Weekly Assignment 1

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MATH 117: Advanced Linear Algebra

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Some hints for this assignment are written in the footnotes. See the weekly assignment webpage for due dates, templates, and assignment description.

1. Let $V = \mathbb{R}$. For $u, v \in V$ and $\alpha \in \mathbb{R}$, define vector addition by $u \oplus v := u + v - 1$ and scalar multiplication by $\alpha \odot u := \alpha u - \alpha + 1$. Prove that $(V, \oplus, \odot)$ is a vector space.[1]

Proof. Write your proof here. $\square$

2. Prove Proposition 1.4.3: a subset S of a vector space V is dependent if and only if there exists $v \in S$ such that $v \in \operatorname{span}(S \setminus \{v\})$.

Proof. Write your proof here. $\square$

3. Prove Proposition 1.4.8: a subset B of a vector space V is a basis if and only if B is a minimal[2] spanning set.

Proof. Write your proof here. $\square$

4. Let M and N be finite-dimensional subspaces of a (not necessarily finite dimensional) vector space V. Prove the following equation:

$$\dim(M) + \dim(N) = \dim(M + N) + \dim(M \cap N).$$

Proof. Write your proof here. $\square$

5. Prove Proposition 2.1.6: if $L : V \to W$ is a linear map, then $\operatorname{im}(L)$ is a subspace of W and $\ker(L)$ is a subspace of V.

Proof. Write your proof here. $\square$

[1] You need to specify a zero vector 0 and the additive inverse $\ominus u$ of $u \in V$, and then verify the several defining conditions of a vector space.

[2] A minimal spanning set is a spanning set that does not properly contain any other spanning set.