MATH 117: Daily Assignment 6

WRITE YOUR NAME HERE

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See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. Solve Problem 1 from Daily 5 using Theorem 3.2.2 from today's lecture.

2. Let V be a vector space. In Friday's lecture, we defined a map $L : V \to V^{**}$ via the rule $L(v) = \mathrm{ev}_v$, where $\mathrm{ev}_v : V^* \to F$ is the function satisfying $\mathrm{ev}_v(f) = f(v)$ for any $f \in V^*$.[1]

(a) Verify that $\mathrm{ev}_v \in V^{**}$ for all $v \in V$. Thus, L defines a function from V to V^{**}.

(b) Verify that L is a linear map.

3. Let $V = \mathbb{R}^2$ and let $E = (e_1, e_2)$ denote the standard basis for V. In can be shown that $E \otimes E := (e_1 \otimes e_1, e_1 \otimes e_2, e_2 \otimes e_1, e_2 \otimes e_2)$ is a basis for $V \otimes V$. Compute $[(5,3) \otimes (-1,2)]_{E \otimes E}$.[2]

4. Let V be a vector space. Use the Universal Property of the Tensor Product to show that there is a linear map $L : V \otimes V^* \to F$ satisfying $L(v \otimes f) = f(v)$ for all $v \in V$ and $f \in V^*$.

[1] The proof of Theorem 3.1.6 from the lecture actually shows that (assuming the Axiom of Choice, so that every independent set can be extended to a basis in case V has infinite dimension) $L : V \to V^{**}$ is injective, regardless of whether or not V the dimension of V is finite. So V is always isomorphic to a subspace of V^{**}. If V is finite-dimensional, then L is an isomorphism because $dim V = \dim V^{**}$.

[2] Hint: Take advantage of the relations in Proposition 3.3.3.