TEACHING PORTFOLIO

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Contents

1 Teaching Statement & Philosophy

1.1 Introduction

How does a person learn something new? If you ask my calculus students, you will find that the most frequent answer is *practice*. Intuitively, students know how learning works, and yet a significant number hold the common—and incorrect—belief that mathematical ability is an innate talent rather than a skill that can be developed and improved over time. On the first day of class, I make these underlying beliefs visible by asking this question and generating a word-cloud based on their responses. As the visualization forms on the screen, words like *practice*, *making mistakes*, *curiosity*, and *communication* swell in size, dwarfing passive verbs like *listening* or *watching*. I use this activity to start a dialogue on the learning process where my ultimate goal is to show my students that an active, inquiry-based classroom is not just my preference, but a reflection of their own intuition about how learning works.

This anecdote provides a window into the foundation of my pedagogy. There is a substantial body of recent scholarship showing that students learn more effectively when they are actively engaged in the learning process. You cannot learn to play the electric guitar by watching Buckethead shred, nor can you become a chess Grandmaster by watching Magnus Carlsen on Twitch. In the same way, you cannot learn mathematics by simply watching—you must do mathematics! My responsibility as an instructor is to design a classroom environment where students do not merely mimic prefabricated examples, but construct their own understanding through activities that reflect authentic mathematical practice.

1.2 Teaching Philosophy

My teaching is based on the belief that *students learn mathematics by doing mathematics*. Students develop a deeper understanding when they actively engage with mathematics by building examples, asking questions, making conjectures, and communicating their reasoning. Activities like these, which mirror what mathematicians actually do, allow students to experience mathematics as an active and creative process rather than a black box of rules to be followed. As students make their own discoveries and construct their own understanding, they begin to see themselves as creators, rather than consumers. This shift helps them develop the ownership and confidence that supports deeper learning.

When students are engaged in authentic mathematical thought, mistakes and struggle are inevitable. This is a feature of the process, not a bug—*mistakes and productive struggle are essential components of learning*. Mistakes are not a sign of failure; they are simply evidence that a person is grappling with a challenging idea. When students try something new, make a mistake, and revise their thinking, they are practicing the same iterative reasoning that mathematicians engage in on a daily basis. Struggle and refinement are normal parts of the learning process, and meaningful learning frequently happens when we analyze our earlier unsuccessful attempts. Fostering an environment where productive failure is valued helps students to tackle challenges with curiosity and resilience.

If we want students to approach mathematics in this way, then we must recognize that *students' beliefs about learning influence how they learn*. A student who views struggle as a normal part of the learning process is going to be more likely to try a new strategy on a tough problem, or share their reasoning with the class. A student who believes that mathematical ability is a skill to be developed is going to be more likely to put in the work. An important part of teaching is making students aware of their own beliefs and encouraging the adoption of empowering habits of mind that support their long-term success, both in mathematics and in other parts of their lives.

1.3 Teaching Experience

As a graduate student at the University of California, Santa Cruz, I gained over six years of teaching experience, which spans multiple roles, courses, classes, and modalities.

I was the *instructor of record* for seven courses, ranging from service courses like precalculus and multivariable calculus to upper-division courses like complex analysis and advanced linear algebra. In each appointment, I was responsible for the full design and delivery of the course. Additionally, I worked closely with a TA for each course to coordinate their duties, communicate expectations, and ensure consistency in grading and course implementation. These roles have given me substantial experience in course design, classroom leadership, and teaching team management.

I also have extensive experience as a *teaching assistant* spanning twenty appointments across a wide range of lower- and upper-division mathematics courses. In these roles, I led weekly discussion sections, designed supplemental course materials, held office hours, and performed grading on a wide variety of assignments and exams. I also served several times in an informal *Lead TA* role for the large online calculus courses (MATH 19B and 23A), which included additional responsibilities such as coordinating the duties of the other TAs and working with the instructors to manage logistical elements of these large, multi-section courses. My work as a Lead TA was recognized by the UCSC Mathematics Department with the *Outstanding TA Award* for the 2020-2021 academic year. In the nomination, the online calculus faculty highlighted my leadership and instructional reliability, writing

> "This quarter he is the lead teaching assistant in a large lecture course....He does an equally outstanding job engaging students in our Piazza discussion forum, teaching his sections, organizing a group of TAs to get the grading work done efficiently, fairly and timely. Jadyn is an enormous asset for our department and he should be recognized for his consistently outstanding work."

My teaching is also informed by formal pedagogical training through our departments two-course sequence, *MATH 288A/B Pedagogy of Mathematics*. These courses introduced me to research-based instruction strategies, approaches to equitable course design, and reflective teaching practices that I continue to implement in my courses. I also completed the UCSC Teaching and Learning Center's *Preparing for Inclusive Teaching* professional development program (see Appendix **??**), a one-day workshop for STEM graduate students that focuses on equity-minded strategies to enhance student learning.

1.4 Teaching Practice

The clearest example of my pedagogy in practice is my development of MATH 22: Introduction to Calculus of Several Variables, which I designed in an active-learning format and refined over three iterations. Students came to class having completed structured reading assignments. These reading assignments were carefully aligned with learning objectives which I identified as prerequisites for meaningful participating in class. Class time was devoted to guided inquiry through thoughtfully chosen activities from the textbook *Active Calculus Multivariable* by Steven Schlicker [7]. These activities were scaffolded by short mini-lectures that linked the pre-class reading work to more advanced learning objectives, clarified high-level concepts, and formalized ideas students discovered through the activities. This structure created a classroom environment where students were consistently and deeply engaged with multivariable calculus. A student from my MATH 22 course noted that

> "The instructor helped me feel engaged with the course very frequently because the majority of class time was spent doing activities... we were actively participating... instead of just listening to a lecture." —*MATH 22 student*, *Summer 2025*

In my upper-division courses such as complex analysis and advanced linear algebra, I have primarily taught in a more traditional lecture-based format. The main reason was practical: unlike multivariable calculus, I did not have access to well-developed, freely available inquiry-based materials for these subjects, and developing a full active-learning curriculum from scratch was not feasible within the constraints of a ten-week quarter. In these courses, I therefore relied on highly structured, interactive lectures and supplemented them with daily inquiry-oriented tasks. A student from my MATH 117 course said this about the daily tasks:

> "I thought the daily homework was brilliant. It got me to be able to understand how to play with these abstract ideas by myself." —*MATH 117 student*, *Summer 2023*

Looking ahead, I intend to transition my design of these courses toward more active and student-centered approaches, either by incorporating established inquiry-based resources or by developing my own.

1.5 Undergraduate Mentorship

One of the great joys of mathematics comes from sharing it with others. I have been extremely fortunate to have come in contact with numerous great teachers, mentors, colleagues, and students, many of whom have changed my life and influenced my career as a mathematician. For this reason, I believe that learning partnerships between undergraduates and graduates, and faculty-led undergraduate research form an integral part of any successful mathematics program. I have served as a mentor for undergraduates numerous times, both officially through structured programs, and unofficially for students who regularly interacted with me through office hours or by chance. In these roles, I acted not only as a mathematical guide, but also as an anchor of support.

