Chapter 6

How Language and Graphs Support Conversation in a Bilingual Mathematics Classroom

JUDIT MOSCHKOVICH

In this chapter, I examine a mathematical discussion about two graphs that took place in an eighth grade (13-14 years old) bilingual mathematics classroom. The discussion involved multiple ways of seeing and talking about the scales on the vertical axes. In particular, the teacher and students used multiple meanings for the phrase 'I went by . . .' to describe the scales (e.g. 'I went by ones' or 'I went by twos'). The graphs, verbal descriptions, gestures and the multiple meanings generated during the discussion were all resources for socially constructing interpretations of the graphs. I use a situated perspective on learning and discussing mathematics. I use the term 'situated' to mean 'local, grounded in actual practices and experiences' (Gee, 1999: 40). More specifically, I take a situated view of language and meanings: I assume that in all discussions, whether they occur in one or two languages or among native speakers of a language, the meanings for utterances are situated or grounded in the local situation. I also assume that participants interpret utterances and representations in multiple ways. I view these multiple interpretations as part and parcel of sense making in the local situation, rather than mistakes. Lastly, rather than focusing on obstacles, I focus on describing how utterances and representations provide resources for discussions.

Using this perspective, I address the following questions: What are students' multiple interpretations of graphs? What kinds of resources do bilingual students use to discuss graphs? I use transcript excerpts to illustrate multiple meanings for the phrase 'I went by' and multiple views of the scales on two graphs.
Multiple Interpretations in Bilingual Mathematics Classrooms

Why focus on mathematical discussions that involve multiple interpretations, meanings and ways of talking? Researchers have suggested that an important function of productive classroom discussions is to bring different ways of talking and points of view into contact (Ballenger, 1997; Warren & Rosebery, 1996). Such discussions may be particularly important in mathematics instruction because they contradict pervasive beliefs about mathematical activity, such as the idea that there is no room for interpretation (Borasi, 1990).

A focus on mathematical discussions is particularly important for bilingual classrooms. On the one hand, we could imagine that mathematical discussions would be difficult to create and maintain in bilingual classrooms. After all, are bilingual students not struggling with language? Should we not be concerned that an instructional emphasis on mathematical discussions will make bilingual students look less competent than traditional computational work? The discussion about graphs examined below is a counter example to the imagined difficulties that bilingual students might face in discussing mathematical concepts. This mathematical discussion shows that bilingual students are, in fact, able to participate in mathematical discussions that are conceptual and involve multiple interpretations. The question is not whether bilingual students can engage in these types of discussions but how instruction can support bilingual students in participating in discussions and in learning to communicate about mathematical concepts. Too often, descriptions of bilingual students focus on the obstacles they face in understanding text or utterances in English and these misunderstandings are invariably ascribed to their lack of proficiency in their second language. In contrast, the discussion shown below shows that multiple interpretations, rather than being seen as caused by language difficulties, can be seen as reflecting how a student was grappling with a mathematical concept.

Linguistic and Educational Setting

Carlos and David are students in an eighth-grade bilingual class in an urban area in Massachusetts in the United States. They are both native Spanish speakers who are bilingual. In their school, there is a ‘two-way’ or ‘dual immersion’ bilingual programme for Grades K–6. This means that in Grades K–6 (5–12 years old), students spend half their instructional time in English and half in Spanish. In Grades 7 and 8 (12–14 years old), classes are no longer two-way bilingual. Instead, teachers and students use both languages depending on the setting and participants. Most of the students...
in Carlos and David’s class have been in the programme for several years, many since elementary school. Some of the students are recent immigrants, several students are Spanish dominant and most students are proficient in both Spanish and English.

Carlos and David arrived in the United States from Central America as young children and both have been in the bilingual programme since the early grades in elementary school. They report sometimes speaking Spanish at home, and in the classroom they seem to switch easily and fluidly between monolingual and bilingual modes (Grosjean, 1999). When discussing a mathematics problem together, they will throw in words, phrases or extended talk in Spanish. When talking to the teacher, they tend to use mostly English. Thus, they represent an important and significant segment of the Latino/a population in the United States: those students who would not be labelled as Spanish dominant.

