thought was best at mathematics in the group (according to their responses to a survey question) or who had the correct answer to the exercise under discussion.

We observed that during their peer mathematics discussions, these students frequently alternated between using Spanish and English (which may be thought of as a hybrid discourse practice). The students’ use of two languages, while not surprising, led to questions about how the two languages functioned in these peer discussions. In our initial analysis, we did not focus on this practice because numerous writers have already addressed this issue (see Moschkovich, 2007b, for more details). However, one of the fundamental assumptions of conversation analysis (and ethnomethodology in general) is that the microlevel details of interaction can reveal information about people’s interpretations of the social worlds they inhabit and discursively create. Therefore, no details of interaction can be dismissed a priori (Schiffrin, 1994). This assumption, together with questions from numerous audiences of our presentations and readers of our manuscripts, has led us to this analysis focusing on the students’ use of Spanish and English in more detail.

**OVERVIEW OF EXCERPTS 3.1 AND 3.2**

To ground our discussion of code switching in examples of students’ mathematical discussions, we start with a summary of two excerpts from our study of students’ mathematical discussions in peer groups. Here we summarize the mathematical ideas that the students were discussing and provide our summary of the students’ interaction patterns in both excerpts. We briefly introduced Excerpt 3.1 at the start of this chapter. A second, more
the end of the excerpt, she appeared to agree with Claudia and disagree with Francisco. For example, in lines 16, 18, and 21 Amber either disagreed with Claudia’s answer or with Claudia’s agreement with Francisco (it is not precisely clear):

16   Amber: “No, you could go.”
17   Francisco: “It has to be [two point].”
18   Amber: “[It’s six times four is forty, + is twenty-four.”
19   Francisco: “It has to be like two point something. It can’t be twenty-four, Claudia.” (watching as Claudia approaches table.)
20   Claudia: “Yes.”
21   Amber: “[No: porque Claudia quatro por seis es veinticuatro.”
           [“No, because Claudia, four times six is twenty-four.”]

The confusing part of line 21 is that Amber named Claudia as her addressee, but her initial “no” was apparently a response to Francisco’s suggestion that the answer was “two point something.” Amber may have been suggesting an alternative way to get the solution without using long division: “Quatro por seis es veinticuatro” represents one possible way to reach the answer to this question

\[
\frac{6}{25} \frac{4}{4} = \frac{24}{100} = 24%.
\]

Despite this point of ambiguity, Amber appeared to align with Claudia for the majority of this discussion and she ended her contribution by helping Claudia complete the computations in the long division method of finding the solution.

These students used the long division algorithm to solve the percent problems. Given the students’ focus on using the algorithm correctly (rather than on explaining why this was the best solution method), we describe these discussions as reflecting a calculational orientation.

COMPETING HYPOTHESES ABOUT CODE SWITCHING

In this section, we consider five hypotheses about students’ use of two languages during mathematical discussions. Some of these hypotheses relate to code switching in general, while others are specific to code switching during a mathematics discussion. In response to each hypothesis, we consider relevant theory and use examples from excerpts 3.1 and 3.2 to analyze how code switching between Spanish and English functioned.
Hypothesis 1: Students use two languages due to a linguistic or cognitive deficit; children use their first language for words they do not remember.

This hypothesis about the reason for code switching has the advantage of being intuitive. Those of us who have tried to learn a new language can imagine forgetting a word. Unfortunately, this hypothesis has the liability of being wrong, at least in this bilingual setting. Moschkovich (2007b) noted that the "missing word" (p. 139) explanation for code switching reflects a deficit view of children's linguistic or mathematical proficiency. To avoid making deficit-based assumptions about the students' knowledge of mathematical vocabulary, a more nuanced interpretation of this use of language starts with the observation that code switching is different in bilingual communities than in settings in which people are learning a second language. With that caveat in mind, we think it is also important to recognize that in bilingual communities in which schooling is conducted exclusively in a language different from the language used outside of school (a common arrangement for students in former colonies and some communities in the United States), then children are likely to develop technical, academic vocabulary in the language of instruction, but not in their home language. For example, antibilingual education initiatives have resulted in many California schools systematically denying access to content instruction in students' home language (Gándara & Contreras, 2009). In such contexts, the missing word interpretation of code switching may be more valid.

