MATH 117: Daily Assignment 8

WRITE YOUR NAME HERE

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See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. Let $V = \mathbb{R}^2$ and let $E = (e_1, e_2)$ denote the standard basis for V. Then $E \otimes E := (e_1 \otimes e_1, e_1 \otimes e_2, e_2 \otimes e_1, e_2 \otimes e_2)$ is a basis for $V \otimes V$. Compute $[(5,3) \otimes (-1,2)]_{E\otimes E}$.[1]

2. (a) Show that the cross product (from calculus) defines a bilinear map $- \times - : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$.

(b) By the Universal Property of the Tensor Product, there is a unique linear map $L : \mathbb{R}^3 \otimes \mathbb{R}^3 \to \mathbb{R}^3$ satisfying $L(u \otimes v) = u \times v$. Compute $[L]^E_{E\otimes E}$ where is the standard basis $E = (e_1, e_2, e_3)$ and $E \otimes E = (e_i \otimes e_j)$ is ordered lexicographically.

3. (Optional, in case you want more practice)

(a) Show that matrix multiplication $- \cdot - : \mathbb{R}^{2\times 1} \times \mathbb{R}^{1\times 2} \to \mathbb{R}^{2\times 2}$ defines an $\mathbb{R}$-bilinear map.[2]

(b) By the Universal Property of the Tensor Product, there is a unique linear map $L : \mathbb{R}^{2\times 1} \to \mathbb{R}^{1\times 2} \to \mathbb{R}^{2\times 2}$ satisfying $L\left(\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) \otimes \left(\begin{smallmatrix} z & w \end{smallmatrix}\right)\right) = \left(\begin{smallmatrix} xz & xw \\ yz & yw \end{smallmatrix}\right)$. Compute $[L]^E_B$ where

$$B = \{\left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) \otimes \left(\begin{smallmatrix} 1 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) \otimes \left(\begin{smallmatrix} 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right) \otimes \left(\begin{smallmatrix} 1 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right) \otimes \left(\begin{smallmatrix} 0 & 1 \end{smallmatrix}\right)\}$$

and E is the standard basis for $\mathbb{R}^{2\times 2}$.

[1] Hint: Take advantage of the relations in Proposition 3.3.3.

[2] More generally, matrix multiplication defines an F-bilinear map $- \cdot - : F^{m\times n} \times F^{n\times k} \to F^{m\times k}$ where F is any field.