# MATH 117: Daily Assignment 12

## WRITE YOUR NAME HERE

### August 27, 2023

See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. For each matrix $X$, find a Jordan matrix $J$ and an invertible matrix $P$ such that $J = P^{-1}XP$. If it is not possible to do this, explain why.

   (a) $F = \mathbb{Z}_2$ and $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \in F^{3\times 3}$.

   (b) $F = \mathbb{Z}_3$ and $B = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \in F^{3\times 3}$.

   (c) $F = \mathbb{Z}_2$ and $C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \in F^{4\times 4}$.

2. Determine all the eigenvalues, their algebraic and geometric multiplicities, and bases for each generalized eigenspace of the matrix $A = PJP^{-1} \in \mathbb{R}^{7\times 7}$ where

$$P = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 & -1 & -2 & -3 \\ 9 & 4 & 1 & 0 & 1 & 4 & 9 \\ 27 & 8 & 1 & 0 & -1 & -8 & -27 \\ 81 & 16 & 1 & 0 & 1 & 16 & 81 \\ 243 & 32 & 1 & 0 & -1 & -32 & -243 \\ 729 & 64 & 1 & 0 & 1 & 64 & 729 \end{pmatrix} \text{ and } J = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

3. Determine all possible Jordan forms for a linear operator with characteristic polynomial $(x-2)^4(x+1)^2$ and minimal polynomial $(x-2)^2(x+1)$.