

Midterm Exam Review

Midterm Exam Topics: The midterm exam will cover Sections 9.1-9.7 and Sections 10.1-10.2 of the Active Calculus textbook. Anything covered in the reading or discussed in class is fair game. Here is my (non-exhaustive) summary of the specific things you should know and be able to do from each section of the book. You should also look at the learning objectives from Reading 1-5.

1. Multivariable functions:

- (a) Find the domain.
- (b) Find and graph the traces and level curves (simple examples only, no graphing calculator).
- (c) Use traces and level curves to understand what the graph of a function looks like.
- (d) Read a table of values for a function. Read the traces from a table of values. Read a contour diagram.

2. Vectors:

- (a) Know the geometric and algebraic definition of a vector.
- (b) Compute position and displacement vectors, both algebraically (i.e. using a formula) and geometrically (i.e. drawing a picture).
- (c) Compute the sum, difference, scalar multiple of a vector, both algebraically (i.e. using a formula) and geometrically (i.e. drawing a picture).
- (d) Be able to determine when two vectors are equal.
- (e) Compute the magnitude of a vector. Find a unit vector in a specific direction.

3. Dot product:

- (a) Know both definitions of the dot product and how to compute the dot product.
- (b) Know the relationship between dot product and magnitude.
- (c) Find the angle between two vectors.
- (d) Determine whether the angle between two vectors is acute, obtuse, or a right-angle, using the dot product.
- (e) Compute the projection of one vector onto another.

4. Cross Product:

- (a) Compute the cross product of two vectors using the determinant formula.
- (b) Compute the magnitude of the cross product.
- (c) Determine the direction of the cross product geometrically (orthogonal to both inputs, right-hand rule!)
- (d) Know the geometric definition of the cross product.
- (e) Compute the area of a parallelogram using vectors and the cross product.
- (f) Compute the volume of a parallelepiped using vectors and the cross product.
- (g) Determine when three vectors or points are coplanar (lie in the same plane).
- (h) Determine when two vectors are parallel using the cross product.

5. Lines and planes:

- (a) Find a vector equation of a line. Find parametric equations for a line.
- (b) Determine when two lines are parallel or equal. Determine whether two lines intersect. Find the point of intersection when they do.
- (c) Find a scalar equation of a plane given various geometric information. For instance, a plane containing three specified points or a plane containing a point and a line, or a plane containing a specified point and orthogonal to some line, etc.
- (d) Determine when two planes are equal, parallel, orthogonal. Find the line of intersection of two planes when they intersect, but are not equal.

6. Vector-Valued Functions

- (a) Know what a vector-valued function is and what a parametrization for a vector-valued function is.
- (b) Parametrize traces and level curves for simple examples of functions.

7. Derivatives and Integral of Vector-Valued Functions

- (a) Know the definition of the derivative of a vector-valued function.
- (b) Know the direction of $r'(t)$ relative to the curve traced out by $r(t)$.
- (c) Compute derivatives of vector valued functions using differentiation rules from single-variable calculus.
- (d) Compute the tangent line to a curve at a specified point.
- (e) Know the definition of an antiderivative and an indefinite integral of a vector-valued function.
- (f) Compute antiderivatives and indefinite integrals of vector-valued functions using single-variable calculus.

8. Limits

- (a) Have basic intuition about the limit of a multi-variable function. You do not need to know the precise definition.
- (b) Compute a limit when it exists, using properties of limits and continuity.
- (c) Show that a limit does not exist using "limits along different paths".
- (d) Show that a limit does not exist by reading a contour diagram.
- (e) Determine whether a function is continuous at a point or a specified set. Find points of discontinuity of a function.

9. First-Order Partial Derivatives

- (a) Know the limit definition of a partial derivative.
- (b) Know what a partial derivative measures geometrically.
- (c) Compute partial derivatives using differentiation rules from single-variable calculus.
- (d) Interpret partial derivatives in specific contexts (e.g. units and physical interpretations)
- (e) Estimate partial derivatives using a table of data or the level curves of a function.
- (f) Determine whether a partial derivative is positive, negative, or zero at a point using a contour diagram.

Practice Problems: Here is a sample of the type of problems you might encounter on the exam.

1. Answer the following questions. You must provide justification for your answer.
 - (a) Is the angle between the vectors $\mathbf{u} = \langle 1, 2, 0, -4 \rangle$ and $\mathbf{v} = \langle 0, 2, 1, 2 \rangle$ obtuse?
 - (b) Do the lines $\mathbf{r}(t) = \langle 1 + 2t, -t, 7 - 6t \rangle$ and $\mathbf{s}(t) = \langle 1 + 4t, 1 - 2t, 7 - 12t \rangle$ intersect?
 - (c) Are the planes $2x + y - z = 2$ and $-6x - 3y + 9z = 0$ parallel?
 - (d) Are the vectors $\mathbf{u} = \langle 1, 3, 2 \rangle$, $\mathbf{v} = \langle 1, 0, 1 \rangle$, and $\mathbf{w} = \langle 0, -3, -1 \rangle$ coplanar?
 - (e) Is the line $\mathbf{r}(t) = \langle 1 - t, 2 - t, 3 + 6t \rangle$ contained in the plane $6x + 6y + 2z = 24$?
 - (f) Is the line $\mathbf{r}(t) = \langle 1 + 3t, 1 + 4t, t \rangle$ tangent to the curve $\mathbf{s}(t) = \langle t^2, t^3, \ln t \rangle$ at the point $(1, 1, 0)$?