The class was conducted mostly in English, with some discussions and explanations in Spanish. The teacher used Spanish mostly when addressing students who were seen as Spanish dominant. Some students spoke mainly English, some students used both languages and some students spoke mainly in Spanish.

This is a classroom where students expect to make sense of their work, discuss their work with peers and also use the teacher as a resource in their discussions. Students took on some of the responsibility for explaining and understanding solutions. Students engaged in serious and extended discussion of their solutions. The group discussions seemed to be important to the students. Nevertheless, while the students shared responsibility for explaining solutions, they sometimes also tended to rely on the teacher as the authority for evaluating a solution.

In the next section, I examine a discussion between Carlos and David. Although both students are bilingual, I have selected a discussion that transpires in only one language, English, on purpose and for several reasons. First, their discussion reminds us that many conversations in bilingual classrooms take place in only one national language. Second, and perhaps more importantly, their discussion highlights how multiple interpretations are not tied to the use of more than one national language but are connected to the negotiation of mathematical meanings. And lastly, I chose this example because, in the United States, bilingual Latino/a students who are labelled as ‘English dominant’ represent an important and significant segment of the Latino/a population. I will use the discussion between Carlos and David to ground deeper consideration of these broad issues. First, however, I need to present their discussion in some detail in order to make visible what is taking place, what each participant means and where the discussion is going (a more detailed analysis of some aspects of this transcript appears in Moschkovich, 2008).
The transcript comes from a larger set of data collected in Carlos and David's class. Classroom observations and videotaping were conducted during two curriculum units from *Connected Mathematics* (Lappan et al., 1998), 'Variables and Patterns' and 'Moving Straight Ahead'. The discussion involving Carlos and David occurred during the unit 'Moving Straight Ahead'. Data collected included video recordings of whole-class discussions and at one student group for every lesson, as well as videotaped problem-solving sessions in pairs.

Carlos and David's discussion occurred towards the beginning of a classroom period. The teacher usually started the 90-min class with a brief whole-class discussion about a mathematics problem. Students then worked in groups of two to four, discussing the problem at their tables. The teacher moved from small group to small group, asking and answering questions in each group. Towards the end of the class period there were usually reports or presentations by each group as well as whole-class discussions led by the teacher. On the day of this discussion, students expected that each group would at some point be asked to go to the front of the classroom to explain their graphs or charts and would be expected to describe how and why they solved a problem as they had, as well as be prepared to answer questions from other students and the teacher.

Making Sense of Two Graphs

The class had been working on several problems about a five-day bicycle tour. In the story, while some riders rode bicycles, other riders rode in a van and recorded the total distance from the starting point for the van every half-hour. The problem below (Figure 6.1) refers to the second day of the tour.

Carlos and David often worked together in a small group. We join their discussion as they compare their answers to this problem and review the graphs each created independently for homework. Carlos first read his written answer. Then David read his answer. Carlos and David agreed in two ways in their written answers. First, they both wrote that the bikers travelled a total of 45 miles in 5 h. Second, they both wrote that the half-hour time interval in which the bikers made the least progress was the one between 2.0 and 2.5 h. However, they did not write the same answer for which half of the trip they made the most progress and for the half-hour interval during which they made the most progress.

As the discussion began, David and Carlos noticed that they had different answers for this problem. They looked at their graphs and noticed that these graphs looked different (Figures 6.2 and 6.3). When reading the transcript, it is important to continue to look at the graphs to understand
On the second day of their bicycle trip, the group left Atlantic City and rode five hours South to Cape May, New Jersey. This time, Sidney and Sarah rode in the van. From Cape May, they took a ferry across the Delaware Bay to Lewes, Delaware. Sarah recorded the following data about the distance traveled until they reached the ferry.

<table>
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<tr>
<th>Time (hours)</th>
<th>Distance (miles)</th>
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<td>0.5</td>
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<td>5.0</td>
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1. Make a coordinate graph of the (time, distance) data given in the table 2. Sidney wants to write a report describing the day 2 of the tour. Using information from the table and the graph, what would she write about the days travel? Be sure to consider the following questions:
A. How far did the group travel in the day? How much time did it take them?
B. During which interval(s) did the riders make the most progress? The least progress?
C. Did the riders go further in the first half or the second half of the days' ride?
2. By analyzing the table, how can you find the time intervals when the riders made the most progress? The least progress? How can you find these intervals by analyzing the graph?