However, the data in this study is from a dual-immersion bilingual school in California, where the children learned mathematics in both Spanish and English. Given this unique setting, we did not see the children systematically using English or Spanish for technical mathematical language. For example, in excerpts 3.1 and 3.2, the students used both the Spanish and English words for the mathematical concepts and operations they were discussing: percent/porcentaje, divided/dividido, times/multiplica, decimal/punto decimal. One instance of a word that was only said in Spanish is "primero," which was part of Joaquin's suggestion in lines 29 and 32 of Excerpt 3.2 of when to add a decimal in long division. Joaquin's use of the diminutive form may indicate he was intentionally playing with words or descending to Francisco (which is consistent with Joaquin and Francisco's other interactions). We think it is likely that Joaquin knew how to express this idea in English as well as Spanish although, in this excerpt, we do not have direct evidence of Joaquin using technical vocabulary in both languages. We are more interested in investigating his language choice and the effect that it had, rather than speculating about Joaquin's English skills.

We end with two final notes about the missing word hypothesis to explain code switching. First, assuming a bilingual person is completely balanced in their two languages is unrealistic; a person who participates in multiple language communities will not develop the same fluency in both languages (Grosjean, 2001; MacSwan, 2000; Moschkovich, 2007b; Zentella, 1997). Languages are learned and used in specific situations and often for specific functions. The second major point about the missing word hypothesis is that even when children do use their first language to fill in vocabulary they do not know in the language of instruction, there is no indication that doing so is necessarily detrimental for learning mathematics. Rowland's (2000) conclusions about the utility of using pronouns for making mathematical generalizations seem relevant here. In his study of language and mathematics (in monolingual English settings), Rowland noted that students, as well as expert mathematicians, used hedges, estimators, and ambiguous words like "it" or "you" to talk about mathematical objects they could not yet define precisely. Rowland argued that using vague language facilitated, rather than hindered, making mathematical generalizations (he also noted that the context matters—using vague language serves an important role during the learning process, but vague language is not generally allowed in completed, formal mathematical generalizations). In terms of using two languages, as long as students also have opportunities to use and learn the meaning of words in both languages, then we argue that using a word from Spanish or English in the middle of mathematical discussions could even facilitate learning the corresponding word in the other language. If bilingualism is the goal (rather than English acquisition), then using primary language vocabulary may be an effective way to facilitate vocabulary development in a new language, assuming that children have the opportunities to learn and use both languages.

Hypothesis 2: Children use their primary language to do computations.

That people tend to recall bits of information such as addition and multiplication facts in the language in which they memorized those facts appears to be true. Moschkovich (2007b) reviewed several studies of computation time in bilingual students and adults (Marsh & Maki, 1976; McClain & Huang, 1982; Secada, 1991; Tamamaki, 1998). The major points made in those studies were (1) bilingual participants appeared to recall arithmetic facts in the language in which they had learned those facts; and (2) despite the very small time lag in carrying out arithmetic computations, bilingualism had no significant effects on computational accuracy. Although this phenomenon is interesting, doing computations in one language or another is not likely to have profound effects on the ability of bilingual students to reason and communicate mathematically. Mathematical communication involves far more than doing computations (Pimm, 1987), and this hypothesis has little to say about other practices associated with math-
ematical reasoning such as conjecturing, proving assertions, and imagining (Moschkovich, 2007a).

Some of the students in these examples expressed computations in both Spanish and English, while others appeared to rely on one language for computation. Amber and Claudia provide us with contrasting cases of language use in their computational private speech (Zahner & Moschkovich, 2010). In both excerpts 3.1 and 3.2, the numbers in Amber's computational private speech were all uttered in English (line 3 of Excerpt 3.1 and lines 8, 40, 44, 53, and 59 of Excerpt 3.2). However, Amber used Spanish for numbers when she was explicitly communicating with other group members (e.g., in line 21: “no porque Claudia quarto por seis es veinticinco”). Claudia, on the other hand, appeared to carry out and voice computations in both languages. In lines 50, 59, and 42 of Excerpt 3.2, she set up the long division 256 while speaking English. Then, in lines 47, 50, and 54 of Excerpt 3.2, she talked through the steps of that same computation in Spanish. In line 57, she used both the Spanish and English forms of the word for the digit 0 in rapid succession.

