Final Exam Review

Final Exam Topics: The final exam will be based on Sections 10.3-10.8, Sections 11.1-11.3, and Sections 11.5-11.6 of the Active Calculus textbook. The final exam is not cumulative in the sense that I will not explicitly test you on topics that were already covered on the midterm. However, some prerequisite knowledge from the midterm topics is required. Anything covered in the reading or discussed in class is fair game. Here is my (non-exhaustive) summary of the specific things you should know and be able to do from each section of the book.

1. **Second-Order Partials:**
   (a) Compute second-order (or higher-order) partial derivatives using differentiation rules from single-variable calculus.
   (b) Estimate second-order partials using a table of values or a contour diagram.
   (c) Understand what the second-order partials measure geometrically.
   (d) Determine the sign (positive,negative,zero) of a second-order partial derivative at a point by reading the contour diagram. Identify points where the second-order partials have a certain sign.
   (e) Use traces and level curves to understand what the graph of a function looks like.
   (f) Read a table of values for a function. Read the traces from a table of values. Read a contour diagram.
2. **Linearization:**
   (a) Compute an equation of a tangent plane to the graph of a differentiable function.
   (b) Use the tangent plane as a linear approximation of a function.
   (c) Find a linear approximation of a function using a table of value or a contour diagram. This requires that you estimate the first-order partial derivatives. Estimate the value of the function using the linear approximation.
3. **Chain rule:**
   (a) Construct a tree diagram and use the tree diagram to write down a formulas for the chain rule.
   (b) Use the chain rule to compute derivatives and partial derivatives of compositions of functions.
4. **Directional Derivative:**
   (a) Compute the gradient of a multivariable function.
   (b) Compute the directional derivative using the gradient and the dot product.
   (c) Determine the instantaneous rate of change of a function in a certain direction.
   (d) Determine the direction of steepest ascent or descent using the gradient.
   (e) Determine the rate of change in the direction of steepest ascent.
   (f) Compute the equation of a tangent plane using the gradient.
5. **Lines and planes:**
   (a) Find a vector equation of a line. Find parametric equations for a line.

(b) Determine when two lines are parallel or equal. Determine whether two lines intersect. Find the point of intersection when they do.

(c) Find a scalar equation of a plane given various geometric information. For instance, a plane containing three specified points or a plane containing a point and a line, or a plane containing a specified point and orthogonal to some line, etc.

(d) Determine when two planes are equal, parallel, orthogonal. Find the line of intersection of two planes when they intersect, but are not equal.

6. **Optimization**

   (a) Find critical points of a function of two variables.

   (b) Know the various types of extrema: local max/min, global max/min. Know what a saddle or "pringle" point is.

   (c) Classify critial points using the Second-Derivative test.

   (d) Know what the extreme-value theorem says.

   (e) Optimize a function on a closed and bounded region. This involved finding critical points on the interior of the region and on the boundary.

   (f) Optimize a function of two or more variables subject to a constraint equation, by using the method of Lagrange multipliers.

   (g) Determine which method makes sense for a given optimization problem. I might not tell you which method to use!

7. **Double Riemann sums**

   (a) Compute a double Riemann sum over a rectangle.

   (b) Estimate the integral of a function over a rectangle using a double Riemann sum. The function could be given as a table of values, or a contour diagram.

   (c) Interpret the value of a double Riemann sum or a double integral in context.

8. **Iterated Integrals**

   (a) Compute an integral over a rectangle.

   (b) Use Fubini's theorem to switch the order of bounds of integration over a rectangle.

   (c) Set up and evaluate iterated integrals over more general regions.

   (d) Given an iterated integral, sketch the region of integration described by the bounds. Evaluate the integral by swapping the order of integration.

   (e) Recognize when it might be advantageous to swap the order of integration.

   (f) Compute area of a region using a double integral.

9. **Double Integrals in Polar Coordinates**

   (a) Convert between rectangular and polar coordinates.

   (b) Describe a curve or a region using polar coordinates, when it makes sense to do so.

   (c) Convert a double integral in rectangular coordinates to a double integral in polar coordinates, using the change of variable formula.

   (d) Recognize when an integral would be easier to compute in polar coordinates.

10. **Surface area**
    (a) Given a parametrized surface, compute the surface area over a region using an integral. (I am not going to ask you to come up with the parametrization of a surface from scratch.)

11. **Triple Integrals (Optional)**
    (a) Set up and evaluate a triple integral over a rectangular prism.
    (b) Set up and evaluate a triple integral over a more general solid region.
    (c) Given a triple integral, identify and sketch the solid region of integration. Swap the order of integration of a triple integral.
    (d) Compute the volume of a solid region using a triple integral.

12. **Change of Variables (Optional)**
    (a) Compute the Jacobian of a given transformation.
    (b) Given a region and a change of variables, find the region which corresponds to it.
    (c) Compute a double or triple integral using a change of variables.

Practice Problems:

1. The problems from Weekly 4 are good practice. Complete them prior to the exam!

2. Suppose that $4x + 6y - 2z = 0$ is an equation of the tangent plane to the graph of a function $g(x, y)$ at the point $(1, 2)$. Each of the following questions are worth **6 points**.

(a) Find $g(1, 2)$, $g_x(1, 2)$, and $g_y(1, 2)$.

(b) Estimate $g(1.1, 1.9)$.

3. Suppose that z is a function such that (a) z depends on the variable x and y; (b) x and y depend on the variables s and t.

(a) Write down the appropriate tree diagram and label it with the appropriate derivatives.

