Assessing Mathematical Proficiency
MSRI Publications
Volume **53**, 2007

Chapter 20
Beyond Words to Mathematical Content: Assessing English Learners in the Mathematics Classroom

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Introduction

In the U.S., approximately 4.5 million (9.3%) students enrolled in K–12 public schools are labeled English learners [NCES 2002]. In California, during the 2000–2001 school year 1.5 million (25%) K–12 public school students were labeled as having limited English language skills [Tafoya 2002]. For numerous reasons, the instructional needs of this large population warrant serious consideration.

Assessment is particularly important for English learners because there is a history of inadequate assessment of this student population. LaCelle-Peterson and Rivera [1994] write that English learners "historically have suffered from disproportionate assignment to lower curriculum tracks on the basis of inappropriate assessment and as a result, from over referral to special education [Cummins 1984; Durán 1989; Ortiz and Wilkinson 1990; Wilkinson and Ortiz 1986]."

Previous work in assessment has described practices that can improve the accuracy of assessment for this population [LaCelle-Peterson and Rivera 1994]. Assessment activities should match the language of assessment with language of instruction and "include measures of content knowledge assessed through the medium of the language or languages in which the material was taught." Assessments should be flexible in terms of modes (oral and written) and length of time for completing tasks. Assessments should track content learning through

This work was supported in part by a grant from the National Science Foundation and faculty research grants from the University of California at Santa Cruz Senate.

oral reports and other presentations rather than relying only on written or one-time assessments. When students are learning a second language, they are able to display content knowledge more easily by showing and telling, rather than through reading text or choosing from verbal options on a multiple-choice test. Therefore, discussions with a student or observations of hands-on work will provide more accurate assessment data than written assessments. Lastly, evaluation should be clear as to the degree to which "fluency of expression, as distinct from substantive content" is being evaluated. This last recommendation raises an interesting and difficult question for assessing English learners' mathematical proficiency. For classroom assessments that are based on mathematical discussions, how can we evaluate *content* knowledge as distinct from fluency of expression in English? The next section presents two examples of how assessment during classroom discussions can, in fact, focus on mathematical content rather than on fluency of expression in English.

Moving from Assessing Words to Assessing Mathematical Content

Example 1: Describing a pattern. The first example [Moschkovich 2002] is from a classroom of sixth-to-eighth grade students in a summer mathematics course. The students constructed rectangles with the same area but different perimeters and looked for a pattern to relate the dimensions and the perimeters of their rectangles. Below is a problem similar to the one students were working on:

1. Look for all the rectangles with area 36 (length and width are integers). Write down the dimensions.
2. Calculate the perimeter for each rectangle.
3. Describe a pattern relating the perimeter and the dimensions.

In this classroom, there was one bilingual teacher and one monolingual teacher. A group of four students were videotaped as they talked with each other and with the bilingual teacher (mostly in Spanish). As the four students attempted to describe the pattern in their group, they searched for the word for rectangle in Spanish. The students produced several suggestions, including ángulo (*angle*), triángulo (*triangle*), rángulos, and rangulos. Although these students attempted to find a term to refer to the rectangles neither the teacher nor the other students provided the correct word, rectángulo (*rectangle*), in Spanish.

Following the small-group discussion, a teacher — a monolingual English speaker — asked several questions from the front of the class. In response, Alicia, one of the students in this small group, attempted to describe a relationship between the length of the sides of a rectangle and its perimeter.

TEACHER: [Speaking from the front of the class] Somebody describe what they saw as a comparison between what the picture looked like and what the perimeter was? ....

ALICIA: The longer the ah, ... the longer [traces the shape of a long rectangle with her hands several times] the ah, ... the longer the, rángulo,[1] you know the more the perimeter, the higher the perimeter is.

If assessment of this student's mathematical knowledge were to focus on her failed attempt to use the right word, "rectangle," and her lack of fluency in expressing her ideas in English, we would miss the mathematical content and competence in her description. Focusing on the missing vocabulary word would not do justice to how this student successfully communicated a mathematical description. If we were to focus only on Alicia's inaccurate use of the term "rángulo," we might miss how her statement reflects important mathematical ideas. If we move from a focus on words and English fluency, then we can begin to assess this student's mathematical competence. This move is important for assessment because it shifts the focus from a perceived deficiency in the student that needs to be corrected (not using the word "rectangle") to competencies that are already evident and can be refined through instruction. This move also shifts our attention from words to mathematical ideas, as expressed not only through words but also other modes. This shift to considering multiple modes of expression is particularly important to assess the competencies of students who are learning English.

