MATH 117: Daily Assignment 1

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Some hints/comments for this assignment may be written in the footnotes. See the daily assignment webpage for due dates, templates, and assignment description.

1. Sign up for the Zulip discussion forum. Check your @ucsc.edu email for an invite to join or use the invite link on the canvas page. Then complete the following tasks:

(a) Introduce yourself to the class on Zulip. Make a post on the introductions stream using your first and last name as the title. Respond to at least two other posts on this stream

(b) Create a LaTeX example for your classmates. It can be as simple as mine (how to: create a matrix), but the topic should be unique. Post your example on the LaTeX stream. Make sure to use the Zulip latex code block to display your raw code (click view source on my post to see the syntax). Title your post as follows: "how to: (your topic here)".

2. Consider the set $\mathbb{Z}_7 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\}$ of residue classes of integers modulo 7.

(a) Construct the multiplication table for the group $(\mathbb{Z}_7 \setminus \{\overline{0}\}, \cdot)$ where $\cdot$ is defined using representatives: $\overline{m} \cdot \overline{n} := \overline{mn}$.

(b) Use part (a) to find the multiplicative inverse of every nonzero element of $\mathbb{Z}_7$.

3. Let V be a vector space over a field F. Using only the definitions, prove Proposition 1.2.2: for all $v \in V$ and $a \in F$,

(a) $0v = 0$;

(b) $(-a)v = -(av)$;

(c) $a0 = 0$; and

(d) $av = 0$ implies $a = 0$ or $v = 0$.

4. Let $C(\mathbb{R})$ be the real vector space[1] of all continuous functions $f : \mathbb{R} \to \mathbb{R}$. Determine which of the following are subspaces of $C(\mathbb{R})$. Make sure to justify your reasoning.

(a) $\{f : f$ is twice differentiable and $f''(x) - 2f'(x) + 3f(x) = 0$ for all $x \in \mathbb{R}\}$.

(b) $\{g : g$ is twice differentiable and $g''(x) = g(x) + 1$ for all $x \in \mathbb{R}\}$.

(c) $\{h : h$ is twice differentiable and $h''(0) = 2h(1)\}$.

[1] $C(\mathbb{R})$ is a subspace of the real vector space $\mathbb{R}^{\mathbb{R}} = \mathrm{Maps}(\mathbb{R}, \mathbb{R})$