These students’ language choice for verbalizing computation and for communication with group-mates seems intimately tied to the communicative function of each utterance. Both Amber and Claudia used number words from both languages in their computations and in the group discussions. Amber vocalized computational private speech in English, but sometimes switched to Spanish when addressing her group-mates. Claudia appeared more fluid in her language choice while voicing computations, sometimes expressing them in Spanish, and at other times expressing the same computation in English. Rather than systematically switching languages for arithmetic recall, the communicative function of each utterance appears far more important in terms of language choice. Generally, when the children wanted to get the speaking floor, they used both languages (or switched between languages) to command attention (we consider this phenomenon further in Hypothesis 4, below).

**Hypothesis 3:** Bilingualism allows for cognitive flexibility and children switch languages when they encounter cognitive roadblocks

Vygotsky’s (1978, 1986) genetic theory of human development gave a primary role to language and culturally shared sign systems as mediators of thinking. Cognitive psychologists (Bialystok, 2001) and, to a lesser extent, cultural psychologists (John-Steiner, 1991) have used Vygotsky’s focus on language as a mediator of thought to investigate the impact of bi/multilingualism on thought. Early research in this area attempted to show the detrimental effects of bilingualism on intelligence although, for many of these researchers, their biases against bilingual immigrants permeated the study (Bialystok, 2001). Since Peal and Lambert (1962) showed a bilingual advantage on nonverbal intelligence tests, more recent research based on cognitive theories has shown mixed results for the hypothesis that bilingualism has significant effects on cognition. In her review of cognitive research of nonlinguistic effects of bilingualism, Bialystok (2001) concluded that, in general, there are no differences between bilingual and monolingual problem solvers although, on some tasks, bilingual children were able to ignore misleading clues more effectively than their monolingual peers. This has led some researchers to hypothesize that bi/multilingualism allows for greater cognitive flexibility.

One question arising from the cognitive flexibility hypothesis is whether the students in our study switched languages when they got stuck on a problem or when they experienced other difficulties. If the students systematically switched from English to Spanish (or vice versa) as moments of confusion, then we might see some indirect evidence for bilingualism allowing for cognitive flexibility. However, we do not see evidence of this practice in our data—in fact, even specifying which language the students spoke is difficult because the children frequently used both Spanish and English in the same utterance. One example is Claudia’s statement in lines 52–57 in Excerpt 3.2: “Y luego tra ser dos son cinco porque veinte y cinco. Tu tove one, and that’s zero. Zero bring down the zero, es cero, cien.” (She appears to be doing the intermediary steps 2 * 25 = 50 and 60 - 50 = 10 for the long division 256.) Claudia’s fluid use of Spanish and English echoes the use of languages in Zentella’s (1997) study among Puerto Rican children in New York. For the children in this group, the norm is speaking both Spanish and English at the same time.

**Hypothesis 4:** Using two languages facilitates interactions, specifically management of the conversational floor and face saving

This hypothesis appears to focus on the social, rather than cognitive, functions of code switching. However, we argue that because cognition is socially distributed (Cole & Engeström, 1993; Hutchins, 1996), making a clear distinction between the social elements and the cognitive elements of a mathematical discussion is a false dichotomy. The social functions of code switching facilitate mathematical reasoning during group discussions. Our analysis focuses on two primary issues: the management of “face” during a mathematical discussion and the issue of turn taking. In both instances, students used code switching as a resource.