One specific instance of mentorship I have done is through the mathematics department Directed Reading program (DRP). In this program, mentors are pairs with undergraduate mentees, helping the students learn an advanced topic that is not normally encountered in the curriculum. I have mentored several students on topics such as the representation theory of the symmetric group, Jordan canonical form, the Sylow theorems, and finite fields. At the end of each project, students give a final presentation on what they've learned. Getting to work with motivated students one-on-one is very fulfilling and allows me to give back to the community by fostering enthusiasm and excitement for mathematics. Several of my mentees have gone on to pursue graduate studies in mathematics or nearby fields.

2 Courses and Teaching Roles

2.1 Courses as Instructor of Record

Math 22: **Introduction to Calculus of Several Variables** **Summer 2025**
Students: 22, TAs: 1, Modality: In-Person. Designed the entire 5-week course.

Math 22: **Introduction to Calculus of Several Variables** **Summer 2024**
Students: 18, TAs: 1, Modality: In-Person. Designed the entire 5-week course.

Math 117: **Advanced Linear Algebra** **Summer 2023**
Students: 18, Readers: 1, Modality: Remote. Designed the entire 5-week course.

Math 117: **Advanced Linear Algebra** **Summer 2022**
Students: 14, TAs: 1, Modality: Remote. Designed the entire 5-week course.

Math 3: **Pre-calculus** **Fall 2021**
Students: 30, TAs: 1, Modality: In-Person. Designed the entire 10-week course.

Math 22: **Introduction to Calculus of Several Variables** **Summer 2021**
Students: 38, TAs: 1, Modality: Remote. Designed the entire 5-week course.

Math 103A: **Complex Analysis** **Winter 2021**
Students: 44, TAs: 1, Modality: Remote. Designed the entire 10-week course.

2.2 Courses as Teaching Assistant

Math 117:	**Advanced Linear Algebra**	**Fall 2024**
Math 160:	**Mathematical Logic**	**Fall 2023**
Math 111B:	**Algebra**	**Spring 2022**
Math 105A:	**Real Analysis**	**Winter 2022**
Math 24:	**Ordinary Differential Equations**	**Summer 2022**
Math 23B:	**Vector Calculus**	**Spring 2022**
Math 23A:	**Vector Calculus**	**Spring 2025** **Spring 2023** **Fall 2022** **Fall 2020** **Summer 2020** **Spring 2020**
Math 22:	**Intro to Calculus of Several Variables**	**Winter 2025**
Math 21:	**Linear Algebra**	**Summer 2023**
Math 19B:	**Calculus for Scientists & Engineers**	**Spring 2024** **Winter 2024** **Winter 2023** **Spring 2021** **Winter 2020** **Fall 2019**

3 Course Design – MATH 22: Intro to Calculus of Several Variables

3.1 Overview of MATH 22

This course serves as the foundational introduction to multivariable calculus for non-math STEM majors at UC Santa Cruz. As the **Instructor of Record** for three iterations of MATH 22: Introduction to Calculus of Several Variables, I was solely responsible for the design of the *entire course*. Rather than relying on a departmental template, I designed and created all course components, based on a student-centered philosophy of *active learning* [6] and *structured inquiry*.

- **Pedagogical Approach:** I utilize the free and open-source *Active Calculus Multivariable* textbook to facilitate an active and inquiry-based learning environment. A significant amount of class time is devoted to collaborative problem-solving rather than extended lectures.
- **Course Webpage:** I maintain a comprehensive course website that serves as the central hub for all materials, including a detailed schedule, daily meeting notes, and assignment archives: MATH 22 Course Webpage (Summer 2025).
- **Syllabus:** The complete syllabus, outlining the grading contract and course policies, is available in Appendix A.

3.2 Cultivating Student Buy-In: "Setting the Stage"

When I am teaching via active or inquiry-based learning methods, it is important to foster student buy-in. My approach is to guide students to conclude *for themselves* that something like active learning is exactly what we should be doing. On the first day of class, I use a modified version of Dana Ernst's "Setting the Stage" presentation [4] which uses interactive questions to guide a discussion centered on the nature of learning itself. We address fundamental questions such as:

- How does a person learn something new?
- What do you reasonably expect to remember from your courses in 20 years?
- What is the value of making mistakes in the learning process?

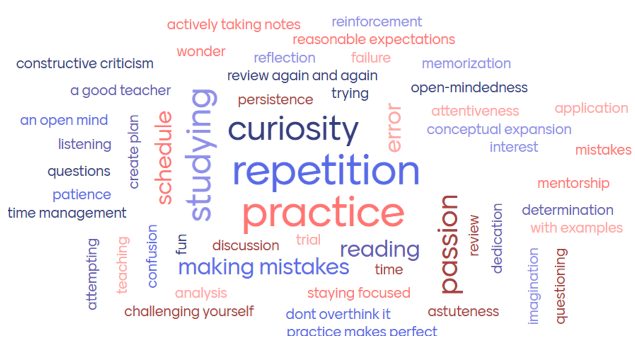

Figure 1: Student responses emphasize *practice* and *mistakes* over innate ability.

When asking the first question, I utilize a word-cloud generator to create a visualization of the students' intuitive beliefs about learning. The results from Summer 2025 are displayed in Figure 1. A larger font size indicates a higher frequency of that response. *Practice* is always one of the most frequent responses, and I use it to spark a conversation about growth mindset [3]. I use this opportunity to challenge the common—and incorrect—belief that mathematical ability is innate. Some version of *making mistakes* usually shows up, and I use this to start a discussion about productive failure and the value of making mistakes in the learning process. To transition from philosophy to evidence, I facilitate a memorization experiment adapted from *The Talent Code* by Daniel Coyle [2]. Students view two lists of word pairs—one complete (e.g., ocean/breeze) and one with missing letters (e.g., bread/b_tter). Then they are instructed to try to recall as many pairs as possible. When the class data reveals that students recall significantly more (on average three times as many according to

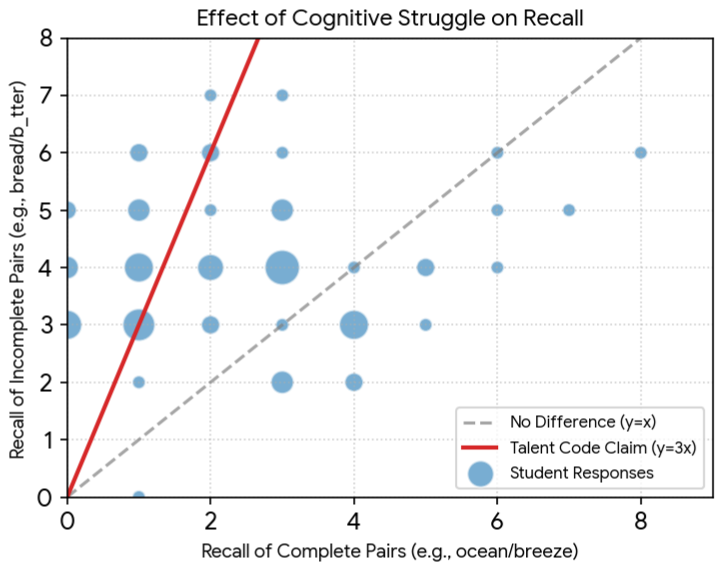

Figure 2: Visual representation of MATH 22 student responses to The Talent Code activity

The Talent Code [2]) pairs from the incomplete list, we discuss how that "microsecond of struggle" is essential for retention. The aggregate data collected from three iterations of this activity in MATH 22 is displayed in the scatter plot in Figure 2. This evidence validates the active-learning approach and allows me to re-frame my role in the classroom as a guide and moderator, emphasizing that students should be (as much as possible) responsible for guiding the acquisition of knowledge and validating the ideas presented.

