MATH 117: Daily Assignment 1

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Some hints for this assignment are written in the footnotes. See the daily assignment webpage for due dates, templates, and assignment description.

1. Sign up for the Zulip discussion forum and then post a short introduction on the introductions stream. Check your @ucsc.edu email for an invite to join.

2. Consider the set $\mathbb{Z}_7 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\}$ of residue classes of integers modulo 7.

    (a) Construct the multiplication table for the group $(\mathbb{Z}_7 \setminus \{\overline{0}\}, \cdot)$ where $\cdot$ is defined using representatives: $\overline{m} \cdot \overline{n} := \overline{mn}$.

    (b) Use part (a) to find the multiplicative inverse of every nonzero element of $\mathbb{Z}_7$.

3. Let V be a vector space over a field F. Using only the definitions, prove Proposition 1.2.2: for all $v \in V$ and $a \in F$,

    (a) $0v = 0$;

    (b) $(-a)v = -(av)$;

    (c) $a0 = 0$; and

    (d) $av = 0$ implies $a = 0$ or $v = 0$.

4. Let $C(\mathbb{R})$ be the real vector space[1] of all continuous functions $f : \mathbb{R} \to \mathbb{R}$. Determine which of the following are subspaces of $C(\mathbb{R})$. Make sure to justify your reasoning.

    (a) $\{f : f$ is twice differentiable and $f''(x) - 2f'(x) + 3f(x) = 0$ for all $x \in \mathbb{R}\}$.

    (b) $\{g : g$ is twice differentiable and $g''(x) = g(x) + 1$ for all $x \in \mathbb{R}\}$.

    (c) $\{h : h$ is twice differentiable and $h''(0) = 2h(1)\}$.

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[1]$C(\mathbb{R})$ is a subspace of the real vector space $\mathbb{R}^{\mathbb{R}} = \mathrm{Maps}(\mathbb{R}, \mathbb{R})$