Figure 6.1 Problem: From Atlantic City to Lewes (Connected Mathematics, Lappan et al., 1998)

what Carlos and David are referring to and to get a sense of the discussion. What aspects of the graph is each participant focusing on? (For transcription conventions, see Note 1.)

**Episode 1: David and Carlos compare their graphs**

22 David: Here's my graph. Did it come out like yours?
23 David's gesture: ((Turning his paper towards Carlos.))
24 Carlos: I don't know.
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25 Carlos's gesture: (He looks at both papers and compares graphs.) ((0.2 sec.))
26 David: I'll do mine. =
27 Carlos: = It's because you did it upwards.
28 Carlos's gesture: ((Sweeping his right hand, with his finger pointing up.))
29 David: Oh, you did. ((0.2 sec.)) ((Teacher approaches the group.))
30 Carlos: Were we supposed to do the graph upwards? Or to--
31 Carlos's gesture: ((Sweeping his hand upwards again and then moving his hand horizontally in the air.)) [
32 David: Or, or or crooked like this? Whatever.
33 Carlos: Or horizontally? ((0.2 sec.))
34 Teacher: Doesn't matter. ((shaking her head))
35 Carlos: 'Cause like, like when we look at our graphs his is going up and mine is going towards the side.
36 Carlos's gesture: ((Moving his right hand upwards and then moving his right hand horizontally in the air.)) ((0.2 sec))
37 Teacher: Did you have the same, did you put both of the same things on the x-axis and on the y-axis?
38 Teacher's gesture: ((Looking through students’ graphs.))
39 Carlos: No, ((0.2 sec)) yes, actually.
40 Teacher: You both put time here. OK, so what's different about these? I don't think it's, I don't think it's the positioning of them. Look at your numbers, the way you placed your numbers.
41 Teacher’s gesture: ((Points at the x-axis on both papers, back and forth. Then turns David’s paper in the same direction as Carlos’s paper.))

In this first episode, Carlos and David began the discussion of their solutions to the homework problem by considering whether their graphs were the same or not. They apparently seemed to share the expectation that their graphs would or should look the same. The teacher then joined their group. Carlos asked her how they were supposed to do the graph. She responded first by asking them to compare their axes (Line 37). The teacher’s participation in the discussion began by addressing the issue of what is the same and what is different about the two graphs. She first considered whether David and Carlos had the same variable assigned to each axis (Line 37) and concluded that they both had put time on the
Figure 6.2 Carlos's graph

Figure 6.3 David's graph

Next, she considered what was different about the two graphs and suggested that it was the way that David and Carlos had 'placed their numbers' (Line 40).

In Episode 1, the teacher set the goal of finding similarities and differences between the two graphs. The students seemed to be focusing on the graph at a global level, describing the overall shape of the curve. In contrast, the teacher defined the goal as sorting out 'sameness' and
‘difference’ at a local level focusing on the axes, not the shape of the curves. She did this through the following questions and statements: ‘Did you put both of the same on the x-axis and the y-axis?’, ‘You both put time here (on the x-axis)’, ‘So what’s different about these?’ She thus shifted the discussion and the students’ focus from the ‘positioning’ of the graphs (whether each graph goes ‘upwards’ or not, as described by the students in Lines 27 and 30) to the way each student had placed numbers on the axes.

In the next episode, the teacher considered the impact of the scale on the shape of the graph, using a global view of the graphs, and then began to construct and share her descriptions of the students’ scales on the y-axis. Carlos began to describe how he labelled the axes of his graph (see Figure 6.1) making tick marks every two segments. In contrast, David had labelled the axes of his graph (see Figure 6.2) with tick marks on every one segment. Again, when reading the transcript, it is important to continue to look at the graphs to understand the referents of utterances. In the next episode, what does each participant mean by the phrase ‘I went by’?

**Episode 2: Describing the shape of the curve**

42 Carlos: Oh, that’s true, ‘cause I went by twos, I went 1, 2 (0.2 sec) and then I put that one (---) he went by one.

43 Carlos’s gesture: ([Begins counting with his right finger following the numbers on his paper.])

44 Teacher: Aha, You skipped one (referring to a segment on the scale on Carlos’s graph). So how does that change how it looks?