3.3 Pre-Class: Reading Assignments

A significant amount of class time is reserved for *doing* mathematics. In order to ensure that our meetings are productive, I incentivize students to engage with the material in a meaningful way *before* they walk through the door.

Assignment Description: Reading Assignments are due before every class meeting. Students read specified sections of the text, complete carefully chosen preview activities, and answer reflection questions.

Purpose: Instead of encountering definitions and ideas for the first time in a lecture, students arrive ready to discuss them.

Evaluation: Students receive credit by making a good-faith effort to complete the reading. This low-stakes approach encourages risk-taking and honest self-assessment.

3.3.1 Sample Reading Assignment

See Appendix B for a sample Reading Assignment (Reading Assignment 2 from Summer 2025). A complete archive of Reading Assignments from Summer 2025 is also available for review on my course webpage MATH 22 Reading Assignments (Summer 2025).

3.3.2 Learning Objectives

I include learning objectives within each Reading Assignment in order to provide students with clear goals for the next course meeting and beyond. I organize learning objectives into two distinct categories in order to clarify expectations for student preparation versus mastery. Table 1 provides several examples of learning objectives from Reading Assignment 2 (Appendix B).

Basic Objectives (Pre-Class)	**Advanced Objectives (Post-Class)**
Describe the algebraic relation between the dot product $\mathbf{u} \cdot \mathbf{v}$ and the angle between vectors.	Determine when two vectors are perpendicular and when the angle between them is acute or obtuse.
Compute the cross product of any pair of the standard unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ using the right-hand rule.	State a geometric definition of the cross product $\mathbf{u} \times \mathbf{v}$ by specifying its magnitude and direction (relative to $\mathbf{u}$ and $\mathbf{v}$).
Compute the dot product of two vectors.	Use the dot product to compute the projection $\text{proj}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ onto $\mathbf{v}$ and identify this vector geometrically.

Table 1: Comparison of Basic vs. Advanced Learning Objectives (from Appendix B)

The *Basic Learning Objectives* provide expectations for what students should be able to do *before they arrive* at the next class meeting. These typically focus on definitions and basic computations. On the other hand, the *Advanced Learning Objectives* set expectations for what students should be able to do *after class and with sufficient practice*. These typically focus on conceptual understanding and application, and serve as guidelines for our in-class activities and summative assessments.

3.4 In-Class: Meeting Structure and Daily Assignments

I structure class time to prioritize active student inquiry, using the Daily Assignments as the primary vehicle for engagement.

Assignment Description: Daily Assignments are assigned during every class meeting and typically consist of specific activities from the *Active Calculus Multivariable* textbook, which students complete during class meetings in small groups. Students submit individual write-ups of their work at the end of class.

Purpose: These assignments are designed to facilitate active learning and scaffold the transition between the *Basic* and *Advanced Learning Objectives*. Instead of listening to a lecture, students actively engage in the learning process by doing mathematics in a collaborative and supportive environment.

Evaluation: Students receive credit by making a good-faith effort to complete the activities and contribute to class discussion. The low-stakes nature of the assignments encourages exploration without fear of grade penalties.

A typical class meeting follows a cycle designed to scaffold the activities and provide opportunity for collaboration. We begin with a reading debrief to address questions and confusion from the pre-class Reading Assignment. Then we transition into a brief mini-lecture where I clarify high-level concepts and connect the reading to the day's topics and learning objectives. For instance, in Reading Assignment 2 (Appendix B) students learned how the cross product is defined for pairs of standard unit vectors. My corresponding mini-lecture explains how this extends to arbitrary vectors and culminates in the formula for the cross product in terms of determinants, which directly aligns with Advanced Learning Objective 5 from Reading 2 (Appendix B). Then students work on an associated activity such as Activity 9.4.2 from the *Active Calculus Multivariable* book [7].

3.4.1 Sample Activity

This is Activity 9.4.2 from *Active Calculus Multivariable*.

> Suppose $\mathbf{u} = \langle 0, 1, 3 \rangle$ and $\mathbf{v} = \langle 2, -1, 0 \rangle$.
>
> a. Find the cross product $\mathbf{u} \times \mathbf{v}$.
>
> b. Evaluate the dot products $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$ and $\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v})$. What does this tell you about the geometric relationship among $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} \times \mathbf{v}$?
>
> c. Find the cross product $\mathbf{v} \times \mathbf{i}$.
>
> d. Multiplication of real numbers is associative, which means, for instance, that $(2 \cdot 5) \cdot 3 = 2 \cdot (5 \cdot 3)$. Is it true that the cross product of vectors is associative? For instance, is it true that $(\mathbf{u} \times \mathbf{v}) \times \mathbf{i} = \mathbf{u} \times (\mathbf{v} \times \mathbf{i})$?

This activity was chosen because it scaffolds Advanced Learning Objective 5 from Reading Assignment 2 and builds towards Advanced Learning Objective 6. Following the group work, we reconvene for a class discussion to synthesize the results, such as establishing the orthogonality relation between $\mathbf{u}$ and $\mathbf{u} \times \mathbf{v}$ alluded to in part (b) of Activity 9.4.2. Then we rinse and repeat.

3.4.2 Archive of Daily Assignments & Meeting Notes

I take detailed notes during all course meetings. These notes reflect the structure of the meeting, contain mini-lectures, and contain summaries of class discussions for all activities. This is intended to be a resource for students so that they can spend class time *thinking* rather than copying things down. The meeting notes from Summer 2025 are available for review on my course webpage: MATH 22 Daily Assignments & Notes (Summer 2025).

3.5 Post-Class: Practice via Edfinity

To reinforce the concepts explored in class, students engage in independent practice using the Edfinity online homework platform.

Assignment Description: Students complete problem sets corresponding to each course section. These problems are algorithmically generated to ensure individual practice while allowing for collaboration.

Purpose: While in-class work focuses on conceptual understanding and discovery, post-class work focuses on fluency and computational proficiency. This ensures students have the skills necessary to support their conceptual knowledge. Edfinity was chosen as it provides immediate feedback and is a low-cost alternative to other similar homework systems.

Evaluation: Assignments are graded for accuracy, but students are given multiple attempts to encourage persistence and mastery-oriented learning.

