Midterm Exam Review

Midterm Exam Topics: The midterm exam will cover Sections 9.1-9.7 and Sections 10.1-10.2 of the Active Calculus textbook. Anything covered in the reading or discussed in class is fair game. Here is my (non-exhaustive) summary of the specific things you should know and be able to do from each section of the book.

1. **Multivariable functions:**
   (a) Find the domain.
   (b) Find and graph the traces and level curves (simple examples only, no graphing calculator).
   (c) Use traces and level curves to understand what the graph of a function looks like.
   (d) Read a table of values for a function. Read the traces from a table of values. Read a contour diagram.

2. **Vectors:**
   (a) Know the geometric and algebraic definition of a vector.
   (b) Compute position and displacement vectors, both algebraically (i.e. using a formula) and geometrically (i.e. drawing a picture).
   (c) Compute the sum, difference, scalar multiple of a vector, both algebraically (i.e. using a formula) and geometrically (i.e. drawing a picture).
   (d) Be able to determine when two vectors are equal.
   (e) Compute the magnitude of a vector. Find a unit vector in a specific direction.

3. **Dot product:**
   (a) Know both definitions of the dot product and how to compute the dot product.
   (b) Know the relationship between dot product and magnitude.
   (c) Find the angle between two vectors.
   (d) Determine whether the angle between two vectors is acute, obtuse, or a right-angle, using the dot product.
   (e) Compute the projection of one vector onto another.

4. **Cross Product:**
   (a) Compute the cross product of two vectors using the determinant formula.
   (b) Compute the magnitude of the cross product.
   (c) Determine the direction of the cross product geometrically (orthogonal to both inputs, right-hand rule!)
   (d) Know the geometric definition of the cross product.
   (e) Compute the area of a parallelogram using vectors and the cross product.
   (f) Compute the volume of a paraellelepiped using vectors and the cross product.
   (g) Determine when three vectors or points are coplanar (lie in the same plane).
   (h) Determine when two vectors are parallel using the cross product.

5. **Lines and planes:**

(a) Find a vector equation of a line. Find parametric equations for a line.

(b) Determine when two lines are parallel or equal. Determine whether two lines intersect. Find the point of intersection when they do.

(c) Find a scalar equation of a plane given various geometric information. For instance, a plane containing three specified points or a plane containing a point and a line, or a plane containing a specified point and orthogonal to some line, etc.

(d) Determine when two planes are equal, parallel, orthogonal. Find the line of intersection of two planes when they intersect, but are not equal.

6. **Vector-Valued Functions**

(a) Know what a vector-valued function is and what a parametriziation for a vector-valued function is.

(b) Parametrize traces and level curves for simple examples of functions.

7. **Derivatives and Integral of Vector-Valued Functions**

(a) Know the definition of the derivative of a vector-valued function.

(b) Know the direction of $r'(t)$ relatives to the curve traced out by $r(t)$.

(c) Compute derivatives of vector valued functions using differentiation rules from single-variable calculus.

(d) Compute the tangent line to a curve at a specified point.

(e) Know the definition of an antiderivative and an indefinite integral of a vector-valued function.

(f) Compute antiderivatives and indefinite integrals of vector-valued functions using singe-variable calculus.

8. **Limits**

(a) Have basic intuition about the limit of a multti-variable function. You do not need to know the precise definition.

(b) Compute a limit when it exists, using properties of limits and continuity.

(c) Show that a limit does not exist using "limits along different paths".

(d) Show that a limit does not exists by reading a contour diagram.

(e) Determine whether a function is continuous at a point or a specified set. Find points of discontinuity of a function.

9. **First-Order Partials**

(a) Know the limit definition of a partial deriviative.

(b) Know what a partial derivative measures geometrically.

(c) Compute partial derivatives using differentation rules from single-variable calculus.

(d) Interpret partial derivatives in specific contexts (e.g. units and physical interpretations)

(e) Estimate partial derivatives using a table of data or the level curves of a function.

(f) Determine whether a partial derivative is positive, negative, or zero at a point using a contour diagram.

Practice Problems: Here is a sample of the type of problems you might encounter on the exam.

1. Answer the following questions. You must provide justification for your answer.

(a) Is the angle between the vectors $\mathbf{u} = \langle 1, 2, 0, -4 \rangle$ and $\mathbf{v} = \langle 0, 2, 1, 2 \rangle$ obtuse?

(b) Do the lines $\mathbf{r}(t) = \langle 1 + 2t, -t, 7 - 6t \rangle$ and $\mathbf{s}(t) = \langle 1 + 4t, 1 - 2t, 7 - 12t \rangle$ intersect?

(c) Are the planes $2x + y - z = 2$ and $-6x - 3y + 9z = 0$ parallel?

(d) Are the vectors $\mathbf{u} = \langle 1, 3, 2 \rangle$, $\mathbf{v} = \langle 1, 0, 1 \rangle$, and $\mathbf{w} = \langle 0, -3, -1 \rangle$ coplanar?

(e) Is the line $\mathbf{r}(t) = \langle 1 - t, 2 - t, 3 + 6t \rangle$ contained in the plane $6x + 6y + 2z = 24$?

