MATH 117: Daily Assignment 2

Carl Friedrich Gauss

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Some hints for this assignment are written in the footnotes. See the daily assignment webpage for due dates, templates, and assignment description.

1. Let F be any field. A *Fibonacci sequence* in F is a function $f : \mathbb{N} \to F$ defined recursively by letting $f(0), f(1)$ be elements of F and then setting $f(n+2) = f(n+1) + f(n)$ for all $n \in \mathbb{N}$. Show that the set $\mathcal{F}$ of all Fibonacci sequences in F is a subspace of $F^{\mathbb{N}}$. Then compute the dimension of this space.

Solution. Your solution can go here. □

2. For each part, determine whether the set of vectors S is a spanning set for the vector space V over the field F. If S is a spanning set, determine whether or not it is a basis[1]. Justify your answers.

(a) $F = \mathbb{Q}$, $V = \mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \in \mathbb{R} : a, b \in \mathbb{Q}\}$, $S = \{1 - \sqrt{2}, 4\}$.

(b) $F = \mathbb{Z}_2$, $V = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in F^{2\times 2} : a_{11} + a_{22} = 0 \right\}$, $S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right\}$

(c) $F = \mathbb{Z}_2$, $V = F_2[x] = \{a_0 + a_1 x + a_2 x^2 : a_0, a_1, a_2 \in F\}$, $S = \{1 + x, 1 + x^2\}$

Solution. Your solution can go here. □

3. Let $M := \mathrm{Mag}_3(\mathbb{R})$ denote the set of 3×3 magic squares with entries from $\mathbb{R}$.

(a) Show that M is a subspace of $\mathbb{R}^{3\times 3}$.

(b) Find a basis for M.[2]

(c) What is the dimension of M?

Solution. Your solution can go here. □

4. (optional) Let $M := \mathrm{Mag}_3(F)$ denote the set of 3×3 magic squares with entries from a field F.

(a) Convince yourself that the set M_0 of magic square with magic sum 0 is a subspace of M.

(b) Find a basis for M_0 when $F = \mathbb{Z}_p$, the field with p-elements. Does your answer depend on p?

(c) What is the dimension of M_0?

Solution. Your solution can go here. □

[1] Hint: use a Proposition from Section 1.4!

[2] You will need to solve a large system of equations - you may use a computer algebra system (CAS) to do this part of the computations.