# MATH 117: Daily Assignment 5

WRITE YOUR NAME HERE

August 9, 2023

Some hints may be written in the footnotes. See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

**1.** For each part, you are given a vector space $V$ over a field $F$ with ordered bases $B$ and $C$ and a linear operator $L : V \to V$. Compute $[L]_B^C$.[1]

(a) $F = \mathbb{Z}_3$, $V = F_2[x]$, $B = (1 + x, 1 + x^2, x + x^2)$, $C = (2, x^2, 1 + x)$, $L(a + bx + cx^2) = b + cx + ax^2$. Note: I am using the notation 0,1,2 for the elements of $F = \mathbb{Z}_3$. So for instance $1 + 2 = 0$ in $F$.

(b) $F = \mathbb{Z}_5$, $V = F^2$, $B = ((1, 4), (2, 4))$, $C = ((1, 1), (2, 1))$, $L((a, b)) = (2a - b, 3a)$. Note: I am using the notation 0,1,2,3,4 for the elements of $F = \mathbb{Z}_5$. So for instance, $3 \cdot 4 = 2$ in $F$.

(c) $F = \mathbb{Z}_2$, $V = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in F^{2\times 2} : a + d = 0 \right\}$, $B = (\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right))$,
$C = (\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix}\right))$, $L\left(\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)\right) = \left(\begin{smallmatrix} a+b+c & c \\ b & a+b+c \end{smallmatrix}\right)$.

*Solution.* Your solution can go here. □

[1] The transition matrices from Daily 4 might be useful....