45 Carlos: ‘Cause it doesn’t go up as far, it only goes, it’s more steeper. It looks more steeper.

46 Carlos’s gesture: ([Moving his right hand outward. Then moving his right hand straight up.])

47 Teacher: Remem-.- Similar to the difference between this one and (0.1 sec) and this one here. Right?

48 Teacher’s gesture: ([Makes a sign with right thumb and index of her hand to show interval differences on their papers. Then she points to a graph on the blackboard. Next she points to a second graph on blackboard. The first and second graphs have different scales on the x-axis so that the second graph is compressed along the x direction.])

49 David: That one.
50 Carlos: Yeah.
51 Teacher: Here the numbers are closer together so it looks looks steeper. Other than that are they the same graph?

52 Teacher's gesture: ((Makes a sign with thumb and index finger. Then she gestures upwards with her right hand.)) ((0.1 sec.))

53 Carlos: No, also here in the x-axis.
54 Carlos's gesture: ((Carlos point to the x-axis on his paper.))

55 David: (the distance) ((Points to the axis on his paper.))

56 David: I went by twos. =
57 Carlos: = This is the x-axis. Right?
58 ((Carlos points to the axis.))
59 Teacher: This is the y-axis, ((0.1 sec.)) this is the x-axis.
60 Teacher's gesture: ((Sweeps her pencil vertically to represent the y-axis and then horizontally to represent the x-axis))

During Episode 2, Carlos introduced the phrase ‘I went by twos’ (Line 42) to describe his own y-axis scale and the phrase ‘he went by one’ (Line 42) to describe David’s scale. Carlos continued to use these phrases during Episodes 2 and 3 as he described how he had labelled the axes of his own graph. Turning to the graphs, we can see that Carlos had labelled his axes by making a tick mark at every two-grid segment. In contrast, David had labelled the axes of his graph making tick marks on every grid segment.

In Episode 3, below, the teacher and Carlos clarify the meanings for ‘I went by …’ What are the claims each participant is making about the scales?

**Episode 3: Using and clarifying ‘I went by …’**

61 David: I went by twos.
62 Teacher: You went by twos and you went by y- ((0.2 sec.))
63 Carlos: I went by twos. You (didn’t) you went by ones! What are you talking about.

64 Teacher: No, here on the y-axis.

65 Teacher's gesture: ((Points to the axis.))

66 Carlos: Oh, I went by fives.
67 Teacher: You went by fives. ((0.2 sec)) No, actually you didn’t go by fives. You actually went by two
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and a halves because you’d, you did every 2
spaces was five

68 Teacher’s gesture: ((She points to Carlos’s paper while she
explains.)) ((0.3 sec.))

69 Carlos: Then he only went by one.

70 Carlos’s gesture: ((Carlos points to David’s paper.)) ((0.2 sec))

71 Teacher: Every one space was two of his. You’d see,
they’re almost the same. If you look at the next
two-((puts down her notebook and points to
the graphs)) [Wait!

72 Carlos: but I don’t get what you’re saying.

73 Teacher: OK.

74 Carlos: ‘Cause I went by fives. ((David stands up))

During Episode 3, although David and Carlos had labelled their
axes differently, David initially claimed that he also ‘went by twos’
(Line 61). The teacher first accepted this claim and proceeded to describe
Carlos’s scale. Carlos disagreed with David, insisting that while he
‘went by twos’, David ‘went by ones’ (Line 63). At this point Carlos
seemed to notice that they might not all be talking about the same
thing, saying, ‘What are you talking about?’ (Line 63). Carlos then
changed the description of his own scale, saying, ‘Oh, I went by fives’
(Line 66). The teacher first agreed with this claim saying, ‘You went by
fives’ (Line 67), but then, after a short pause, she disagreed, proposing
that Carlos had not gone by fives but, rather, had gone by ‘two and a
halves’ (Line 67). In response, Carlos proposed that, if that were the
case, then David ‘went by one’ (Line 69). The teacher explained that on
David’s scale, each space had a value of two units, ‘Every one space
was two of his’ (Line 71). At this point, Carlos said that he did not
understand the teacher’s explanation and returned to claiming that he
‘went by fives’ (Line 74).

