

# MATH 117: Daily Assignment 11

WRITE YOUR NAME HERE

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See the [daily assignment webpage](#) for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. Let  $A \in \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in F^{3 \times 3}$ .

(a) Let  $F = \mathbb{Z}_2$ . Find a matrix  $J$  in Jordan form and an invertible matrix  $P$  such that  $J = P^{-1}AP$ .

(b) Repeat problem 1(a) with  $F = \mathbb{Z}_3$ .

2. Determine all the eigenvalues, their algebraic and geometric multiplicities, and bases for each generalized eigenspace of the matrix  $A$ .

(a)  $\begin{pmatrix} 2 & 1 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ -1 & 1 & 2 & 1 \\ -1 & 1 & -1 & 4 \end{pmatrix} \in \mathbb{R}^{4 \times 4}$

(b)  $A = PJP^{-1}$  where

$$P = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 & -1 & -2 & -3 \\ 9 & 4 & 1 & 0 & 1 & 4 & 9 \\ 27 & 8 & 1 & 0 & -1 & -8 & -27 \\ 81 & 16 & 1 & 0 & 1 & 16 & 81 \\ 243 & 32 & 1 & 0 & -1 & -32 & -243 \\ 729 & 64 & 1 & 0 & 1 & 64 & 729 \end{pmatrix} \text{ and } J = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

3. Determine all possible Jordan forms for a linear operator with characteristic polynomial  $(x - 2)^4(x + 1)^2$  and minimal polynomial  $(x - 2)^2(x + 1)$ .