Midterm Exam Cover Page

Overview: This exam consists of one cover page and **three pages of questions**. There are **7 questions** worth a total of **117 points**. Your grade on this exam will be calculated as

(**number of points earned**)/100.

Therefore, you need not feel pressured to complete every problem (but if you do, you can earn some extra credit). This exam is worth **15% of your final grade** in the course.

Directions:

- Solve as many of the following problems as you can in 90 minutes. Try to start with problems that you know how to solve right away, and save the others for last.
- You are **required** to provide sufficient justification for **every** problem you submit. You should show all of your computations and explain in detail your reasoning. If you do not provide sufficient justification, as judged solely by the grader, you will receive **NO CREDIT** for that problem, even if your answer is correct.
- Submit your work via the Midterm Exam quiz on Canvas. **You will only be able to submit 1 file and it MUST be a .pdf.** You can use CombinePDF to combine your files into a single .pdf for free.
- Read the exam protocol in the syllabus.

The simple rules for the exam are:

1. You may freely consult the Active Calculus: Multivariable textbook or any notes from our class meetings.
2. You are **NOT** allowed to consult any other resources, including, but not limited to, other textbooks, the internet, Chegg, and math.stackexchange.
3. You are **NOT** allowed to discuss the exam with any other person until after the due date (7/10 at 11:59pm).
4. Follow the Exam Protocol in the syllabus.

I will vigorously pursue anyone suspected of breaking these rules.

Good luck, and have fun!

Midterm Exam Problems

Problems:

1. **(36 points)** Answer the following questions, each of which is worth **6 points**. You must provide justification for your answer.
   - **(a)** Is the angle between the vectors $\mathbf{u} = \langle 1, 2, 0, -4\rangle$ and $\mathbf{v} = \langle 0, 2, 1, 2\rangle$ obtuse?
   - **(b)** Do the lines $\mathbf{r}(t) = \langle 1 + 2t, -t, 7 - 6t\rangle$ and $\mathbf{s}(t) = \langle 1 + 4t, 1 - 2t, 7 - 12t\rangle$ intersect?
   - **(c)** Are the planes $2x + y - z = 2$ and $-6x - 3y + 9z = 0$ parallel?
   - **(d)** Are the vectors $\mathbf{u} = \langle 1, 3, 2\rangle$, $\mathbf{v} = \langle 1, 0, 1\rangle$, and $\mathbf{w} = \langle 0, -3, -1\rangle$ coplanar?
   - **(e)** Is the line $\mathbf{r}(t) = \langle 1 - t, 2 - t, 3 + 6t\rangle$ contained in the plane $6x + 6y + 2z = 24$?
   - **(f)** Is the line $\mathbf{r}(t) = \langle 1 + 3t, 1 + 4t, t\rangle$ tangent to the curve $\mathbf{s}(t) = \langle t^2, t^3, \ln t\rangle$ at the point $(1, 1, 0)$?

2. **(12 points)** Let $P = (1, 2, -3)$, $Q = (1, 2, 4)$, $R = (1, 0, 7)$, $S = (-1, -1, -2)$. Determine if the points P, Q, R and S are coplanar. That is, do P, Q, R and S lie in the same plane? You must justify your claim.

3. **(15 points)** Let p denote the plane defined by the scalar equation $3x + 2y - z = 1$ and let q denote the plane defined by $7x - y - 2z = 4$. Complete the following tasks, each of which is worth **5 points**.
   - **(a)** Show that the planes p and q intersect.
   - **(b)** Show that the planes p and q are not parallel.
   - **(c)** It follows from (a) and (b) that the intersection of the planes is a line. Find a vector-valued function $\mathbf{r}(t)$ that describes this line.

4. **(18 points)** Let $f(x, y) = 3x^2 + 2y^2$. Complete each of the following tasks, each of which is worth **3 points**.

   (a) Write down any nonzero point $P_0 = (x_0, y_0, z_0)$ that lies on the graph of the function $f(x, y)$.