3.6 Summative Assessments

3.6.1 Sample Weekly Assignment

The Weekly Assignments serve as high-stakes summative assessment. They primarily assess conceptual understanding, mathematical reasoning, and written communication skills. A sample Weekly Assignment from Summer 2025 can be found in Appendix C. These assignments consist of several multi-step problems. Students are evaluated based on the following rubric from [5] which can be found in Table 2.

Score	Criteria
4	This is correct and well-written mathematics!
3	This is a good piece of work, yet there are some mathematical errors or some writing errors that need addressing.
2	There is some good intuition here, but there is at least one serious flaw.
1	I don't understand this, but I see that you have worked on it; come see me!
0	I believe that you have not worked on this problem enough or you didn't submit any work.

Table 2: Weekly Assignment grading rubric.

3.6.2 Sample Exam The course includes one midterm exam and one comprehensive final exam. A sample midterm from Summer 2025 can be found in Appendix D.

3.6.3 Constructive Alignment I design my summative assessments using a framework of *constructive alignment* [1]. My goal is to ensure that high-stakes assessments prioritize the Advanced Learning Objectives shared with the students (Appendix B). Table 3 demonstrates specific examples of alignment between my summative assessment samples (Appendix C & D) and the Advanced Learning Objectives from Reading Assignment 2.

Advanced Learning Objective from Appendix B	Assessment Source	Specific Problem Content
Objective 2: Determine when two vectors are perpendicular.	Weekly Assignment 1 (Appendix C)	**Problem 2(b):** "Find a unit vector **u** in $\mathbb{R}^2$ such that **u** is perpendicular to **v**. How many such vectors are there?"
Objective 7: Utilize vectors to compute areas of triangles and parallelograms.	Weekly Assignment 1 (Appendix C)	**Problem 3(b):** "Observe that the area of triangle PQR is half of the area of the parallelogram formed by PQ and PR. Hence find the area of triangle PQR."
Objective 6: State a geometric definition of the cross product specifying its magnitude and direction.	Midterm Exam (Appendix D)	**Question 5(d):** "Find a vector that is perpendicular to both ℓ_1 and ℓ_2." *(Requires synthesis of the geometric definition of the cross product).*
Objective 7: Utilize vectors to compute... volumes of tetrahedrons and parallelepipeds.	Midterm Exam (Appendix D)	**Question 3(d):** "Do the vectors **u**, **v**, and **w** lie in a common plane?" *(Requires computation of a volume using vectors).*

Table 3: Constructive Alignment of Advanced Objectives with Assessments

4 Evidence of Effective Teaching and Student Learning

4.1 Quantitative Evaluations

Table 4 presents a comparative analysis of my Student Experience of Teaching Survey (SETS) data for my MATH 22 (Active Learning) course against a benchmark which includes traditional lecture-based courses. The comparison of results allows for an assessment of how pedagogical choices and environmental factors affect student outcomes. The accompanying self-reflections demonstrate my commitment to reflective practice by linking quantitative results to specific strengths or challenges.

Context for Student Experience of Teaching Survey Data

Benchmark: The All Courses benchmark aggregates data from a total of 153 students across all seven of my courses, including my traditional lecture-based courses (MATH 3, 103A, 117). The benchmark for the questions "Used class time effectively to support student learning" and "In-class activities were well structured and had clear goals" has a significantly lower sample size because these questions were added to the SETS Survey in 2023.

MATH 22: This course was designed around an active-learning curriculum (detailed in Section 3). The data allows for a direct comparison of this method across three distinct environments:

- **2021:** Taught 35 students remotely via Zoom. Group work was handled using breakout rooms.
- **2024:** Taught 15 students in a small (capacity: 23) seminar-style classroom with movable tables, ideal for group work and class discussion.
- **2025:** Taught 18 students in a large (capacity: 58) lecture hall with individual desks. The large room introduced physical distance between students and was not conducive to the active learning model.

Data Validity: I regularly communicate to my students about the value of their feedback and how I use it to improve my teaching practices. Consequently, I maintain an average SETS response rate of approximately 90% across all courses for which I was the instructor of record. This exceptionally high participation rate ensures that my SETS data accurately reflects the collective student experience in my classes.

Table Key: The letter n refers to the number of student respondents. Percentages displayed represent the ratio of responses of the form "Very Frequently" and "Frequently" (% VF + F) to the total number of responses n. Responses marked "Unable to comment" were excluded to accurately reflect the views of students who engaged with that specific aspect of the course. The symbol "N/A" indicates that the specific prompt was not included in the SETS survey for that course.

Raw SETS data for any of my courses is available by request.

Student Experience of Teaching Survey Prompts	**MATH 22 (2021) % VF + F n=33, 94% rate**	**MATH 22 (2024) % VF + F n=15, 88% rate**	**MATH 22 (2025) % VF + F n=18, 86% rate**	**All Courses Benchmark % VF + F n=153, 90% rate**
Used class time effectively to support student learning	N/A	80%	67%	72%
Reflection: Benchmark ($n = 48$) includes one lectured-based course. The high score in 2024 (small room) and the low score in 2025 (large room) relative to the benchmark reflect the physical environment as a variable that needs to be taken into account when planning class meetings.				
The instructor explained concepts in ways that supported my learning.	67%	80%	50%	64%
Reflection: Relative to the benchmark, active-learning offers a much higher ceiling for clarity (80% in 2024) but carries greater risk. The 2025 score (50%) shows that my active-learning format can become less clear than a standard lecture when external factors (like the large room) conflict with my pedagogy.				
The instructor provided useful feedback on my assigned work	63%	67%	50%	61%
Reflection: This metric is complicated by the fact that a significant portion of feedback was communicated by TAs or Readers. To improve the low score (50%) in the future, I plan to better communicate grading expectations to ensure that TA feedback aligns more closely with my own standards.				
The instructor clearly communicated how assignments would be evaluated.	91%	93%	83%	83%
Reflection: My consistently high scores in this metric demonstrate my commitment to maintaining transparency in grading and validate the effectiveness of my grading rubric and syllabus design (Appendix A).				
The instructor helped me feel engaged with the course.	64%	93%	61%	60%
Reflection: The benchmark (60%) reflects the passive nature of traditional lectures. In contrast, my active learning curriculum consistently outperforms this baseline, even via Zoom (2021) or in a large lecture hall (2025). The surge to 93% in 2024 supports my observation that the small seminar-style classroom is more conducive to the active-learning format.				
Lectures were well structured and had clear goals.	79%	77%	77%	75%
Reflection: My consistently high scores here demonstrates my ability to provide clear instruction, independent of modality.				
In-class activities were well structured and had clear goals	N/A	87%	83%	81%
Reflection: The benchmark ($n = 43$) includes my lecture-based MATH 117 course from 2023, which scored lower on this metric. The consistent high scores for MATH 22 validates that my active-learning modules are more effective than standard lectures.				
Problem sets and homework helped me feel prepared for examinations	61%	73%	71%	60%
Reflection: The upward trend suggests that my implementation of Constructive Alignment (Section 3.6.3) is succeeding.				

