111B Section Week 8

**Overview:** Work on the following problems one at a time, either by yourself or in small-groups. After a sufficient amount of time has passed, we will discuss the solutions as a class. Attending section counts toward your participation grade.

1. Let \( I = (2, x) \) be the ideal of \( \mathbb{Z}[x] \) generated by 2 and \( x \).
   
   (a) Show that a polynomial \( \sum_{i=0}^{n} a_i x^i \in \mathbb{Z}[x] \) belongs to \( I \) if and only if \( a_0 \) is even.
   
   (b) Show that \( I \) is a maximal ideal of \( \mathbb{Z}[x] \).

2. Let \( R \) be a ring and \( M \) an ideal.
   
   (a) Prove that if \( R/M \) is a field, then \( M \) is a maximal ideal.
   
   (b) Use 2(a) and the First Isomorphism Theorem to give an alternative proof of 1(b).