With the series of actions and utterances at the end of Episode 3, the
teacher showed her commitment to focus on a detailed comparison of the
two scales. By putting her own notebook down, looking at the two graphs,
pointing to the axes on each graph, touching the papers and orienting the
two graphs so that they are facing her, she called on the students to focus
their attention on the two y-axis scales. In response to these actions, David
reoriented himself, standing up (Line 74), perhaps so that he could read
both graphs.

During Episode 4, the teacher responds to Carlos’s claim that he ‘went
by fives.’ What is the teacher’s role in the discussion?
Episode 4: Teacher responds to Carlos’s claim that he ‘went by fives’

75 Teacher: OK, your numbers, right, the numbers you have are by five ((0.2 sec)) OK ((0.1 sec)) If you look at one line here, what number is he at?

76 Teacher’s gesture: ((Takes David’s paper and places it next to Carlos’s paper. Then points to David’s graph.))

77 Carlos: Two.
78 Teacher: What number would you be at if you had a number here?

79 Teacher’s gesture: ((Points to Carlos’s graph.))
80 Carlos: Three.
81 Teacher: Almost, two and a half.
82 Carlos: Yeah.
83 Teacher: Because that’d be half way to five. OK. ((0.1 sec)) At this point, after 1, 2, 3, he’s got 6. For you after three, 1, 2, 3, you’d be at 7 and a half. ((She counts the squares with her pencil))
84 Teacher’s gesture: ((Points to the graph.))
85 Carlos: O.K.

86 Teacher: See what I mean? So it’s actually two and a half. The numbers you wrote are by fives but since you skipped a line in between, each one is two and a half.

87 Teacher’s gesture: ((Raises her hand and in the air uses thumb and index to show interval.))

Multiple Ways of Talking

There are at least two perspectives of the students’ graphs evident in their discussion. The statement ‘I went by twos’ can be interpreted as describing the action taken to construct the scale, so that ‘I went by twos’ means ‘I went by two segments’ (see Figure 6.4).

It could also be interpreted as describing the quantity represented by the chunk created between two tick marks, as in ‘I went by two units’. For David’s scale this means ‘I made tick marks at every two units’ (see Figure 6.5).

In both of these interpretations, there is an actor, ‘I’, who is constructing the graph. Indeed, Carlos and David repeatedly refer to their scales as ‘I went by’. The teacher also initially included an active subject, referring to the scales as ‘you went by’ (Lines 62 and 67). She then moved to referring to the action as ‘you did every 2 spaces was 5’ (Line 67). She then moved from referring to the action that constructed the graph to focusing on the quantitative relationship between the spaces on each of the two
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Figure 6.4 'I went by twos' describing number of segments

Figure 6.5 'I went by twos' describing number of units

graphs and making a comparison between the two graphs as in 'every one space was two of his (Line 71)'. And lastly, she described the quantitative relationship between the numbers on the scale within one graph, as in 'the numbers you have are by five' (Line 75). Her descriptions were thus moving away from a personal action perspective to an impersonal nominalization based on the numbers and quantitative relationships.

In Lines 75 and 78, the teacher resumed taking a personal action view, saying 'If you look at one line here, what number is he at?' and 'What number would you be at if you had a number here?'. But these were not personal views of the actions taken to construct the graphs. Instead, they were statements about where one would be on the graph if one were moving on the graph at the present moment, some time after constructing the graph.

The teacher alternated between referring to Carlos's actions and to the quantitative meaning of the marks on the scales. She referred to the actions Carlos would take, saying 'you have', 'is he at', 'would you be at', 'he's got', 'you wrote' and 'you skipped' (Lines 75, 78, 83 and 86). She referred to the quantities saying, 'that'd be half way to five', 'it's actually two and a half' and 'each one is two and a half' (Lines 83 and 86). This
Figure 6.6 Carlos describes his scale as ‘I went by fives’

point of view, combined a view of the actions taken to construct the scale with a quantitative view of the meaning of the marks on the scales.