Table 4: Comparative analysis of SETS data for three years of MATH 22 (active learning) versus the All-Courses Benchmark, with self-reflections.

4.2 Qualitative Evaluations

Students' written feedback from the SETS surveys consistently highlights the effectiveness of the active-learning format in MATH 22, as well as my commitment to clarity and student success across all my courses.

Evidence of Effectiveness of Active Learning in MATH 22 Students consistently identify specific active-learning design elements of the course as primary drivers of their success.

> "I thought this made the course very engaging because we were actively participating and able to solve problems to learn the material instead of just listening to a lecture."
>
> *MATH 22 student, Summer 2025*

> "I also liked the reading assignments because they made me think about the concepts before class while being low stakes... I could just focus on understanding it and making mistakes."
>
> *MATH 22 student, Summer 2025*

> "I think the structure is good because we are able to learn through doing and not just listening to a lecture for hours on end."
>
> *MATH 22 student, Summer 2021*

Evidence of Clarity, Organization, and Effective Lecturing The following comments support my effective teaching in my upper-division and lecture-based courses.

> "I liked how Jadyn explained the abstract topics in a very clear and understandable way during the lectures, and he wrote up very helpful detailed notes that us students could easily refer back to."
>
> *MATH 117 (Advanced Linear Algebra) student, Summer 2023*

> "The lecture notes were incredibly organised [sic] and the problem sets were immensely useful and interesting."
>
> *MATH 103A (Complex Analysis) student, Winter 2021*

> "Great class overall. I learned so much. I feel I can confidently say what the universal property is, what a tensor is, what a free vector space is, what a dual space is, and much more. This is a class I can feel confident about when I move on."
>
> *MATH 117 (Advanced Linear Algebra) student, Summer 2023*

> "The lectures were very helpful, they followed the textbook but had much better examples. None were rushed or poorly structured."
>
> *MATH 3 (Precalculus) student, Fall 2021*

Evidence of Fostering a Supportive Learning Environment The following comments support my commitment to fostering a supportive and inclusive learning environment.

> "I liked the whole vibe of this class and how it was kinda like a high school class where it was small and personal and easier to make friends."
>
> *MATH 22 student, Summer 2025*

> "10/10 professor very engaged and cared for his students."
>
> *MATH 22 student, Summer 2024*

> "I really liked the professor... he got to know all of our names in the beginning which was cool and made the class feel more personal."
>
> *MATH 22 student, Summer 2025*

> "I could tell you were invested in my success."
>
> *MATH 103A (Complex Analysis) student, Winter 2021*

Appendix A MATH 22 Syllabus (Summer 2025)

MATH 22 (Summer 2025) Syllabus

Welcome to the Summer '25 manifestation of MATH 22 at UC Santa Cruz! This syllabus contains important information about the course. If you are a student, I highly recommend you read the document in it's entirety. Additional information about the course can be found on my webpage.

Basic Course Information

Instructor: Jadyn V. Breland (he/him/his.) **Office Hours:** Tu/Th 4:45-5:45PM in McHenry 4117.

Email: jbreland@ucsc.edu **Personal Webpage:** http://jadynbreland.com

Prerequisites: MATH 11B or MATH 19B or MATH 20B or AM 15B or AP calculus BC exam score of 4 or 5.

Meetings: M/W/F 5:00-7:15PM in Cowell 131. Attendance is factored into your grade via the Daily Assignment.

Textbook: *Active Calculus - Multivariable*, Steve Schlicker. In the spirit of reducing the cost of your education, I have chosen to use this free and open source textbook. You can download the textbook for free or view the .html version. This textbook is by no means traditional: as the title suggests, the student is expected to actively engage with the textbook. There are very few worked examples in the texts, with there instead being 3-4 activities per section that engage students in connecting ideas, solving problems, and developing understanding of key calculus concepts. Everyone will be expected to read the textbook and digest the material in a meaningful way, outside of class. Class meetings will typically be reserved for discussing key ideas and completing the activities, either individually or in groups.

Course Webpage: The course web page is located at https://people.ucsc.edu/~jbreland/teaching/SM25_MATH22.html. Most of the course content will be posted here, including: this syllabus, assignments, due dates, notes, and other resources.

Edfinity: We will use the online homework management system edfinity. You are required to enroll in our course at the following link: https://edfinity.com/join/K3LFRUY6. The cost is $35 per student.

Canvas: The Canvas webpage will be used for hosting grades and making announcements.

Discussion Sections: Tu/Th 3:30-4:30PM in Physical Sciences 130. Sections begin the first week of class.

TA: Joseph Immel **TA e-mail:** jhimmel@ucsc.edu **TA Office Hours:** TBD

Study Hall Tutoring: LSS is supporting our course through the Math Study Hall Program. You can sign-up for tutoring sessions on TutorHub.

Tutors: Kate Wang (she/her) e-mail: kafwang@ucsc.edu

Megan Tallcott (they/she) e-mail: mtallcot@ucsc.edu

Accessibility: I am strongly committed to making my course as accessible as possible. If you encounter materials that are not accessible to you, or experience a barrier to your participation, please bring this to my attention and I will gladly work with you to ensure accessibility. I am also happy to honor any accommodations letters from the Disability Resource Center (DRC) that you would like to confidentially bring to my attention.

Course Content: Functions of several variables. Continuity and partial derivatives. The chain rule, gradient and directional derivative. Maxima and minima, including Lagrange multipliers. The double and triple integral and change of variables. Surface area and volumes. Applications from biology, chemistry, earth sciences, engineering, and physics.

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Learning Outcomes: Upon successful completion of the course, students will be able to do the following within the topic of multivariable calculus:

1. Recall the basic definitions, theorems, and techniques of multivariable calculus.
2. Distinguish truth from falsehood and create examples and counterexamples.
3. Competently and confidently solve a variety of problems that require techniques from multivariable calculus.
4. Communicate mathematical ideas and arguments in clear, convincing, and concise language, both written and oral.

In addition to mastering course content, students will further develop as independent, self-directed learners with the confidence to explore unfamiliar problems and ideas.

Assessment

Assessment Distribution: Your final score in the course will be calculated as the weighted average of the following assessments.

- READING ASSIGNMENTS (15%)

  There will be 12 Reading assignments. Reading assignments will be assigned shortly after each class meeting and will be due before the next class meeting. Reading assignments and due dates will always be posted on the course website.

  Generally, you will read assigned sections of the textbook, complete the assigned preview activities, and write a brief summary of things that you learned or still have questions about. I may add additional tasks as I see fit. The work to be turned in will be a write-up of the assigned tasks, submitted via gradescope.

  For each reading assignment, you will either receive 1 point (a "Pass") or 0 points (a "No Pass"). You will receive 1 point if you:

  – submit your assignment on time;
  – attempt all assigned activities; and
  – make a good faith effort to complete each activity correctly.

  Otherwise, you will receive 0 points. Your lowest 2 scores will be dropped.

