

111B Section Week 4

Overview: Work on the following problems one at a time, either by yourself or in small-groups. After a sufficient amount of time has passed, we will discuss the solutions as a class. Attending section counts toward your participation grade.

1. Let $\mathbb{F} = \{p(x) \in \mathbb{Z}/2\mathbb{Z}[x] : \deg p(x) < 3\}$.
 - (a) List all elements of \mathbb{F} .
 - (b) Show that \mathbb{F} is a subgroup of $\mathbb{Z}/2\mathbb{Z}[x]$.
 - (c) Show that \mathbb{F} is not a subring of $\mathbb{Z}/2\mathbb{Z}[x]$.
 - (d) The abelian group \mathbb{F} can be made into a ring as follows: given two polynomials $p(x), q(x)$, compute the product $p(x)q(x)$ in $\mathbb{Z}/2\mathbb{Z}[x]$, and then reduce using the relation $x^3 = x + 1$. Compute the multiplication table for \mathbb{F} under this operation.
 - (e) Do you notice anything special about the ring \mathbb{F} ?

If we have extra time, we will define the group ring and play with some examples.