What competencies in mathematical practices did Alicia display? Alicia described a pattern, a paradigmatic practice in mathematics, so much so that mathematics is often defined as "the science of patterns" [Devlin 1998, p. 3]. And Alicia described that pattern correctly. The rectangle with area 36 that has the greatest perimeter is the rectangle with the longest possible length, 36, and shortest possible width, 1 (if the dimensions are integers). As the length gets longer, say in comparing a rectangle of length 12, width 3, and perimeter 30 with a rectangle of perimeter 74, the perimeter does in fact become greater. Although Alicia was not fluent in expressing these ideas, she did appropriately (in the right place, at the right time, and in the right way) describe this pattern.

What language resources did Alicia use to communicate mathematical ideas? She used her first language as a resource for describing a pattern. She interjected an invented Spanish word into her statement. Even though the word that she used for rectangle does not exist in either Spanish or English, it is quite clear from looking at the situation that Alicia was referring to a rectangle. What modes of expression other than language did this student use? Alicia used gestures

[1] Although the word does not exist in Spanish, it might be best translated as "rangle," perhaps a shortening of the word "rectángulo."

to illustrate what she meant and she referred to the concrete objects in front of her, the drawings of rectangles, to clarify her description. It is clear from her gestures that even though she did not use the words "length" or "width," she was referring to the length of the side of a rectangle that was parallel to the floor.

Seeing mathematical competence as more than words shifts assessment from focusing on fluency in English expression to focusing on mathematical practices such as describing patterns, generalizing, and abstracting. Shifting the focus of assessment to mathematical ideas, practices, competencies, and multiple modes of expression has important implications for how assessment informs instruction. Certainly, Alicia needs to learn the word for rectangle, ideally in both English and Spanish, but future instruction should not stop there. Rather than only correcting her use of the word "rángulo" and recommending that she learn vocabulary, future instruction should also build on Alicia's use of gestures, objects, and description of a pattern.

Example 2: Describing parallel lines. The lesson excerpt presented below [Moschkovich 1999] comes from a third-grade bilingual classroom in an urban California school. In this classroom, there are thirty-three students who have been identified as Limited English Proficiency (this was done using a local assessment of English Proficiency) In general, this teacher introduces students to topics in Spanish and then later conducts lessons in English. The students have been working on a unit on two-dimensional geometric figures. For several weeks, instruction has included technical vocabulary such as "radius," "diameter," "congruent," "hypotenuse" and the names of different quadrilaterals in both Spanish and English. Students have been talking about shapes and the teacher has asked them to point, touch and identify different shapes. The teacher identified this lesson as an English as a Second Language mathematics lesson, one where students would be using English in the context of folding and cutting to make tangram pieces:

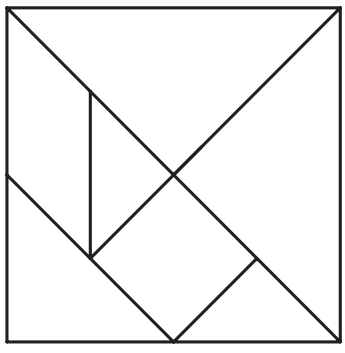

Figure 1. Tangram puzzle

1. TEACHER: Today we are going to have a very special lesson in which you really gonna have to listen. You're going to put on your best, best listening ears because I'm only going to speak in English. Nothing else. Only English. Let's see how much we remembered from Monday. Hold up your rectangles ... high as you can. (Students hold up rectangles) Good, now. Who can describe a rectangle? Eric, can you describe it [a rectangle]? Can you tell me about it?
2. ERIC: A rectangle has ... two ... short sides, and two ... long sides.
3. TEACHER: Two short sides and two long sides. Can somebody tell me something else about this rectangle, if somebody didn't know what it looked like, what, what ... how would you say it.
4. JULIAN: Paralela [holding up a rectangle, voice trails off].
5. TEACHER: It's parallel. Very interesting word. Parallel. Wow! Pretty interesting word, isn't it? Parallel. Can you describe what that is?
6. JULIAN: Never get together. They never get together [runs his finger over the top side of the rectangle].
7. TEACHER: What never gets together?
8. JULIAN: The paralela ... they ... when they go, they go higher [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines], they never get together.
9. ANTONIO: Yeah!
10. TEACHER: Very interesting. The rectangle then has sides that will never meet. Those sides will be parallel. Good work. Excellent work.