  If you do not complete the reading prior to class, you will be unprepared to discuss the activities during class meetings. This will severely limit your ability to learn the material.

- DAILY ASSIGNMENTS (15%)

  There will be 13 Daily assignments, which are to be completed during class meetings. You will complete assigned activities from the textbook during class meetings, usually in small groups. You will submit (individually) a write-up of the activities completed in class via gradescope. The deadline to submit is 11:59PM the same day.

  For each daily assignment, you will either receive 1 point (a "Pass") or 0 points (a "No Pass"). You will receive 1 point if you:

  – attend the class meeting;
  – actively participate in discussions;
  – attempt all assigned activities; and

– make a good faith effort to complete each activity correctly.

Otherwise, you will receive 0 points. Your lowest 3 scores will be dropped.

- **Weekly Assignments (25%)**

  There will be 4 weekly assignments. Weekly assignments and due dates will always be posted on the course website.

  Weekly assignments are writing assignments which will typically consist of solving a few challenging problems and carefully writing up a detailed solution. Each problem is graded out of 4 points. Your score for each problem is determined by the grader using the Weekly Assignment Rubric.

- **Edfinity Exercises (15%)**

  Excercises from Edfinity will be assigned each time we cover a section of the text and will typically be due a week later. See Edfinity for due dates.

- **Midterm (15%)**

  The midterm exam will happen during class on **Wednesday, August 13**.

- **Final Exam (15%)**

  The final exam will happen during class on **Friday, August 29**. The final exam is *not* cumulative and won't directly test material from the midterm, but since calculus builds on itself, a strong grasp of earlier topics is essential for success.

- **Discussion Sections (5%)**

  There will be 10 discussion sections held by our TA. You will receive 1 point for each section you attend. During the section, the activities from the most recent reading assignment will be discussed. I recommend that you work on the reading assignment prior to the section.

Submitting Assignments: All Reading, Daily, and Weekly assignments must be submitted via gradescope. When you submit your files, you will be prompted to select, for each specified problem or activity, the pages on which the associated work/solution are located. You are required to accurately identify the pages associated to each problem. If you fail to do so, you may receive a "No Pass" (if it is a daily or reading assignment) or you may receive no credit for each problem for which the pages are not correctly identified (if it is a weekly assignment).

It is your responsibility to make sure your submission is legible and easy to read. If you submit work that is difficult or impossible to read, you will not receive credit for it, and you will not be allowed to resubmit. There are numerous free smart phone apps that allow you scan your work and save it as a `.pdf`.

Use of Generative AI: Use of generative AI (such as ChatGPT or Gemini) is permitted as a learning and study aid in this course, provided that their use aligns with the academic integrity policies at UCSC. All work submitted for grading must be your own original work. Any AI use on graded assignments must be documented and properly cited.

Generative AI programs create responses to prompts by predicting what text comes next based on patterns in its training data. It does not understand math or reason logically. This means it can produce convincing "answers" that are completely wrong, especially in technical subjects like mathematics.

Students must be aware that generative AI programs frequently generate misinformation and are generally unreliable for mathematical problem solving.

MATH 22 (Summer 2025) Syllabus

Weekly Assignment Guidelines: (See also: Weekly Assignment Webpage)

- Discussing the problems with your classmates is allowed and *encouraged*.
- You are **NOT** allowed to copy someone else's work.
- You are **NOT** allowed to let someone else copy your work.
- I expect your submissions to be well-written, neat, and organized. Do not turn in rough drafts or scratch work. What you turn in should be the "polished" version of potentially several drafts.
- Pay close attention to the presentation and clarity of your reasoning in your answers. The ability to communicate effectively is just as important as solving a problem correctly.
- You may freely consult the textbook or any notes from our class meetings. However, you are forbidden from consulting any other resources, including, but not limited to, other textbooks, the internet, Chegg, and math.stackexchange.

Exams: Exams are timed tests which will be administered during class. You will have the entire time to work on the exam.

- **Note sheets:** I do not expect you to memorize all the material. On each exam, you are allowed to use one two-sided 8.5in by 11in sheet of notes. There is no restriction on what you may write on your note sheet - you might include examples, definitions, theorems, or whatever else seems important to you.
- **Missed Exams:** If you miss an exam, you will receive a zero. Make-up exams will not be administered except in extreme circumstances, as determined by the instructor. Extreme circumstances must be adequately documented and confidentially brought to my attention as soon as possible.

Late Work Policy: I will not, under any circumstance, accept late submissions for reading assignments, daily assignments, or Edfinity exercises. Late submissions of weekly assignments and exams are only accepted, at my sole discretion, in extreme circumstances. Extreme circumstances must be adequately documented and confidentially brought to my attention as soon as possible.

Letter Grades: Your final letter grade depends on your score. Final letter grades are assigned according to the following score ranges:

A+	96-100	B+	86-89	C+	76-79	D+	66-69	F	0-59
A	93-95	B	83-85	C	73-75	D	63-65		
A-	90-92	B-	80-82	C-	70-72	D-	60-62		

Score ranges may be adjusted (to your advantage) according to class performance. Scores falling in between two ranges will be rounded up. For example, according to the ranges above a final score of 75.1 will earn the letter grade C+ (rounded up), whereas a final score of 74.9 will earn the letter grade C (no rounding).

P/NP Grading: A passing grade (P) will be awarded if your score would earn a letter grade of C or higher. Otherwise, you will not receive a passing grade (NP). **Warning**: a score earning the letter grade of C- at UCSC is NOT passing, contrary to popular belief.

I RESERVE THE RIGHT TO CHANGE ANY PART OF THE SYLLABUS ABOVE.

YOU WILL BE PROMPTLY NOTIFIED OF ANY CHANGES VIA EMAIL.

Other Important Information

Summer Session Calendar:
https://summer.ucsc.edu/summer-edge/dates-deadlines/

Mathematics Department's Enrollment Info:
https://www.math.ucsc.edu/courses/enrollment-info.html

DRC Accommodations: UC Santa Cruz is committed to creating an academic environment that supports its diverse student body. If you are a student with a disability who requires accommodations to achieve equal access in this course, please affiliate with the DRC. I encourage all students to benefit from learning more about DRC services to contact DRC by phone at 831-459-2089 or by email at drc@ucsc.edu. For students already affiliated, make sure that you have requested Academic Access Letters, where you intend to use accommodations. You can also request to meet privately with me during my office hours or by appointment, as soon as possible. I would like us to discuss how we can implement your accommodations in this course to ensure your access and full engagement in this course.

CAPS (Counseling and Psychological Services): This is a stressful time, so if you are in distress, managing heightened stress and anxiety, or want to get more support and a counselor's perspective on something you're going through, CAPS provides a variety of services for your needs, please visit their website for more information https://caps.ucsc.edu.

