Midterm Exam Cover Page

Overview: This exam consists of 1 cover page and **4 pages of questions**, for a total of 5 pages. There are **8 questions** worth a total of **52 points**. Each question has multiple parts. Your grade on this exam will be calculated as

$$(\textbf{number of points earned})/50.$$

This exam is worth **15% of your final grade** in the course.

Directions:

- **Write your NAME and STUDENT ID at the top of page 2.**
- Solve as many of the following problems as you can. Try to start with problems that you know how to solve right away, and save the others for last.
- Provide justification and show your work whenever possible. This makes it easier to give you partial credit. I want to give you partial credit!
- Write the work and the answer that you want graded in the provided answer box.
- You may use one two-sided 8.5inch by 11inch sheet of notes. No other resources are allowed. You do not need a calculator.

Good luck, and have fun!

Name: | Student ID:

Midterm Exam Problems

1. **(7 points)** Determine whether the following statements are **TRUE** or **FALSE**. No justification is required: if you don't know, just guess! Each question is worth **1 point**.

   (a) Two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are parallel if and only if $|\mathbf{n}_1 \times \mathbf{n}_2| = 0$.

   Answer:

   (b) For all vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{v} \times \mathbf{u}|$.

   Answer:

   (c) If all the level curves of a function $f(x, y)$ are lines, then the graph of $f(x, y)$ is a plane.

   Answer:

   (d) The slope of the tangent line to the $y = b$ trace of a function $f(x, y)$ at the point $(a, b, f(a, b))$ is equal to $\lim_{h\to 0} \frac{f(a,b+h)-f(a,b)}{h}$.

   Answer:

   (e) Any two lines which are perpendicular to the same plane are parallel.

   Answer:

   (f) Any three points $P, Q, R \in \mathbb{R}^3$ are contained in a *unique* plane.

   Answer:

   (g) The cross product of two unit vectors is a unit vector.

   Answer:

2. **(8 points)** Answer the following questions, each of which is worth **2 points**. Briefly justify your answer.

   (a) Are the vectors $\mathbf{u} = \langle 1, 2, 0, -4\rangle$ and $\mathbf{v} = \langle 0, 2, 1, 2\rangle$ perpendicular?

   (b) Does the line $\mathbf{r}(t) = \langle t, t, -2t\rangle$ intersect the plane $x + y + z = 1$?

   (c) Are the planes $x + y + z = 2$ and $-2x - 2y - 2z = 4$ parallel?

   (d) Are the vectors $\mathbf{u} = \langle 1, 0, 3\rangle$, $\mathbf{v} = \langle 1, 1, 1\rangle$, and $\mathbf{w} = \langle 0, 1, 1\rangle$ coplanar?

3. (10 points) Consider the lines $\boldsymbol{\ell}_1(t) = \langle t, t, 1+t\rangle$ and $\boldsymbol{\ell}_2(t) = \langle 2-t, t, 3-t\rangle$. Complete the following tasks.

(a) **(3 points)** Show that $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$ intersect.

(b) **(3 points)** Show that $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$ are not parallel.

(c) **(2 points)** Find a vector that is perpendicular to both $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$.

(d) **(2 points)** Find a scalar equation for the plane that contains both lines $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$.

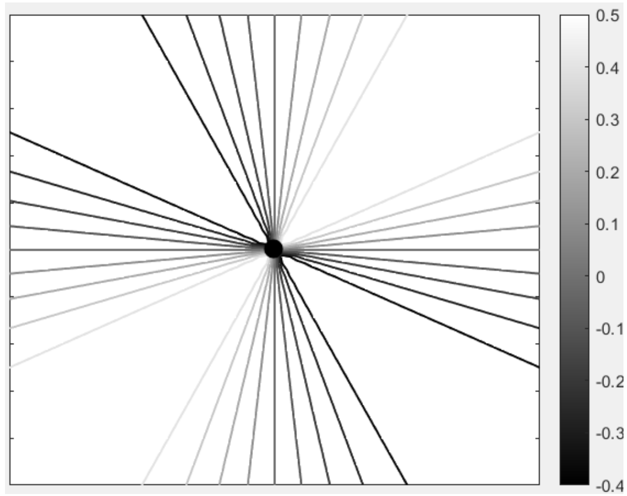
4. (6 points) Consider the curve defined by $\mathbf{r}(t) = \langle \sin(t), \cos(t), t\rangle$.

(a) **(2 points)** Compute $\mathbf{r}(\pi)$.

(b) **(2 points)** Compute $\mathbf{r}'(\pi)$.

(c) **(2 points)** Find a vector-valued function that parametrizes the line that is tangent to the curve $\mathbf{r}(t)$ when $t = \pi$.

5. (4 points) Consider the following contour diagram of a function $f(x, y)$:



The black dot in the center is the point $(0, 0)$. All of the lines in the contour diagram intersect at $(0, 0)$. Compute the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ or explain why it does not exist.

6. (5 points) The table below gives the number $F(d, v)$ of liters (L) of fuel burned by a Ferrari after traveling a distance of d kilometers (km) at an average velocity of v kilometers per hour (km/h).

d \ v	**25**	**50**	**75**	**100**	**125**	**150**
10	25	50	75	100	125	150
20	50	100	150	200	250	300
30	75	150	225	300	375	450
40	100	200	300	400	500	600

For example, $F(20, 125) = 250L$. Answer the following questions.

(a) (3 points) Estimate the partial derivative $F_v(30, 75)$. Make sure to include the units.

(b) (2 point) Explain the meaning of the partial derivative that you computed in part (c).

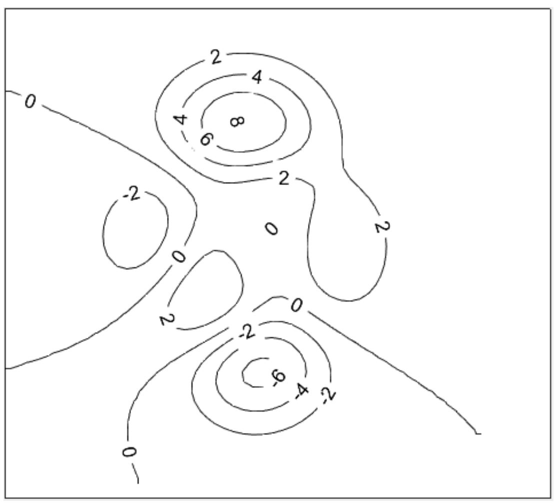
7. (6 points) Consider the function $f(x, y, z) = x^2 \sin(ze^{y+z})$. Complete the following, each of which is worth **2 points**.

(a) Compute $f_x(x, y, z)$.

(b) Compute $f_y(x, y, z)$.

(c) Compute $f_z(x, y, z)$.

8. (6 points) Consider the following contour diagram of a continuously differentiable function $f(x, y)$:



Complete the following, each of which is worth **3 points**.

(a) Find a point B where $f_x(B) > 0$ and $f_y(B) < 0$. Mark your point in the diagram with a dot and the letter B. Explain your reasoning.

(b) Find a point C at which $f_x(C) < 0$ and $f_y(C) = 0$. Mark your point in the diagram with a dot and the letter B. Explain your reasoning.