Rowland’s (2000) study of vagueness in mathematical discourse noted that mathematical discussions are especially face-threatening events. Drawing on Lakoff’s (1973) framework for politeness and the management of face in conversations (Brown & Levinson, 1987), Rowland argued that the
perception that there are right and wrong answers in mathematics and the potential for publicly being identified as incorrect leads to the frequent use of estimators, hedges, and vague pronouns in mathematical discussions. One linguistic resource these students drew upon for managing face was code switching—which was also one of the functions of code switching identified by Sánchez (1994). For example, in lines 18 and 21 of Excerpt 3.2, Amber appears to contradict first Francisco and then Claudia's apparent agreement with Francisco. In lines 16–18, Amber disagreed with Francisco's proposition that the answer was 2.4: "No, you could go, it's six times four is forty, it is twenty-four." After Claudia apparently agreed with Francisco in line 20, Amber said, "No, porqué Claudia quiera por seis seis veinticuatro." Amber essentially repeated the same message in both Spanish and English.

In Excerpt 3.2, there are at least two more examples of students switching between Spanish and English in potentially face-threatening moments. In line 33, Amber contradicted Claudia's "Six" from line 30 with "no, six no vale." While this utterance is difficult to interpret mathematically, by adding the tag "no vale" in Spanish, Amber was skillfully using her languages to navigate a disagreement with Claudia and Francisco. With the Spanish phrase "no vale," Amber softened the effect of her direct contradiction of Claudia (Sánchez, 1994). Claudia repeated this same pattern in lines 52–57 when she used both languages in her direct contradiction of Francisco's assertion that the answer should be 2.4.

Besides management of face, other relevant conversational issues evident in these discussions were managing turn-taking and controlling the conversational floor (Sacks et al., 1974). Management of the floor is an important issue in mathematical discussions. For example, Barron (2008) studied group discussions among 16 triads of monolingual middle school students in the southeastern United States, and she observed that management of turn-taking was one issue that led to breakdowns in group coordination. While we do not claim that these discussions show perfect coordination, we do see evidence in these excerpts that the students used Spanish and English in their bids to gain and control the conversational floor. Claudia's use of Spanish in Excerpt 3.1 is an example of such a bid. Francisco started the discussion with a question in English (line 1), and Claudia's initial response was also in English, reflecting the bilingual pattern of responding in the language of initiation (Sánchez, 1994). In line 3, Amber attempted to take the floor after a short pause by Claudia, but Claudia did not cede the floor. In lines 4 and 6, Claudia talked in Spanish over Amber to take back control of the conversational floor. We are not claiming that Claudia's use of Spanish was the only reason Amber ceded the floor (recall that prior to Excerpt 3.1, Claudia had corrected Amber on this exercise), but we note how Claudia used Spanish as a resource for gaining control of the floor in this discussion.

Related to managing turn-taking is the issue of getting one's propositions heard and acknowledged by the group during a mathematical discussion (Barron, 2000, 2008). Moschkovich (2007b) identified that code switching is an important resource for politely repeating a proposition: "[C]ode switching has been documented as a resource for elaborating on a point that is repeated, without repeating the initial utterance word for word" (p. 138). In Excerpt 3.2, there are several instances in which students used both Spanish and English to repeat themselves as they attempted to gain the floor. For example, in lines 18 and 21 of Excerpt 3.2, Amber repeated her assertion that "six times four is twenty-four" in both languages. Likewise, in lines 43, 46, 48, and 51, Francisco made his argument that the answer should be 2.4 in Spanish, and then repeated himself in English.

One idea that was not repeated in both languages was Joaquin's assertion that the decimal is inserted in the quotient only the first time a zero is added (see Excerpt 3.2, lines 29 and 32). This idea was not taken up by any of the group members. This is one example of a correct proposition that was not taken up by the group (Barron, 2008), and it raises the question of how Joaquin's language choice was related to the group's lack of uptake of his idea because Joaquin did not repeat his idea in English. However, when interpreting this interaction, we must be cautious because Joaquin also spent much of the time during mathematics group work engaging in behaviors that appeared "off task" and he did not regularly contribute to the group's mathematical discussions. Thus, it is difficult to know whether Francisco's lack of uptake of Joaquin's suggestion was due to Joaquin's language choice, his use of the diminutive form (which may be interpreted as condescending), or a general pattern of interaction between Joaquin and his group-mates.

