MATH 117: Daily Assignment 8

WRITE YOUR NAME HERE

August 14, 2022

See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. (a) Compute the determinant of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix} \in (\mathbb{Z}_5)^{3\times 3}$ using the Liebniz formula.

(b) Compute the determinant of $B = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 3 & 4 \\ 1 & 0 & 1 \end{pmatrix} \in (\mathbb{Z}_5)^{3\times 3}$ using row operations.

(c) Compute the determinant of $C = \begin{pmatrix} 2 & 0 & 0 & 2 \\ 1 & 1 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 3 & 1 & 0 & 1 \end{pmatrix}$ using Cofactor expansion along the first row.

2. Let $F = \mathbb{Z}_2$ and let $V = F^2$. Let $L_A : V \to V$ be the linear map given by left multiplication with the matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Similarly, defined $L_B : V \to V$ via left multiplication with the matrix $B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

(a) Compute $\mathrm{cp}_{L_A \otimes L_B}(x)$. You may use the "Cofactor expansion" algorithm, even though we have not proved it.

(b) Find the eigenvalues of $L_A \otimes L_B$

(c) Find a basis for each eigenspace of $L_A \otimes L_B$.

3. Let $F = \mathbb{Z}_2$ and $V = F^2$. Find all linear operators $L : V \to V$ for which $\mathrm{cp}_L(x) = x^2 + 1$.