Weekly Assignment 1

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MATH 117: Advanced Linear Algebra

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Some hints for this assignment are written in the footnotes. See the weekly assignment webpage for due dates, templates, and assignment description.

1. Let F be a field and let $V = F$. Denote the additive and multiplicative identities of F by 0_F and 1_F, respectively. For $u, v \in V$ and $\alpha \in F$, define vector addition by $u \oplus v := u + v - 1_F$ and scalar multiplication by $\alpha \odot u := \alpha u - \alpha + 1_F$. Prove that $(V, \oplus, \odot)$ is an F-vector space.[1]

Proof. Write your proof here. □

2. Suppose that $W_1, \ldots, W_n$ are subspaces of a vector space V over a field F. Prove that

$$\sum_{i=1}^{n} W_i = \left\{ \sum_{i=1}^{n} w_i : w_i \in W_i \text{ for all } i = 1, \ldots, n \right\}.$$

Proof. Write your proof here. □

3. Prove Proposition 1.4.8: a subset B of a vector space V is a basis if and only if B is a minimal[2] spanning set.

Proof. Write your proof here. □

4. Let M and N be finite-dimensional subspaces of a (not necessarily finite dimensional) vector space V. Prove the following equation:

$$\dim(M) + \dim(N) = \dim(M + N) + \dim(M \cap N).$$

Proof. Write your proof here. □

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[1] You need to specify a zero vector 0_V and the additive inverse $\ominus u$ of $u \in F$, and then verify the several defining conditions of a vector space.

[2] A minimal spanning set is a spanning set that does not properly contain any other spanning set.