Gaining and holding the floor during the discussion in Excerpt 3.2 appeared to be difficult. The three main participants in this conversation each appeared to have his or her own idea of how to solve the problem. In this midst of each student's contested bids for the conversational floor, one resource Claudia used to make a bid for the floor was the use of the word "mira"/"mira" ("look" or "watch"). Claudia repeated this word to focus her group-mates' attention on how she solved the problem. For example, in lines 28, 30, and 34 of Excerpt 3.2, Claudia started her computation verbalized in English with "mirem." Later in lines 52–57, Claudia explained the final steps of the division by saying, "Y luego "mira son dos cincuenta por cierto (.)" you take one, and that's zero. Zero, bring down the zero, es cero, cien." Claudia also used this bid in line 4 of Excerpt 3.1. Bilingual speakers from this discourse community reported that this construction (Mira + utterance in English or Spanish) is common in the children's community. The ubiquity of this construction leads us to interpret this practice as one way of making a bid for the floor by using a common bidding device.
In noting that these students used two languages to manage the interactional flow of their mathematical discussions, we are not implying that students in monolingual settings lack such resources. Numerous studies of peer discussions in monolingual settings have shown that monolingual children manage issues such as preserving face and regulating turn taking (Barron, 2000, 2003; Jurow, 2005; Pirie, 1991). Our analysis simply shows that when students are in a bilingual setting, the use of code switching is one additional resource that children can use for these purposes during mathematical discussions.

**Hypothesis 5**: Use of non-dominant language(s) is a way to position self or others vis-à-vis larger social power structures.

Many theorists have studied and written about the intimate relationship between language(s) and power. For example, Fairclough's (2001) critical language studies focused on how mundane details of texts and spoken language (such as using active or passive voice) position readers and authors in specific ways that reinforce social power relations. Gee's (1996) discourse analysis took a broader perspective on language, and he distinguished between the lower-case discourse as a unit of text, and the initial-cap Discourse as socially meaningful ways of using language, other symbolic tools, and artifacts to define identities. Gee argued that Discourses are necessarily related to ideologies and relations of power. At the broadest level, Bourdieu (1991) used economic metaphors to explain how the use of official languages and dialects is a practice that reinforces and reproduces inequitable relations of power. In one salient example, Bourdieu showed how a politician in a rural area of France skillfully used the local dialect in an effort to align himself with the local population when giving a speech, but used the official language in other settings to align with the larger political power structure. While broad-based theories of language and power add a great deal to our understanding of the relationship between using particular languages (or Discourses) and power, can we use these theories to interpret students’ use of two languages in their peer mathematics discussions in the context of our study?

Previous studies in mathematics education have documented how language policies about the use of Spanish and English at the classroom level have an impact on Latino/a students’ participation in school mathematics (R. Gutiérrez, 2002; Gutstein, Lipman, Hernández, & de los Reyes, 1997; Khisty, 1995; Khisty & Vigeo, 1999; Moschkovich, 1999). For example, Rochelle Gutiérrez (2002) studied successful high school mathematics teachers in an urban school with a large number of Latino/a students where the students varied in terms of their use of Spanish and English. She noted that many of the monolingual English-speaking mathematics teachers allowed their students to use both Spanish and English during class, and she related such policies (along with several other factors) to the teachers’ relative success in teaching high-level mathematics to Latino/a students (R. Gutiérrez, 2002). López-Leiva and Khisty (2009) have observed how children and their teacher’s use of Spanish or English in a bilingual after-school program signaled the creation of social positions for participants in mathematical discussions. In López-Leiva and Khisty’s study, using Spanish was generally associated with a lower social positioning.

These mixed findings about the use of Spanish and English in mathematics classrooms point to the complexities of interpreting language practices and policies in the United States, especially in relation to how Spanish and English are used in schools or informal educational settings. Making an ideological claim requires careful documentation of participants’ language practices, as well as a deep understanding of the local context. For example, Sánchez (1994) started her analysis of code switching among Chicano/a students in the U.S. Southwest with an extensive review of the historical and economic relationship between Mexico and the United States. Then, grounding her analysis of everyday dialogues in the broader social and historical setting allowed Sánchez to make ideological observations about the functions of code switching. For example, Sánchez noted how some bilingual Chicano/a students systematically switched to English during disputes in a bid to align with power structures.