Academic Integrity: Academic integrity is the cornerstone of a university education. Academic dishonesty diminishes the university as an institution and all members of the university community. It tarnishes the value of a UCSC degree. All members of the UCSC community have an explicit responsibility to foster an environment of trust, honesty, fairness, respect, and responsibility. All members of the university community are expected to present as their original work only that which is truly their own. All members of the community are expected to report observed instances of cheating, plagiarism, and other forms of academic dishonesty in order to ensure that the integrity of scholarship is valued and preserved at UCSC. For the full policy and disciplinary procedures on academic dishonesty, students and instructors should refer to the Academic Integrity page at the Division of Undergraduate Education.

Title IX: The Title IX Office is committed to fostering a campus climate in which members of our community are protected from all forms of sex discrimination, including sexual harassment, sexual violence, and gender-based harassment and discrimination. Title IX is a neutral office committed to safety, fairness, trauma-informed practices, and due process. Title IX prohibits gender discrimination, including sexual harassment, domestic and dating violence, sexual assault, and stalking. If you have experienced sexual harassment or sexual violence, you can receive confidential support and advocacy at the Campus Advocacy Resources & Education (CARE) Office by calling (831) 502-2273. In addition, Counseling & Psychological Services (CAPS) can provide confidential, counseling support, (831) 459-2628. You can also report gender discrimination directly to the University's Title IX Office, (831) 459-2462. Reports to law enforcement can be made to UCPD, (831) 459-2231 ext. 1. For emergencies call 911.

Appendix B MATH 22 Reading Assignment (Summer 2025)

Reading Assignment 2 (Due Friday 8/1 by 5PM)

Directions: Read the following sections of the book:

- Review Section 9.2.3, Section 9.2.4, Section 9.2.5, and Section 9.2.6 if necessary (we covered the main ideas during class).
- Section 9.3.1
- Section 9.3.2
- The first paragraph of Section 9.4. You can read further if desired.

and complete the following tasks along the way. If an Activity is not listed, you do not need to complete it (although you are welcome to read it). Turn your write up in via gradescope. You do not need to write the questions down, as long as you clearly indicate the question number.

1. Complete Preview Activity 9.3.1.
2. Complete Activity 9.3.2.
3. Click here to view the vectors from Activity 9.3.2.a using GeoGebra. Use Equation 9.3.1 to compute the angle (in radians) between them.
4. Click here to open a GeoGebra applet. Read the instructions and play around with the app for a few minutes. Do you notice anything (or several things) that seem interesting? Do you notice any patterns? Describe your observations.
5. After reading Section 9.3.1 and 9.3.2, write down three things that you learned or that you still have questions about.
6. Complete Preview Activity 9.4.1

Basic learning objectives: These are the things you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated in italics.

1. Compute the *dot product* of two vectors.
2. Describe the algebraic relation between the dot product $\mathbf{u} \cdot \mathbf{u}$ of a vector $\mathbf{u}$ with itself and the magnitude $|\mathbf{u}|$ of the vector.
3. Describe the algebraic relation between the dot product $\mathbf{u} \cdot \mathbf{v}$ of two vectors $\mathbf{u}$ and $\mathbf{v}$ and the *angle between them*.
4. Compute the *cross product* of any pair of the standard unit vectors i, j, k using the right-hand rule.
5. Use properties of the cross product to compute cross products of linear combinations of standard unit vectors.

Advanced learning objectives: In addition to mastering the basic objectives, here are the things you should be able to perform **after class, with sufficient practice:**

1. Utilize the properties of the dot product to perform more advanced computations.

2. Determine when two vectors are perpendicular and when the angle between them is acute or obtuse.
3. Represent a force using a vector and calculate the work required to displace an object using that force.
4. Use the dot product to compute the projection $\text{proj}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ onto $\mathbf{v}$ and identify this vector geometrically.
5. Compute cross products using determinants.
6. State a *geometric* definition of the cross product $\mathbf{u}\times\mathbf{v}$ by specifying its magnitude and direction (relative to $\mathbf{u}$ and $\mathbf{v}$). Compute the magnitude of the cross product algebraically.
7. Utilize vectors, the dot product, and the cross product to compute areas of triangles and parallelograms, and volumes of tetrahedrons and parallelepipeds. Understand why the cross product is related to areas and volumes. Describe various similarities and differences between the dot product and cross product.

Appendix C MATH 22 Weekly Assignment (Summer 2025)

Weekly Assignment 1 (Due Friday 8/8 at 11:59PM)

Overview: This assignment is worth **56 points**. Each question has multiple parts and each part is worth **4 points**. The grader determines your score for each part of each problem using the Weekly Assignment Rubric. Specifically, the grader will be looking for evidence of conceptual understanding, correct mathematical reasoning, and excellent written-communication.

Guidelines: You are required to adhere to the Weekly assignment guidelines and the Use of Generative AI policy, which can be found on pages 3 and 4 of the syllabus. Turn in your assignment via gradescope.

Directions: Complete the following exercises from the Active Calculus textbook and write a detailed solution. You can click the links below to go directly to the exercise.

1. (20 points) Exercise 9.1.15

The Ideal Gas Law, $PV = RT$, relates the pressure (P, in pascals), temperature (T, in Kelvin), and volume (V, in cubic meters) of 1 mole of a gas ($R = 8.314\frac{\text{J}}{\text{mol K}}$ is the universal gas constant), and describes the behavior of gases that do not liquefy easily, such as oxygen and hydrogen. We can solve the ideal gas law for the volume and hence treat the volume as a function of the pressure and temperature:

$$V(P,T) = \frac{8.314T}{P}.$$

(a) Explain in detail what the trace of V with $P = 1000$ tells us about a key relationship between two quantities.

(b) Explain in detail what the trace of V with $T = 5$ tells us.

(c) Explain in detail what the level curve $V = 0.5$ tells us.

(d) Use 2 or three additional traces in each direction to make a rough sketch of the surface over the domain of V where P and T are each nonnegative. Write at least one sentence that describes the way the surface looks.

(e) Based on all your work above, write a couple of sentences that describe the effects that temperature and pressure have on volume.

2. (20 points) Exercise 9.3.11

Let $\mathbf{v} = \langle -2, 5\rangle$ in $\mathbb{R}^2$, and let $\mathbf{y} = \langle 0, 3, -2\rangle$ in $\mathbb{R}^3$.

(a) Is $\langle 2, -1\rangle$ perpendicular to $\mathbf{v}$? Why or why not?

(b) Find a unit vector $\mathbf{u}$ in $\mathbb{R}^2$ such that $\mathbf{u}$ is perpendicular to $\mathbf{v}$. How many such vectors are there? Justify your answers.

(c) Is $\langle 2, -1, -2\rangle$ perpendicular to $\mathbf{y}$? Why or why not?

(d) Find a unit vector $\mathbf{w}$ in $\mathbb{R}^3$ such that $\mathbf{w}$ is perpendicular to $\mathbf{y}$. How many such vectors are there? Justify your answers.

(e) Let $\mathbf{z} = \langle 2, 1, 0\rangle$. Find a unit vector $\mathbf{r}$ in $\mathbb{R}^3$ such that $\mathbf{r}$ is perpendicular to both $\mathbf{y}$ and $\mathbf{z}$. How many such vectors are there? Explain your process.