2. Let $P = (1, 2, -3)$, $Q = (1, 2, 4)$, $R = (1, 0, 7)$, $S = (-1, -1, -2)$. Determine if the points P, Q, R and S are coplanar. That is, do P, Q, R and S lie in the same plane? You must justify your claim.

3. Let p denote the plane defined by the scalar equation $3x + 2y - z = 1$ and let q denote the plane defined by $7x - y - 2z = 4$.
 - (a) Show that the planes p and q intersect.
 - (b) Show that the planes p and q are not parallel.
 - (c) It follows from (a) and (b) that the intersection of the planes is a line. Find a vector-valued function $\mathbf{r}(t)$ that describes this line.

4. Let $f(x, y) = 3x^2 + 2y^2$. Complete each of the following tasks.
 - (a) Write down any nonzero point $P_0 = (x_0, y_0, z_0)$ that lies on the graph of the function $f(x, y)$.
 - (b) Find a vector-valued function $\mathbf{r}(t)$ that describes the $x = x_0$ trace of the function f .
 - (c) Find a vector-valued function $\mathbf{s}(t)$ that describes the $y = y_0$ trace of the function f .
 - (d) Find the direction $\mathbf{v}$ of the line containing P_0 whose direction is tangent to the $x = x_0$ trace at P_0 . Compute the vector form $\mathbf{L}_1(t)$ of this line.
 - (e) Find the direction $\mathbf{w}$ of the line containing P_0 whose direction is tangent to the $y = y_0$ trace at P_0 . Compute the vector form $\mathbf{L}_2(t)$ of this line.
 - (f) The *tangent plane* to the graph of f at P_0 is the plane containing both of the tangent lines $\mathbf{L}_1$ and $\mathbf{L}_2$ from part (d) and (e). Find the scalar equation of this plane.

5. Let $f(x, y) = 3x^2 + 2y^2$ and let $P_0 = (x_0, y_0, z_0)$ be the point you chose in part (a) of the preceding problem. Complete the following tasks.
 - (a) Write down the point $P_0 = (x_0, y_0, z_0)$ that you chose in part (a) of problem 4. Then, compute $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$.
 - (b) Explain the relationship between $f_y(x_0, y_0)$ and $\mathbf{L}_1$.

(c) Explain the relationship between $f_x(x_0, y_0)$ and $\mathbf{L}_2$.

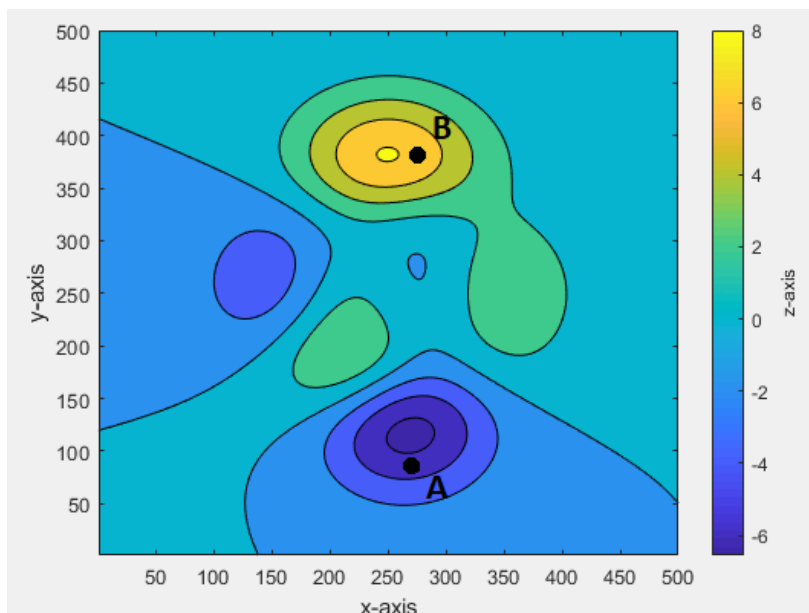
6. (12 points) The table below gives the number $C(W, S)$ of calories burned per hour for a person roller-blading as a function of the person's weight W in pounds and speed S in miles per hour.

$W \backslash S$	8	9	10	11
120	252	348	444	534
140	306	402	498	594
160	366	462	552	648
180	420	516	612	702
200	474	570	666	756

Answer the following questions.

- Estimate the partial derivative $C_W(140, 10)$. Make sure to include the units.
- Describe in words the meaning of the number $C_W(140, 10)$.
- Estimate the partial derivative $C_S(140, 10)$. Make sure to include the units.
- Describe in words the meaning of the number $C_S(140, 10)$.

7. Consider the following contour diagram of a function $f(x, y)$:



I have identified two points A and B in the figure. Complete the following tasks.

- Determine whether f_x is positive, negative, or zero at point A .
- Determine whether f_y is positive, negative, or zero at point A .
- Determine whether f_x is positive, negative, or zero at point B .
- Determine whether f_y is positive, negative, or zero at point B .