In terms of this study, making an ideological interpretation of these children’s use of Spanish and English in the classroom depends on the social and institutional context of the discussions. Given that the school in which excerpts 3.1 and 3.2 were recorded was a dual-immersion bilingual school—a school in which bilingualism was actively valued and encouraged—assigning ideological meaning to the students’ language choices seems dangerous. In California post-Proposition 227, the use of Spanish by students in a classroom might be viewed as resistance against a subtractive school system (Gándara & Contreras, 2009; Valenzuela, 1997). Although it is tempting to make such an interpretation of these students’ language choices during their mathematical discussions, we also need to recall that this school valued and actively promoted bilingualism. Thus, this interpretation may be a step too far for this situation, especially given the type of data and our ethnographic orientation. Making claims about the students’ use of Spanish or English as it relates to ideology would require the incorporation of more ethnographic observational data on language attitudes and practices in the community, at this school, and in this classroom. In future work, we could pursue such questions more explicitly, documenting how historical and institutional factors are related to social norms and microlevel interactions within organizational/institutional settings (Ogawa, Crain, Loomis, & Ball, 2008). Language policies are certainly an important part
of shaping children’s mathematical interactions in schools, and we think investigating the interaction between language policies and students’ mathematical reasoning is a fruitful area for future research.

CONCLUSIONS

In this chapter, we have analyzed excerpts from a mathematical discussion among one group of bilingual sixth-grade students to better understand the functions of their code-switching practices. Our analysis has focused on how the students’ use of multiple languages facilitated their mathematical reasoning, and we have used samples of students’ discussions to evaluate five common hypotheses to explain why bilingual students code switch during mathematical discussions.

Our analysis of one group of students’ code switching during a mathematical discussion in a bilingual middle school classroom gives us comparative power and helps us extend Moschkovich’s (2007b) earlier findings about the function of code switching in children’s mathematical discussions. As Moschkovich proposed, one function of using two languages in a bilingual mathematics discussion is to elaborate on a point or to repeat a proposition. Here we have corroborated this finding and extended this idea by showing that code switching also functioned as a face-saving tool during the students’ discussions. We then related this finding to earlier studies of how students use language to manage face (Rowland, 2000) and bid for the floor during mathematical discussions (Barron, 2000, 2005). We also considered other common hypotheses about the role of code switching during students’ mathematical discussions, but we did not see sufficient evidence to support these hypotheses in our data. Although we did not see evidence for four of the five hypotheses we considered, our conclusions must be interpreted in light of the bilingual school context and the type of data we presented here.

The importance of aligning theory, research questions, methods, data, and conclusions is often the first lesson of social science research methods. It is also a lesson worth repeating, especially when studying something as deceptively simple as language practices. To carefully analyze students’ code-switching during bilingual/multilingual mathematical discussions, we needed to step back from common-sense notions of language or our personal experiences of learning languages to approach students’ interactions and use of language through a systematic scholarly perspective. By adopting systematic methods and relying on scholarly perspectives, we have shown that the missing word/cognitive deficit hypothesis did not explain code switching in these students’ mathematical discussions. Likewise, there was no evidence to support the hypothesis that the students switched languages while doing computations or during moments that required some kind of mental flexibility. What we do see in the data is that students used words and expressions from both languages to describe mathematical operations and concepts. Furthermore, their switches in language did not systematically align with shifts from informal to formal or from everyday to mathematical registers.

Overall, the data do not support the first three hypotheses about code switching that we considered. In terms of the fourth hypothesis, we do not have sufficient ethnographic observations of the students’ language practices in school and out of school that would allow us to make statements about the sociopolitical implications of these students’ use of Spanish. Our data and methods seem best suited to support the fourth hypothesis that code switching functioned as a way for the students to manage the social/interpersonal demands of their mathematical discussions. In the future, we hope to investigate how the skillful management of social interactions facilitates the students’ mathematical reasoning.

