MATH 117: Daily Assignment 9

WRITE YOUR NAME HERE

August 21, 2023

See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

- 1. Read Section 3.4 on your own. You can skip the reading if you are already familiar with the symmetric group. Write each permutation below as a product of transpositions. Then compute the sign of each permutation.
 - (a) $(1,3,4,2) \in S_7$
 - (b) $(1, 4, 5, 7, 6) \in S_7$
 - (c) $(1,2)(3,4,5,7,6) \in S_7$.
- **2.** (a) Compute the determinant of $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix} \in (\mathbb{Z}_5)^{3 \times 3}$ using the Liebniz formula.
 - (b) Compute the determinant of $B = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 3 & 4 \\ 1 & 0 & 1 \end{pmatrix} \in (\mathbb{Z}_5)^{3 \times 3}$ using row operations.
 - (c) Compute the determinant of $C = \begin{pmatrix} 2 & 0 & 0 & 2 \\ 1 & 1 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 3 & 1 & 0 & 1 \end{pmatrix}$ using Cofactor expansion¹ along the first row.
 - (d) Compute the determinant of

$$D = \begin{pmatrix} 1 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 0 \end{bmatrix}$$

without doing any additional work.

- (e) Which matrices above are invertible, if any?
- **3.** Repeat Weekly 3.4(ii) using Proposition 3.3.10. Did you compute the matrix correctly the first time?

 $^{{}^{1}}I$ won't prove this because it takes too long. Feel free to use it whenever you need to compute determinants.