We can see that there were at least three different ways of describing the marks on the axes by looking closely at particular utterances in Episode 4 (Lines 56, 63, 66, and 67). In Line 56, when David said ‘I went by twos’, he could have been referring either to the value of the interval between labelled tick marks on his graph or to the number of units in that interval (see Figures 6.3 and 6.5). Instead, when Carlos said ‘I went by twos’ (Line 63) he seemed to be referring to the number of segments between labelled tick marks along the y-axis (see Figures 6.2 and 6.4).

When Carlos said ‘I went by fives’ (Line 66, see Figure 6.6), he was using a different meaning of the phrase ‘I went by’, this time referring to the value of the interval between labelled tick marks. Using yet another meaning, when Carlos said that David ‘went by one’ (Line 69, see Figure 6.7), he seemed to be describing how David had labelled his scale so that tick marks

Figure 6.7 Carlos describes David’s scale as ‘He went by one’
to construct the scale $s$ on the scales.

A way of describing the
ences in Episode 4
I went by twos', he
interval between units in that interval
went by twos' (Line
nts between labelled
are 6.6), he was using
referring to the value
et another meaning;
, see Figure 6.7), he
ale so that tick marks
appear every one segment, thus referring to the number of segments between labelled tick marks. In contrast, when the teacher said 'you actually went by two and a halves', she was referring to the number of units in the interval between tick marks on Carlos's graph (Line 67, see Figure 6.8).

Table 6.1 lists three ways of using 'I (or you) went by' to describe these two graphs. One meaning refers to the value of the interval between tick marks, the second to the number of segments between tick marks and the third to the value of each segment between tick marks. During Episodes 2 and 3, Carlos used the first and second meaning, David used the first and third meaning and the teacher used the third meaning.

Carlos, David and the teacher seemed to be using the phrase 'went by' with different meanings. David seemed to use 'I went by 2' in Line 56 to mean that the value of each tick mark on the y-axis increased by 2. In David's graph, tick marks corresponded to segments so 'two' is also the value of each segment. In Carlos's graph, tick marks did not correspond to segments since Carlos had labelled only every other segment with a tick mark. Not only did the two graphs look different, Carlos also seemed to use 'I went by —- ' (Lines 42, 63, 66 and 74) in several ways that are sometimes different from David's or the teacher's. On the one hand, Carlos used the phrase in Line 63 to refer to how many segments there were between tick marks on his graph, in this case 2 squares. On the other hand, in Lines 66 and 72, Carlos seemed to be using the phrase to refer to how the value increased for each tick mark on his graph, in this case by 5. There are several ways to interpret Carlos's utterance in Line 69 'Then he only went by ones'. One is that Carlos was referring to how many segments correspond to a tick mark in David's graph. The other is that Carlos was taking the value between tick marks, 2, and dividing by 2 because that was what the teacher had done for Carlos's graph (dividing 5 by 2 to obtain 2.5).
Table 6.1 Multiple meanings for ‘went by’

<table>
<thead>
<tr>
<th>Uutterance</th>
<th>Focus of attention</th>
<th>Coordinated utterance and focus of attention</th>
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</thead>
<tbody>
<tr>
<td>Carlos: ‘I went by fives’</td>
<td>The value of the interval between labelled tick marks</td>
<td>‘I went by fives’</td>
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<tr>
<td>David: ‘I went by twos’</td>
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<td>David’s graph</td>
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<td>10</td>
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<td>Carlos: ‘I went by twos’</td>
<td>The number of segments between labelled tick marks</td>
<td>‘I went by twos’</td>
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<td>Carlos: ‘He ((David)) went by one’</td>
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<td>‘I went by twos’</td>
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<td>David: ‘I went by twos’</td>
<td>The number of units in the interval between tick marks</td>
<td>‘You went by two and a halves’</td>
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<td>Teacher: ‘You ((David)) went by twos …’</td>
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<td>Teacher: No, actually you ((Carlos)) didn’t go by fives, you actually went by two and a halves, because you did every two spaces as five.</td>
<td>‘He went by one’</td>
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<td>‘I went by twos’</td>
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</tbody>
</table>

During the discussion in Episode 5, the teacher explained to Carlos that each segment in his graph had a value of 2.5. Even after the teacher’s explanation, we can see that Carlos used yet another way of describing the graph. Carlos’s utterance ‘three’ in Line 80 is difficult to interpret. If we assume that Carlos knows how to divide 5 by 2 to obtain the correct answer, then his answer that the point on the graph halfway between 0 and 5 is 3, is perhaps evidence that Carlos does not interpret the segments as corresponding to lengths of equal value.