ACKNOWLEDGMENT

This manuscript is based upon work supported in part by the National Science Foundation under Grant No. 0424993 to the Center for the Mathematics Education of Latinos/as (CEMELA). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. We would like to thank the participating students and the classroom teacher.

NOTES

1. We think it is important to note that the school’s survival after Proposition 227 depended on a significant amount of community support and political efforts to maintain the school in a policy environment hostile to bilingual education.

2. Transcript conventions are based on Jefferson’s conventions (Schiffrin, 1994). A key to transcription symbols is given in Appendix A. For utterances where several words are in Spanish, an English gloss of the utterance (our translation) appears directly below the uttered words, in [brackets]. Also, some lines are omitted because they were part of side conversations in parallel to the main discussion.

3. We were intrigued by Amber’s suggestion in line 21 of Excerpt 3.2, “Quatro por seis es veinte y cuatro,” because this suggestion represents an alternative solution to the percent problem. However, Amber did not follow up on this idea with her group and it is possible the 4 was from doing 25 + 6 rather than 6 + 25.
APPENDIX A
TRANSCRIPT CONVENTIONS

(.) short pause
(#) pause of # seconds
[] start of overlapping speech mid-utterance
[ ] two utterances starting simultaneously or two consecutive overlapping turns
italic Spanish utterance or part of an utterance
{(text)} Translation of Spanish utterance or part of an utterance
: rising intonation
: stop with falling intonation
: elongated syllable
= latched phrases
(uncertain transcription)
((transcriber comments or relevant student gestures))

APPENDIX B
EXCERPT 3.2

Prompt: Claudia's calculation:

\[
\frac{5}{25} = 20\%
\]

Claudia: \(0.24\)
25)
25)
100
100
0

Figure 3.2 The exercise discussed in Excerpt 3.2 and our reconstruction of Claudia's computation.

Excerpt 3.2. [Wednesday Session 1 31:39 – 33:50]

1 Claudia: “It's twenty-four percent.” (Looks up at Amber)
2 Joaquin: “\(\text{xxex}\) (Looking off camera)
3 Claudia: “It's twenty-four percent ((erases paper)). Amber, it's twenty-four percent.”
4 Dennis: “What?”
5 Amber: “Quê?”
6 ((“What?”))
7 Claudia: “The number eight is twenty-four percent.”
8 Francisco: “Twenty-four!”
9 Amber: “My bad, twenty-five, six, that's three, no, times four is twenty-four, that's \(\text{xxex}\) zero.” (Claudia walks away from the group, the second half of this utterance appears to be Amber thinking aloud))
10 Amber: “That goes in {} two.” ((Still thinking aloud))
11 Francisco: “[”Oh yeah, twenty-four percent.”
12 Amber: “How is it twenty-four?”
13 Francisco: “OK, ‘cause look, OK, xxx when you put the decimal there, then with that, it will be two, ten, that’ll be fifty, then you have to bring down, then you have—wait—what the heck?”
14 Amber: “No, you could go.”
15 Francisco: “It has to be [two point.”
16 Amber: “[”It’s six times four is forty, \(t\) is twenty-four.”
17 Francisco: “It has to be like two point something. It can’t be twenty-four, Claudia.” (Watching as Claudia approaches table))
18 Claudia: “Yes.”
19 Amber: “[”No porque Claudia quatro por seis é veinte e quatro.”
20 ((“No, because Claudia four times six is twenty-four.”))
21 Claudia: “Miren.”
22 ((“Look.”))
23 (?: Claudia.)
24 Francisco: “But you have to put the decimal in the middle of two.”
25 Francisco: “After two, you have to put a [decimal]
26 Claudia: “[”Miren.”
27 ((“Look.”))
28 Joaquin: “Desde de primerito [no]”
29 ((“From the very first little one, right?”))
30 Claudia: “[”Six.”
31 Francisco: “[”xxx (put a) decimal right after two.”
32 Joaquin: “(On the) primera vez?”
33 ((“On the) first new little one.”))
34 Amber: “[”No, six (no vale).”
35 ((“No, six isn’t good.”))
36 Claudia: “[”Divided (but then).”
37 Joaquin: “This is the way”
38 Dennis: “This is the way [vale].”
39 ((“This is the way, OK.”))
40 Amber: “[”Oh, then.”
41 Claudia: “Twenty-five.”
42 Amber: “Point six.”
43 Claudia: “You put a decimal.”
44 Francisco: “That, see it’s like two point four percent, two point four percent.”
45 Amber: “Twenty-five.”
Claudia: "Por qué fui.
(“Because I went.”)

