MATH 117: Daily Assignment 4

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Some hints for this assignment are written in the footnotes. See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning for every problem. You should be able to do each problem without appealing to theorems we have not proved yet.

1. For each part, you are given a vector space V over a field F with ordered bases B and C. Compute the transition matrix[1] T_B^C in each case.

(a) $F = \mathbb{Z}_3$, $V = F_2[x]$, $B = (1+x, 1+x^2, x+x^2)$, $C = (2, x^2, 1+x)$. Note: I am using the notation 0,1,2 for the elements of $F = \mathbb{Z}_3$. So for instance $1+2=0$ in F.

(b) $F = \mathbb{Z}_5$, $V = F^2$, $B = ((1,4),(2,4))$, $C = ((1,1),(2,1))$. Note: I am using the notation 0,1,2,3,4 for the elements of $F = \mathbb{Z}_5$. So for instance, $3 \cdot 4 = 2$ in F.

(c) $F = \mathbb{Z}_2$, $V = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in F^{2\times 2} : a_{11} + a_{22} = 0 \right\}$, $B = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right)$, $C = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right)$

Solution. □

2. Let $F = \mathbb{Z}_3$. How many isomorphisms are there from $F^2 \to F^2$?

Solution. Your solution can go here. □

[1] You may want to take advantage of Proposition 2.4.2(c) in case V has a nicer basis. The method to compute the inverse of a matrix using row reduction works over any field.