The Teacher’s Role in the Discussion

The analysis presented above shows that, during their discussion, the students used phrases of the form ‘I went by’, which were ambiguous,
had multiple and shifting meanings and were coordinated with different views of the scales. In this last section, I show how the teacher did not respond to these multiple interpretations and ambiguous meanings as obstacles. Instead, she used these multiple interpretations as resources; she built on the students’ interpretations, highlighted a view of the scale anchored in unitizing and provided students an opportunity to use this view.

In their discussion, the students first took some of the responsibility for understanding their solutions and then invoked the teacher’s authority to resolve a question about their graphs. How did the teacher approach this question? Initially she said, ‘they (the graphs) are the same’. That did not seem to work for the students. Next, she engaged students in making sense of their graphs. To support the mathematics in this sense-making activity, she used comparisons of equal intervals. The teacher did not evaluate student products as ‘right or wrong’. Instead, she engaged the students in a discussion detailing and connecting different interpretations. The responsibility for participating, explaining and understanding was distributed across the students and the teacher. The teacher and the students together participated in a mathematical discussion connecting different interpretations of the scales and multiple meanings of the phrase ‘I went by …’.

How did the teacher participate in this discussion? The teacher evaluated how the graphs were the same or different from at least two different perspectives. She then made sense of the two graphs from her own perspective, described to the students how she saw and interpreted the scale and tick marks on the graphs, and then compared the scales on the two graphs. The discussion revolved around what each segment or tick mark represented for each student.

Although the teacher was in a place of authority, she did not produce another graph or evaluate the graphs or the scales. Rather, she based the discussion on the students’ work. The teacher did not correct an error, contradict a misconception or provide the one right answer. Instead, she treated both graphs as correct and accepted a situation where there might be multiple interpretations and meanings. Although the teacher contested Carlos’s description of his scale ‘No, you actually didn’t go by fives’, she also accepted his interpretation saying ‘OK, your numbers, right, the numbers you have are by five’. The teacher did not explicitly define what ‘went by’ meant. Instead, the discussion revolved around what quantity each segment or tick mark represented to each student. The teacher thus supported multiple ways of describing the scales and multiple meanings.

The teacher also used several mathematical concepts as resources. First, she set the goal of looking at the scales rather than looking at the shape of the curves. Initially, the students focused on the curves as objects, comparing the curves in terms of their overall shape, saying that a curve went
'upwards' (Lines 27 and 30) or 'up' (Line 35), in the case of David's graph, or that a curve was 'crooked' (Line 32) or 'towards the side' (Line 35), in the case of Carlos's graph. The teacher proposed and set a new goal, describing the axes rather than describing each curve. This new goal involves implicit knowledge about scales, mainly that the two scales will have an effect on how the slope of a curve appears on the graph. The teacher thus helped the students shift from a view of the curves as objects to a focus on the scales.

The teacher also used the concept of unitizing as a resource. Her descriptions not only moved away from describing a personal action taken to construct the scales towards nominalizations, but they also focused on making comparisons among quantities. In her descriptions, she made a distinction among labels, quantities and measures. The teacher explicitly distinguished between the labels that go 'by fives' and the value of the grid segments as a unit, saying 'You actually went by two and a half' (Line 67). In the second case, the phrase 'you went by' refers to the unit value of one grid segment and is thus an instance of unitizing. The teacher also compared the values of the grid segments on the two scales (Line 71), again an instance of unitizing (Lamon, 1994, 1996).

Lastly, the teacher provided the students an opportunity to use a unitized view of the marks on the scales. She set a new problem, determining the value of the y-coordinate on each graph after moving up one grid segment on the y-axis (Lines 75-82) and after moving up three grid segments on the y-axis (Line 83). As she and Carlos jointly estimated the y-coordinates on the two graphs, she actively engaged him in talking about and viewing the scales from a unitized point of view.

