MATH 117: Daily Assignment 3

WRITE YOUR NAME HERE

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Some hints for this assignment are written in the footnotes. See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. For each part, you are given a vector space V over a field F with ordered bases B and C. Compute the transition matrix[1] T_B^C in each case.

(a) $F = \mathbb{Z}_3$, $V = F_2[x]$, $B = (1+x, 1+x^2, x+x^2)$, $C = (2, x^2, 1+x)$. Note: I am using the notation 0,1,2 for the elements of $F = \mathbb{Z}_3$. So for instance $1+2=0$ in F.

(b) $F = \mathbb{Z}_5$, $V = F^2$, $B = ((1,4),(2,4))$, $C = ((1,1),(2,1))$. Note: I am using the notation 0,1,2,3,4 for the elements of $F = \mathbb{Z}_5$. So for instance, $3 \cdot 4 = 2$ in F.

(c) $F = \mathbb{Z}_2$, $V = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in F^{2\times 2} : a_{11} + a_{22} = 0 \right\}$, $B = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right)$, $C = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right)$

Solution. □

2. For each part, you are given a vector space V over a field F with ordered bases B and C and a linear operator $L : V \to V$. Compute $[L]_B^C$.

(a) $F = \mathbb{Z}_3$, $V = F_2[x]$, $B = (1+x, 1+x^2, x+x^2)$, $C = (2, x^2, 1+x)$, $L(a+bx+cx^2) = b + cx + ax^2$.

(b) $F = \mathbb{Z}_2$, $V = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in F^{2\times 2} : a+d=0 \right\}$, $B = \left(\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right) \right)$, $C = \left(\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix}\right) \right)$, $L\left(\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \right) = \left(\begin{smallmatrix} a+b+c & c \\ b & a+b+c \end{smallmatrix}\right)$

Solution. Your solution can go here. □

[1] You may want to take advantage of Proposition 2.4.2(c) in case V has a nicer basis. The method to compute the inverse of a matrix using row reduction works over any field.