

Reading Seminar on the Torelli map

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This semester the goal is to study the Torelli map and Torodial compactifications of A_g . The goal is to understand Alexeev–Brunyate’s proof that the Torelli map extends to the perfect cone compactifications. Here is a rough outline of the seminar.

1 Background on Torodial compactifications

This is a massive, technically demanding topic, so the goal is to just survey some relevant results. In the context of the seminar, we just need to understand [AB12, Thm. 3.2]. The main goals are: to understand definition of the cone decompositions that give rise to torodial compactifications, to have some idea of how the torodial compactifications are constructed, and to understand roughly how [AB12, Thm. 3.2] is proven. The best references are probably Namikawa’s papers [Nam76a, Nam80, Nam76b]. The introduction to [AMRT75, Chapter 1] is also very nice, but the rest of the book is very technical and assumes a strong background in algebraic groups. I have not read [MV12] closely, but they seem to provide the background necessary to understand Alexeev–Brunyate. **Three lectures.**

2 The cone of positive semidefinite quadratic forms

The cone of positive semidefinite quadratic forms should have been defined in previous lectures. The goal here is understand the combinatorics of the cone better. The main goal is to define the central cone, 1st Voronoi, and 2nd Voronoi decompositions. I do not know the best reference for this material. All the main sources (e.g. [Nam76a, AB12]) give the basic definitions, but it would be great to discuss the cones in greater depth. The last section in [AMRT75, Chapter 2] works out the cones for low dimensions, and these examples would make for a nice presentation. Melo–Viviani [MV12] is another possible source. The bibliographies of [MV12] and [AB12] both provide a guide to the general literature on the cone of semidefinite forms. **Four Lectures**

3 The Torelli map extends to the 2nd Voronoi compactification

Work through Namikawa’s proof that the Torelli map extends to a map from the Deligne–Mumford compactification to the 2nd Voronoi compactification. The reference for this is Namikawa’s papers [Nam76a, Nam80, Nam76b]. State (but do not prove) Alexeev’s result that the Torelli map has a moduli-theoretic interpretation [Ale04]. **Four Lectures.**

4 The Torelli map extends to the Perfect Cone compactifications

Understand Alexeev’s proof that that Torelli map extends to the perfect cone but not to the Igusa blow-up. The references are [AB12] and [MV12]. **Four Lectures.**

5 Further Directions?

Learn about the indeterminacy of the Prym map [ABH02, Vol02]. Study the Torelli map associated to the intermediate Jacobian construction [CML09]. Try to relate the variation of GIT describing the universal compactified Jacobians to a variation of fan decompositions of the cone of semidefinite forms.

References

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