Assessing the mathematical content knowledge in Julian's contributions to this discussion is certainly a complex endeavor. Julian's utterances in turns 4, 6, and 8 are difficult both to hear and interpret. He uttered the word "paralela" in a halting manner, sounding unsure of the choice of word or of its pronunciation. His voice trailed off, so it is difficult to tell whether he said "paralelo" or "paralela"; in any case his pronunciation is that of the Spanish word, with the vowel at the end and a well articulated "e". The grammatical structure of the utterance in line 8 is intriguing. The apparently singular "paralela" is preceded by the word "the", which can be either plural or singular, and then followed with a plural "when they go higher." What stands out clearly is that Julian made several attempts to communicate a mathematical idea in his second language.

What competencies in mathematical practices did Julian display? Julian was participating in three central mathematical practices, abstracting, generalizing, and imagining. He was describing an *abstract* property of parallel lines and making a generalization saying that parallel lines will *never* meet. He was also imagining what happens when the parallel sides of a rectangle are extended. What language resources did Julian use to communicate this mathematical idea?

He used colloquial expressions such as "go higher" and "get together" rather than the formal terms "extended" or "meet."[2] What modes of expression other than language did Julian use? He used gestures and objects in his description, running his fingers along the parallel sides of a paper rectangle.

And lastly, how did the teacher respond to Julian's contributions? The teacher seems to move past Julian's confusing uses of the word "parallela" to focus on the mathematical content of Julian's contribution. He did not correct Julian's English, but instead asked questions to probe what the student meant. This response is significant in that it represents a stance towards student contributions and assessment during mathematical discussion: listen to students and try to figure out what they are saying. When teaching English learners, this means moving beyond vocabulary, pronunciation, or grammatical errors to listen for the mathematical content in student contributions. (For a discussion of the tensions between these two, see [Adler 1998].)

Summary

If classroom assessment only focuses on what mathematical words English learners know or don't know, they will always seem deficient because they are, in fact, learning a second language. If teachers perceive English learners as deficient and only assess and correct their vocabulary use, there is little room for addressing these students' mathematical ideas, building on them, and connecting these ideas to the discipline. English learners thus run the risk of being caught in a repeated cycle of remedial instruction that does not focus on mathematical content.

The two examples in this chapter show that English learners can and do participate in discussions where they grapple with important mathematical content, even if they do not always use the right words and even if they do not express themselves in a fluent manner. One of the goals of mathematics assessment for English learners should be to assess students' mathematical ideas, regardless of their proficiency or fluency in expressing their ideas in English.

Teachers can move towards this goal by learning to recognize the multiple mathematical practices, language resources, and non-language modes that students use to express mathematical ideas. Assessments of how English learners communicate mathematically need to consider more than students' use of vocabulary. Assessments should include how students participate in mathematical practices such as making comparisons, describing patterns, abstracting, gen-

[2]It is important to note that the question of whether mathematical ideas are as clear when expressed in colloquial terms as when expressed in more formal language is highly contested and not yet, by any means, settled. For a discussion of this issue, see Tim Rowland's book [2000] *The Pragmatics of Mathematics Education: Vagueness in Mathematical Discourse*.

eralizing, explaining conclusions, specifying claims, and using mathematical representations. Assessments should also consider how students use language resources such as colloquial expressions and their first language to communicate mathematical ideas. Lastly, assessment should include how students use modes of expression other than language to communicate. Classroom assessments that include the use of gestures, concrete objects, the student's first language, and colloquial expression as legitimate resources for communicating mathematical ideas can support students in both displaying competencies and in further learning to communicate mathematically.

The two examples above also show that assessing the mathematical content of student contributions is a complex task. Three questions that are useful for moving beyond words and uncovering students' mathematical competencies are:

- What competencies in mathematical practices (describing patterns, abstracting, generalizing, etc.) do students display?
- What language resources do students use to communicate mathematical ideas?
- What modes of expression other than language do students use?

Assessment is certainly a complex task, perhaps especially when working with students who are learning English. It may not be possible to examine how a student's utterance reflects the student's conceptual understanding and the student's proficiency in expressing their ideas in English. If the goal is to assess students' mathematical content knowledge, it is important to listen past English fluency and hear students' mathematical ideas.

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