MATH 117: Daily Assignment 7

WRITE YOUR NAME HERE

August 11, 2022

See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. (a) There is a linear map $\Psi : (V \otimes V)^* \to \mathrm{Hom}_F(V, V^*)$ defined as follows. For any linear map $K : V \otimes V \to F$, $\Psi_K : V \to V^*$ is the function given by the rule $(\Psi_K(v))(w) := K(v \otimes w)$ for all $v, w \in V$. Verify that

   (i) $\Psi_K(v) \in V^*$ for all $v \in V$;

   (ii) $\Psi_K \in \mathrm{Hom}_F(V, V^*)$ for all $K \in (V \otimes V)^*$; and that

   (iii) Ψ is a linear map.

   (b) Let $V = \mathbb{R}_1[x]$ and let $L : \mathbb{R}_1[x] \to \mathbb{R}$ be the linear map defined by $L(a + bx) = a + b$. Let $E = (1, x)$ be the standard basis. Compute the matrix $[\Psi_{L\otimes L}]_E^{E^*}$. (Note: here we are identifying $\mathbb{R}\otimes\mathbb{R}$ with $\mathbb{R}$ via the isomorphism $\mathbb{R}\otimes\mathbb{R} \to \mathbb{R}$, $1\otimes 1 \mapsto 1$. So $L\otimes L : V\otimes V \to \mathbb{R}$ is the unique linear map satisfying $(L \otimes L)(v, w) = L(v)L(w)$ for all $v, w \in V$.)

2. Let $L : \mathbb{R}^2 \to \mathbb{C}$ be the $\mathbb{R}$-linear map given by $L(a, b) = (a + b) + (a - b)i$ and let $K : \mathbb{R}^2 \to \mathbb{C}$ be the $\mathbb{R}$-linear map defined by $K(a, b) = 2b + 2bi$. Let $E = ((1, 0), (0, 1))$ be the standard basis for $\mathbb{R}^2$ and let $F = (1, i)$ be the standard basis for $\mathbb{C}$ (over $\mathbb{R}$). Compute the matrix $[L \otimes K]_{E\otimes E}^{F\otimes F}$.