(f) Is the line $\mathbf{r}(t) = \langle 1 + 3t, 1 + 4t, t \rangle$ tangent to the curve $\mathbf{s}(t) = \langle t^2, t^3, \ln t \rangle$ at the point $(1, 1, 0)$?

2. Let $P = (1, 2, -3)$, $Q = (1, 2, 4)$, $R = (1, 0, 7)$, $S = (-1, -1, -2)$. Determine if the points P, Q, R and S are coplanar. That is, do P, Q, R and S lie in the same plane? You must justify your claim.

3. Let p denote the plane defined by the scalar equation $3x + 2y - z = 1$ and let q denote the plane defined by $7x - y - 2z = 4$.

(a) Show that the planes p and q intersect.

(b) Show that the planes p and q are not parallel.

(c) It follows from (a) and (b) that the intersection of the planes is a line. Find a vector-valued function $\mathbf{r}(t)$ that describes this line.

4. Let $f(x, y) = 3x^2 + 2y^2$. Complete each of the following tasks.

(a) Write down any nonzero point $P_0 = (x_0, y_0, z_0)$ that lies on the graph of the function $f(x, y)$.

(b) Find a vector-valued function $\mathbf{r}(t)$ that describes the $x = x_0$ trace of the function f.

(c) Find a vector-valued function $\mathbf{s}(t)$ that describes the $y = y_0$ trace of the function f.

(d) Find the direction $\mathbf{v}$ of the line containing P_0 whose direction is tangent to the $x = x_0$ trace at P_0. Compute the vector form $\mathbf{L_1}(t)$ of this line.

(e) Find the direction $\mathbf{w}$ of the line containing P_0 whose direction is tangent to the $y = y_0$ trace at P_0. Compute the vector form $\mathbf{L_2}(t)$ of this line.

(f) The *tangent plane* to the graph of f at P_0 is the plane containing both of the tangent lines $\mathbf{L_1}$ and $\mathbf{L_2}$ from part (d) and (e). Find the scalar equation of this plane.

5. Let $f(x, y) = 3x^2 + 2y^2$ and let $P_0 = (x_0, y_0, z_0)$ be the point you chose in part (a) of the preceding problem. Complete the following tasks.

(a) Write down the point $P_0 = (x_0, y_0, z_0)$ that you chose in part (a) of problem 4. Then, compute $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$.

(b) Explain the relationship between $f_y(x_0, y_0)$ and $\mathbf{L_1}$.

(c) Explain the relationship between $f_x(x_0, y_0)$ and $\mathbf{L_2}$.

6. (12 points) The table below gives the number $C(W, S)$ of calories burned per hour for a person roller-blading as a function of the person's weight W in pounds and speed S in miles per hour.

W\S	8	9	10	11
120	252	348	444	534
140	306	402	498	594
160	366	462	552	648
180	420	516	612	702
200	474	570	666	756

Answer the following questions.

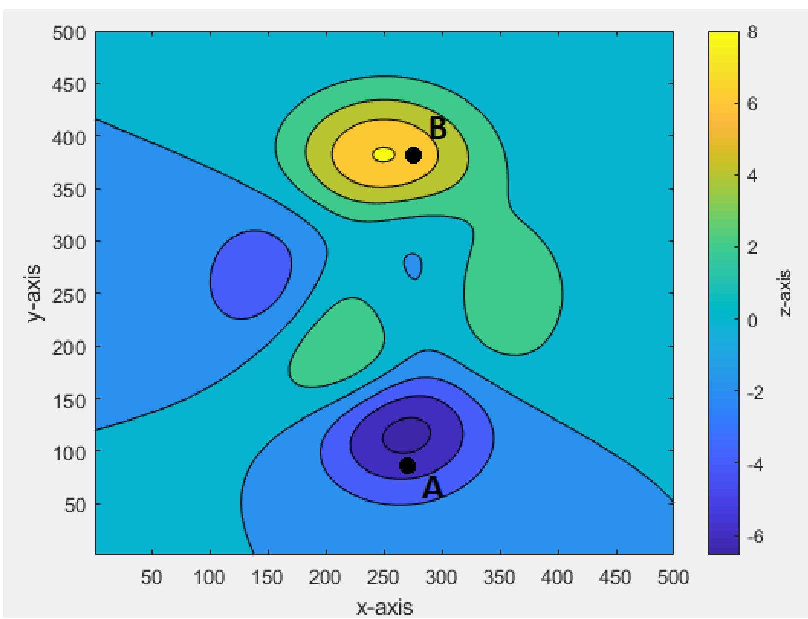
(a) Estimate the partial derivative $C_W(140, 10)$. Make sure to include the units.

(b) Describe in words the meaning of the number $C_W(140, 10)$.

(c) Estimate the partial derivative $C_S(140, 10)$. Make sure to include the units.

(d) Describe in words the meaning of the number $C_S(140, 10)$.

7. Consider the following contour diagram of a function $f(x, y)$:



I have identified two points A and B in the figure. Complete the following tasks.

(a) Determine whether f_x is positive, negative, or zero at point A.

(b) Determine whether f_y is positive, negative, or zero at point A.

(c) Determine whether f_x is positive, negative, or zero at point B.

(d) Determine whether f_y is positive, negative, or zero at point B.