1. Executive Summary

To Whom It May Concern:

We are writing to let you know that Burger Tavern 77 is selling itself short in terms of advertising. Your menu advertises that there are "over 200,000 different possible orders." However, after investigating this number, we have found that the number of possibilities actually exceeds six trillion! That means that every one of the 7.3 billion people on earth could order almost one thousand different burger combinations without anyone ordering the same burger twice.

We realize that this number seems outlandish, but students in the course, "The Magic of Numbers," at the South Carolina Honors College, and as huge fans of your restaurant, we have attempted to calculate the actual number of possibilities. To be realistic, we put a limit on how many options within each category a customer can choose. As a group, we decided that each burger must have one and only one meat, bread, side, and cheese (in our calculations we took into consideration that customers may choose not to have cheese). We also set the limit on "tantalizing toppers" to four, "premium toppings" and "feeling saucy" to two, and "prepare your taste buds" to three. Given these restrictions, we first found out how many ways there were to choose from each of the eight categories. Then, we multiplied all of the numbers together and found that there are over six trillion different burger combinations! (The exact number is 6,158,931,600,000).

We hope that you will consider updating your menu to let customers know just how many unique sandwiches Burger Tavern 77 offers.

Sincerely,
Tayler Belinski, Mae Bradford,
and Georgia Higgins

2. Set-up

To set up the problem, we made a chart that included, for each section, the total possible options and how many of each option a customer can order. We decided that everyone must have one and only one meat, bread, side, and cheese (however, in the

total, we included “no cheese” as an option). The menu specifies that customers can order up to four “tantalizing toppers,” and, in an attempt to be realistic, we decided that customers can order up to two items from “premium toppings” and “feeling saucy,” and up to three items from “prepare your taste buds.”

Category	**Total Number of Options**	**Number of Options Per Sandwich**
Meat	5	Must choose only 1
Bread	4	Must choose only 1
Side	7	Must choose only 1
Cheese	10 (Included the option “no cheese”)	Must choose only 1
Tantalizing Toppers	21	Can choose up to 4
Premium Toppings	6	Can choose up to 2
Feeling Saucy?	9 (Omitted the option “no sauce”)	Can choose up to 2
Prepare Your Taste Buds	15	Can choose up to 3

3. Technical Details

To find out how many total options for burgers exist, you must first find out how many options for each category exist. The first four categories are easy to determine because you can only choose one of each. Therefore, we multiply:

5 (meats) × 4 (breads) × 7 (sides) × 10 (cheeses) = **1400** options

For the next category, "tantalizing toppers," there are twenty-one options and a customer can choose *up to* four of them. This means that the customer may also choose three, two, one, or zero. To solve, we set up the expression as a series of binomial coefficients that we added together.

$$\binom{21}{4}+\binom{21}{3}+\binom{21}{2}+\binom{21}{1}+\binom{21}{0}=7547 \text{ Ways to choose } \textit{tantalizing toppers}$$

For the next category, "premium toppings," there are six options and a customer can choose *up to* two. This means that a customer may also choose one or zero. To solve, we set up the expression as a series of binomial coefficients that we added together.

$$\binom{6}{2}+\binom{6}{1}+\binom{6}{0}=22 \text{ Ways to choose } \textit{premium toppings}$$

For the next category, "feeling saucy," there are nine options and a customer can choose *up to* two. This means that a customer may also choose one or zero. To solve, we set up the expression as a series of binomial coefficients that we added together.

$$\binom{9}{2}+\binom{9}{1}+\binom{9}{0}=46 \text{ Ways to choose } \textit{feeling saucy}$$

For the next category, "prepare your taste buds," there are fifteen options and a customer can choose *up to* three. This means that a customer may also choose two, one, or zero. To solve, we set up the expression as a series of binomial coefficients that we added together.

$$\binom{15}{3}+\binom{15}{2}+\binom{15}{1}+\binom{15}{0}=576 \text{ Ways to pick } \textit{prepare your taste buds}$$

To find the total number of options for burgers, given our specified constraints, we then multiplied the number of ways to choose from each category together.

$$1400 \times 7547 \times 22 \times 46 \times 576 = 6.1589316 \times 10^{12}$$

Therefore, according to our calculations, there are realistically **6,158,931,600,000**, or over **six trillion** different burger combinations.