MATH 117: Daily Assignment 1 Solutions

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Some hints/comments for this assignment may be written in the footnotes. See the daily assignment webpage for due dates, templates, and assignment description.

1. Sign up for the Zulip discussion forum. Check your @ucsc.edu email for an invite to join or use the invite link on the canvas page. Then complete the following tasks:

(a) Introduce yourself to the class on Zulip. Make a post on the introductions stream using your first and last name as the title. Respond to at least two other posts on this stream

(b) Create a LaTeX example for your classmates. It can be as simple as mine (how to: create a matrix), but the topic should be unique. Post your example on the LaTeX stream. Make sure to use the Zulip latex code block to display your raw code (click view source on my post to see the syntax). Title your post as follows: "how to: (your topic here)".

2. Consider the set $\mathbb{Z}_7 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\}$ of residue classes of integers modulo 7.

(a) Construct the multiplication table for the group $(\mathbb{Z}_7 \setminus \{\overline{0}\}, \cdot)$ where $\cdot$ is defined using representatives: $\overline{m} \cdot \overline{n} := \overline{mn}$.

(b) Use part (a) to find the multiplicative inverse of every nonzero element of $\mathbb{Z}_7$.

Solution. (a) Here is the multiplication table:

$\cdot$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

(a) Multiplicative inverses are easily read from the table. For instance, $\overline{4} \cdot \overline{2} = \overline{1}$ implies that $\overline{4}^{-1} = \overline{2}$ and $\overline{2}^{-1} = \overline{4}$.

□

3. Let V be a vector space over a field F. Using only the definitions, prove Proposition 1.2.2: for all $v \in V$ and $a \in F$,

(a) $0v = 0$;

(b) $(-a)v = -(av)$;

(c) $a0 = 0$; and

(d) $av = 0$ implies $a = 0$ or $v = 0$.

Proof. (a) We have to prove that $0v$ is the additive identity in the group V. It suffices to show that $0v + 0v = 0v$ because the identity is the unique group element with this property. We have

$$\begin{aligned} 0v + 0v &= (0+0)v \\ &= 0v. \end{aligned}$$

This proves the claim.

(b) The claim is that $(-a)v$ is the additive inverse of av. We have

$$\begin{aligned} av + (-a)v &= (a + (-a))v \\ &= 0v \\ &= 0. \end{aligned}$$

Since inverses are unique, this proves the claim.

(c) Similar proof to (a).

(d) Suppose $av = 0$ and $a \neq 0$. We will prove that $v = 0$. The key point is that every nonzero element of a field is invertible. We have

$$0 = a^{-1}0 = a^{-1}(av) = (a^{-1}a)v = 1v = v.$$

□

4. Let $C(\mathbb{R})$ be the real vector space[1] of all continuous functions $f : \mathbb{R} \to \mathbb{R}$. Determine which of the following are subspaces of $C(\mathbb{R})$. Make sure to justify your reasoning.

(a) $\{f : f \text{ is twice differentiable and } f''(x) - 2f'(x) + 3f(x) = 0 \text{ for all } x \in \mathbb{R}\}$.

(b) $\{g : g \text{ is twice differentiable and } g''(x) = g(x) + 1 \text{ for all } x \in \mathbb{R}\}$.

(c) $\{h : h \text{ is twice differentiable and } h''(0) = 2h(1)\}$.

Solution. One needs to understand the vector space structure on $C(\mathbb{R})$. Addition and scalar multiplication are defined pointwise: $(f+g)(x) := f(X) + g(x)$ and $(\alpha f)(x) := \alpha f(x)$ for all $f, g \in C(\mathbb{R})$ and $\alpha \in \mathbb{R}$. The zero vector is the zero function $0_{C(\mathbb{R})}$ which sends every real number to zero.

(a) This is a subspace. Clearly $0_{C(\mathbb{R})}$ satisfies the equation. If $f''(x) - 2f'(x) + 3f(x) = 0$ and $g''(x) - 2g'(x) + 3g(x) = 0$, then

$$\begin{aligned} (f+g)''(x) - 2(f+g)'(x) + 3(f+g)(x) &= f''(x) + g''(x) - 2f'(x) - 2g'(x) + 3f(x) + g(x) \\ &= f''(x) - 2f'(x) + 3f(x) + g''(x) - 2g'(x) + g(x) \\ &= 0 + 0. \end{aligned}$$

This shows that the set is closed under addition. Similarly, it is closed under scalar multiplication.

(b) This is not a subpace because the zero function does not satisfy the equation.

[1] $C(\mathbb{R})$ is a subspace of the real vector space $\mathbb{R}^{\mathbb{R}} = \text{Maps}(\mathbb{R}, \mathbb{R})$

(c) This is a subspace. The zero function satisfies the equation. If $f''(0) = 2f(1)$ and $g''(0) = 2g(1)$, then

$$(f+g)''(0) = f''(0) + g''(0) = 2f(1) + 2g(1) = 2(f+g)(1).$$

Scalar multiplication is similar.

□