Francisco: "¿Quieres cuatro por ciento.
(“Two point four percent.”)

Claudia: "Dos.
(“Two.”)

Francisco: "Punto cuatro.
(“Point four.”)

Claudia: "No como como vu el seis como vu el seis (entre veinte y cinco.
(“No, how six goes into twenty-five?”)

Claudia: "En t-o the last zero,
when you’re doing that, you put a point. Oh my go.

Francisco: "Y luego ¿qué?
(“And then what?”)

Amber: "Ten.

Claudia: "Son dos son cinco menos por ciento (-) you take one, and [that’s
zero.
(“It’s two, it’s fifty percent.”)

Claudia: "Zero, bring down the zero, es cero [cien.
(“It’s zero, one hundred.”)

Francisco: "But when you bring
down the zero, you put a decimal.

Amber: "No you don’t.

Francisco: "You have to,” (quietly)

Amber: "No, you just add another zero.

Francisco: "Oh my god.

Amber: "And then that is (by) four.

Joaquín: "Yo.
(“looking away from group”)

Francisco: "Point twenty-four percent.

Amber: "Yes.
(“nodds head”)

Claudia: "Yes.

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CHAPTER 4

STUDENT RESISTANCE IN A FIFTH-GRADE MATHEMATICS CLASS

Heather Cavell
Department of Mathematics
The University of Arizona

INTRODUCTION

Mathematical spaces in classrooms in the United States often confine young people to activities that are devoid of connections to their real social worlds. Relationships between students and teachers are developed from adult-centric views that frame learning as an approximation to adult experience (Andrade, 1994; Cahan, Meckling, Sutton-Smith, & White, 1995). A lack of attention to children's lives can create spaces that suppress student learning. Students of minority or low socioeconomic status are especially vulnerable because they are in educational spaces for extended periods of time with little or no connection to their communities (Trueba, 1998). In an effort to maintain engagement, some students create resistant behavioral forms (Giroux, 1992). The purpose of this study was to observe and analyze how one group of fifth-grade students used resistance in a mathematics classroom.
Latinos/as and Mathematics Education
Research on Learning and Teaching in Classrooms and Communities

Edited by
Kip Téllez
University of California at Santa Cruz

Judit N. Moschkovich
University of California at Santa Cruz

Marta Civil
University of Arizona

INFORMATION AGE PUBLISHING, INC.
Charlotte, NC • www.infoagepub.com
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FOREWORD

Luis C. Moll
University of Arizona

This volume is the result of several years of research on mathematics and education produced by the NSF-sponsored Center for the Mathematics Education of Latino Students (CEMELA). From its beginnings, as conceptualized by Marta Civil of The University of Arizona, the goal of CEMELA was to undertake research on this crucial research topic in a variety of social settings and conditions found in different regions of the country, and with a special focus on Latino students, given the great demographic growth of this population. The work thus required not only the collaboration of colleagues from four different universities, and of administrators, teachers, families, and students at many schools, but also broad agreement on a general framework for the work that the group came to depict as a sociocultural approach.

The great challenge offered by any sociocultural approach is that of bringing to life the study of human learning and development. As Alexander Luria (1982) famously suggested, the key is to locate the study of human thinking not in the "recesses of the human brain or in the depths of the spirit" but "...in the external processes of social life, in the social and historical forms of human existence" (p. 25). This volume seeks to meet this challenge, and does so admirably, through a range of studies of mathematics and education situated in a variety of conditions for learning.