The students were not passive receivers of an explanation, they were active participants in this discussion. How did the students participate in the discussion? Carlos's active engagement is probably easiest to see. He was involved in responding to the teacher's explanations. First, he re-examined his own graph, saying 'I went by twos'. Then he tried to use the teacher's description of David's scale as 'going by twos' but he was not convinced: 'You (didn't) you went by ones!' At one point he even interrupted the discussion, saying 'Wait! But I don't get what you're saying' (Line 72). The teacher took Carlos's interruption seriously and responded with a more detailed explanation and a new approach to explain the differences between the scales. Carlos also persisted until the teacher accepted that his description 'I went by fives' could make sense ('OK, your numbers, right, the numbers you have are by five').

Although David was quieter than Carlos, he was still engaged as seen, for example, when he stood up to look at the two graphs that the teacher had turned. Even though David contributed less talk than Carlos, he continued to look intently at the two graphs during the discussion in Episode 4, so he seemed to still be participating in the discussion.
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Conclusions

In the four episodes, we see two students and a teacher discussing multiple interpretations of a graph and multiple meanings for phrases. During this discussion, the teacher and the two students talked about what quantities and marks on two scales represented. The students and the teacher brought several different ways of talking about the scales on the graphs. The discussion involved different meanings for the tick marks on the axes and different meanings for the phrase 'I went by ....'

The teacher used several instructional strategies to support student participation in a mathematical discussion: she used student-generated products, she used gestures and objects to clarify meaning, she accepted and built on students' responses, and she used a central mathematical concept, unitizing. Although the teacher described how she saw the two graphs and the scales, she did not explicitly address the multiple meanings of 'I went by'. Student contributions were taken seriously, there was time for describing and taking different points of view, and there was room for clarification.

There was no explicit contrast between the students' interpretations and the canonical 'mathematical answer'. Instead the teacher clarified and connected different interpretations of students' products. The teacher did not define a correct interpretation of the scales. Instead, the teacher described how she interpreted the two scales using the concept of unitizing. She described how the scales were the same, how they were different, and she described to the students in detail how she interpreted the scales and tick marks on each of the graphs, using the syntax of the representational system, equal intervals and the concept of unitizing.

This analysis suggests that students' multiple interpretations of the scales and meanings for phrases can be used as resources for a mathematical discussion. The teacher supported this mathematical discussion by, rather than evaluating student work, describing in detail how she understood each student's descriptions. The role of this teacher stands in contrast to more traditional roles for teachers in mathematical discussions (Mehan, 1979; Stodolsky, 1988; Thompson et al., 1994). Discussions that make multiple meanings and interpretations explicit and compare different meanings can provide important opportunities for students to appropriate more mathematical or canonical ways of talking.

Multiple interpretations can serve as resources for instruction in bilingual classrooms. A positive perspective on multiple interpretations is particularly important for bilingual classrooms. This mathematical discussion shows that multiple interpretations need not be seen as obstacles but can be used as resources for explaining and using important mathematical concepts such as unitizing. This positive perspective on multiple interpretations shifts the emphasis from asking what difficulties
bilingual students encounter to how instruction can support students in participating in discussions.

The role of the teacher in supporting this mathematical discussion is also important to consider for instruction in bilingual classrooms. This example, which transpired in only one language, English, can serve as a model for monolingual teachers working with bilingual students. This teacher supported the mathematical discussion using multiple interpretations, building on students' own views of the scales and grounding her explanations in a mathematical concept. These strategies can serve as a model for engaging bilingual students in discussions that simultaneously connect to student interpretations and keep the discussion mathematical.

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Note

1. Transcription conventions: [ ] marks the beginning or end of overlapping utterances; = indicates 'latched' utterances that continue without an intervening pause. Timed pauses (1.8) are measured in seconds. A full stop (period) indicates falling pitch or intonation at the conclusion of an utterance; ? indicates rising vocal pitch or intonation at the conclusion of an utterance; ! marks the conclusion of an utterance delivered with emphatic and animated tone. A comma indicates a continuing intonation with slight upward or downward contour. () indicates talk for which transcriber doubt exists. (()) encloses transcript annotations.
Multilingualism in Mathematics Classrooms
Global Perspectives

Edited by
Richard Barwell

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