   (b) Find a vector-valued function $\mathbf{r}(t)$ that describes the $x = x_0$ trace of the function f.

   (c) Find a vector-valued function $\mathbf{s}(t)$ that describes the $y = y_0$ trace of the function f.

   (d) Find the direction $\mathbf{v}$ of the line containing P_0 whose direction is tangent to the $x = x_0$ trace at P_0. Compute the vector form $\mathbf{L_1}(t)$ of this line.

   (e) Find the direction $\mathbf{w}$ of the line containing P_0 whose direction is tangent to the $y = y_0$ trace at P_0. Compute the vector form $\mathbf{L_2}(t)$ of this line.

   (f) The *tangent plane* to the graph of f at P_0 is the plane containing both of the tangent lines $\mathbf{L_1}$ and $\mathbf{L_2}$ from part (d) and (e). Find the scalar equation of this plane.

5. **(12 points)** Let $f(x, y) = 3x^2 + 2y^2$ and let $P_0 = (x_0, y_0, z_0)$ be the point you chose in part (a) of the preceding problem. Complete the following tasks, each of which is worth **4 points**.

   (a) Write down the point $P_0 = (x_0, y_0, z_0)$ that you chose in part (a) of problem 4. Then, compute $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$.

   (b) Explain the relationship between $f_y(x_0, y_0)$ and $\mathbf{L_1}$.

   (c) Explain the relationship between $f_x(x_0, y_0)$ and $\mathbf{L_2}$.

6. **(12 points)** The table below gives the number $C(W, S)$ of calories burned per hour for a person roller-blading as a function of the person's weight W in pounds and speed S in miles per hour.

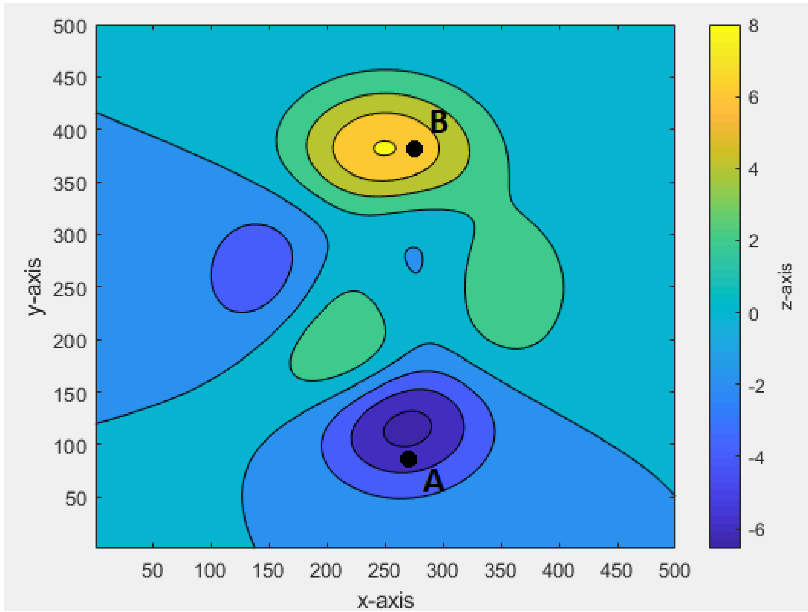
W\S	8	9	10	11
120	252	348	444	534
140	306	402	498	594
160	366	462	552	648
180	420	516	612	702
200	474	570	666	756

   Answer the following questions, each of which is worth **6 points**.

   (a) Estimate the partial derivative $C_W(140, 10)$. Make sure to include the units.

   (b) Estimate the partial derivative $C_S(140, 10)$. Make sure to include the units.

7. (12 points) Consider the following contour diagram of a function $f(x, y)$:



I have identified two points A and B in the figure. Complete the following tasks, each of which is worth **3 points**.

(a) Determine whether f_x is positive, negative, or zero at point A.

(b) Determine whether f_y is positive, negative, or zero at point A.

(c) Determine whether f_x is positive, negative, or zero at point B.

(d) Determine whether f_y is positive, negative, or zero at point B.