(b) Write down the chain rule formula for $\frac{\partial z}{\partial s}$.

(c) Suppose $z(x, y) = x^2 + y^2$, $x(s, t) = \sin(s + t)$, and $y(s, t) = -\cos(s - t)$. Compute $\frac{\partial z}{\partial s}$ and evaluate at the point $(s, t) = (\pi, \pi)$.

4. The function $f(x, y) = \frac{1}{1+x^2+y^2}$ models the elevation of person standing on a volcano. All quantities are measured in meters. Suppose the person is standing at the point $(1, 1)$.

(a) Which direction should they walk in order to maintain their current elevation?

(b) Which direction should they walk to decrease their elevation the fastest? Approximately how many meters will they descend if they walk 1 meter in that direction?

5. Let $f(x, y) = 3x^3 + 2y^2$. Find all critical points of f and classify them using the second derivative test.

6. Find the shortest distance from the point $(0, 0, 0)$ to the plane $2x+4y-6z = 28$ using Lagrange multipliers. Briefly explain why a minimum exists, and why a maximum doesn't. Use this reasoning to classify the critical points you found.

7. The table below gives the number $C(W, S)$ of calories burned per hour for a person roller-blading as a function of the person's weight W in pounds and speed S in miles per hour.

W\S	8	9	10	11
120	252	348	444	534
140	306	402	498	594
160	366	462	552	648
180	420	516	612	702
200	474	570	666	756

Answer the following questions.

(a) Estimate the partial derivative $C_{WW}(160, 10)$. Make sure to include the units.

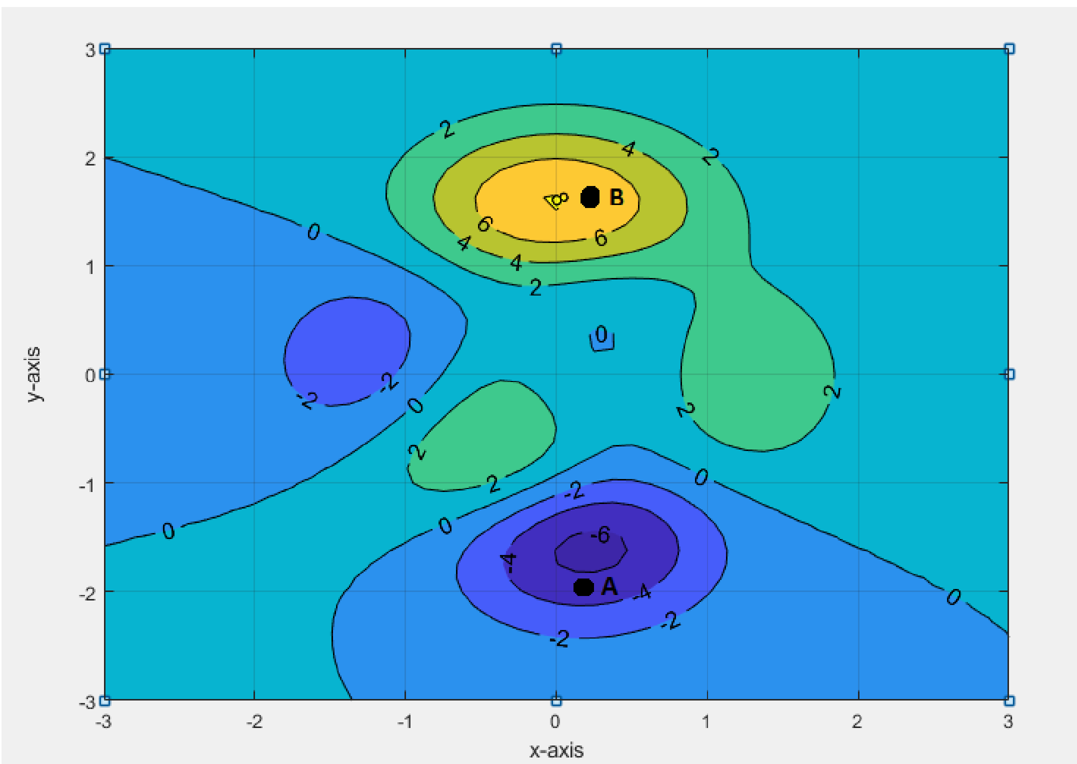
(b) Estimate the partial derivative $C_{WS}(160, 10)$. Make sure to include the units.

(c) Suppose that $R = [140, 180] \times [9, 10]$. Approximate the quantity

$$\frac{1}{\text{Area}(R)} \iint_R C(W, S)\, dA$$

and explain in detail the meaning of number you compute, including units.

8. Consider the following contour diagram of a continuous function $f(x, y)$:



I have identified two points A and B in the figure.

(a) Determine whether f_{xx} is positive, negative, or zero at point A.

(b) Determine whether f_{yy} is positive, negative, or zero at point A.

(c) Determine whether f_{xx} is positive, negative, or zero at point B.

(d) Determine whether f_{yy} is positive, negative, or zero at point B.

(e) Estimate $\iint_R f(x,y)\,dA$ where $R = [-1,2] \times [0,2]$. Use a partition of $[-1,2]$ into 3 subintervals and a partition of $[0,2]$ into 2 subintervals. Give a geometric interpretation of the value you compute.

9. Consider the double integral $\displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}} f(x,y)\,dx\,dy$. Sketch the region of integration and change the order of integration. Do not evaluate the integral.

10. Set up and evaluate a double integral whose value is the volume of the solid enclosed by the three coordinate planes and the plane $2x - y - z = 4$.