

MATH 117: Daily Assignment 11

WRITE YOUR NAME HERE

August 25, 2023

See the [daily assignment webpage](#) for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. Let $F = \mathbb{Z}_5$. Compute the minimal polynomial for the the following matrices in from Daily 10. Use the minimal polynomial to determine if the matrix is diagonalizable. Compare with your solution to Daily 10.

$$(a) A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \in F^{3 \times 3}$$

$$(b) B = \begin{pmatrix} 2 & 4 \\ 3 & 3 \end{pmatrix} \in F^{2 \times 2}.$$

$$(c) C = \begin{pmatrix} 1 & 1 & 0 & 4 \\ 2 & 4 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in F^{4 \times 4}$$

$$(d) D = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{pmatrix} \in F^{3 \times 3}$$

2. We proved in the lectures that similar matrices have the same characteristic polynomial. Do similar matrices have the same minimal polynomial? Explain.
3. Determine whether the matrices $E = \begin{pmatrix} -8 & -10 & -1 \\ 7 & 9 & 1 \\ 3 & 2 & 0 \end{pmatrix} \in \mathbb{R}^{3 \times 3}$ and $F = \begin{pmatrix} -3 & 2 & -4 \\ 4 & -1 & 4 \\ 4 & -2 & 5 \end{pmatrix} \in \mathbb{R}^{3 \times 3}$. are similar¹.
4. Suppose that $A \in \mathbb{C}^{n \times n}$ satisfies $A^3 = A$. Is A diagonalizable? What about if $A \in F^{n \times n}$, where F is an arbitrary field?

¹Hint: similar matrices have a lot of the same invariants - check those first!