3. (16 points) Exercise 9.4.13

Consider the triangle in $\mathbb{R}^3$ formed by $P(3, 2, -1)$, $Q(1, -2, 4)$, and $R(4, 4, 0)$.

(a) Find $\vec{PQ}$ and $\vec{PR}$.

(b) Observe that the area of $\triangle PQR$ is half of the area of the parallelogram formed by $\vec{PQ}$ and $\vec{PR}$. Hence find the area of $\triangle PQR$.

(c) Find a unit vector that is orthogonal to the plane that contains points P, Q, and R.

(d) Determine the measure of $\angle PQR$.

Appendix D MATH 22 Midterm (Summer 2025)

Midterm Exam

MATH 22: Calculus of Several Variables

Summer 2025

Overview: This exam consists of 1 cover page and **5 pages of questions**, for a total of 6 pages. There are **8 questions** worth a total of **55 points**. Each question has multiple parts. Your grade on this exam will be calculated as

(**number of points earned**)/50.

This exam is worth **15% of your final grade** in the course.

Directions:

- **Write your NAME and STUDENT ID at the top of each page.**
- Solve as many of the following problems as you can. Try to start with problems that you know how to solve right away, and save the others for last.
- Provide justification and show your work whenever possible. This makes it easier to give you partial credit. I want to give you partial credit!
- Write the work and the answer that you want graded in the provided answer box.
- You may use one two-sided 8.5inch by 11inch sheet of notes. No other resources are allowed. You do not need a calculator.

Good luck, and have fun!

Name:	**Student ID:**

Midterm Exam Problems

1. **(9 points)** Determine whether the following statements are **TRUE** or **FALSE**. No justification is required: if you don't know, just guess! Each question is worth **1 point**.

   (a) Two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are parallel if and only if $\mathbf{n}_1 \cdot \mathbf{n}_2 = 0$.

   Answer:

   (b) For all vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{v} \times \mathbf{u}|$.

   Answer:

   (c) If all the level curves of a function $f(x, y)$ are parallel lines, then the graph of $f(x, y)$ is a plane.

   Answer:

   (d) The slope of the tangent line to the $y = b$ trace of a function $f(x, y)$ at the point $(a, b, f(a, b))$ is equal to $\lim_{h \to 0} \frac{f(a,b+h)-f(a,b)}{h}$.

   Answer:

   (e) Any two planes which are perpendicular to the same line are parallel.

   Answer:

   (f) If $\overrightarrow{OP} \cdot (\overrightarrow{OQ} \times \overrightarrow{OR}) = 0$, then the three points $P, Q, R \in \mathbb{R}^3$ are collinear (lie on the same line).

   Answer:

   (g) If $\mathbf{r}(t)$ is a vector-valued function, the $\frac{d}{dt}|\mathbf{r}(t)| = |\mathbf{r}'(t)|$.

   Answer:

   (h) If $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$ satisfy $|\mathbf{u} \times \mathbf{v}| = 0$, then either $\mathbf{u} = 0$ or $\mathbf{v} = 0$.

   Answer:

   (i) If $f_x(x, y)$ and $f_y(x, y)$ are constant functions, then the graph of $f(x, y)$ is a plane.

   Answer:

2. **(4 points)** Let $P_0 = (x_0, y_0)$ be a point in $\mathbb{R}^2$ and let $\mathbf{n} = \langle a, b \rangle$ be a vector in $\mathbb{R}^2$. Answer the following questions, each of which is worth **2 points**.

   (a) What does the collection of points $P = (x, y)$ which satisfy the equation $\mathbf{n} \cdot \overrightarrow{P_0P} = 0$ look like geometrically? Draw a picture and explain your reasoning. (Hint: we have seen this equation in 3-dimensions.)

   (b) What does the collection of points $P = (x, y)$ which satisfy the inequality $\mathbf{n} \cdot \overrightarrow{P_0P} > 0$ look like geometrically? How is it related to part (a)? Draw a picture and explain your reasoning.

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Name:	**Student ID:**

3. (8 points) Answer the following questions, each of which is worth **2 points**. *Briefly* justify your answer.

(a) Are the vectors $\mathbf{u} = \langle -1, 2, -12, -4 \rangle$ and $\mathbf{v} = \langle 0, 2, 1, -2 \rangle$ perpendicular?

(b) Does the line $\mathbf{r}(t) = \langle 2t - 1, -t + 1, -t - 1 \rangle$ intersect the plane $x + y + z = 1$?

(c) Do the planes $x + 7y + z = 2$ and $-2x - 2y - 2z = 4$ intersect?

(d) Do the vectors $\mathbf{u} = \langle 1, -1, 3 \rangle$, $\mathbf{v} = \langle 1, -1, 1 \rangle$, and $\mathbf{w} = \langle 2, -2, 4 \rangle$ lie in a common plane?

4. (6 points) Consider the function $f(x, y, z) = x \sin(z^2 e^{y+z})$. Complete the following, each of which is worth **2 points**.

(a) Compute $f_x(x, y, z)$.

(b) Compute $f_y(x, y, z)$.

(c) Compute $f_z(x, y, z)$.

Name: | **Student ID:**

5. **(10 points)** lines $\ell_1(t) = \langle t, t, 1+t \rangle$ and $\ell_2(t) = \langle 2-t, t, 3-t \rangle$. Complete the following tasks.

(a) **(3 points)** Show that ℓ_1 and ℓ_2 intersect and find the point of intersection.

(b) **(1 point)** Show that ℓ_1 and ℓ_2 are not parallel.

(c) **(1 point)** Draw a picture of the situation and use it to explain why there is a *unique* plane containing ℓ_1 and ℓ_2.

(d) **(3 points)** Find a vector that is perpendicular to both ℓ_1 and ℓ_2.

(e) **(2 points)** Find a scalar equation for the plane that contains both lines ℓ_1 and ℓ_2.

Name:	**Student ID:**

6. (5 points) The position of particle in space at time t is given by $\mathbf{r}(t) = \langle \sin(t), \cos(t), t^2 \rangle$.

(a) **(1 points)** Compute $\mathbf{r}(\pi)$.

(b) **(2 points)** Compute $\mathbf{r}'(\pi)$ and explain what it means in context.

(c) **(2 points)** Find a vector-valued function that parametrizes the line that is tangent to the curve $\mathbf{r}(t)$ when $t = \pi$.

7. (7 points) The table below gives the dissolved oxygen concentration $O(T, d)$ in a lake (in milligrams per liter, mg/L) as a function of the water temperature T (in °C) and the depth d (in meters m).

T (°C) \ d (m)	**0**	**5**	**10**	**15**	**20**
5	12.8	12.6	12.5	12.4	12.3
10	11.3	11.1	10.9	10.8	10.6
15	10.1	9.8	9.5	9.3	9.1
20	9.1	8.7	8.4	8.1	7.8
25	8.3	7.8	7.4	7.0	6.6

For example, $O(10, 5) = 11.1$ mg/L. Answer the following questions.

(a) (4 points) Estimate the partial derivative $O_T(15, 10)$. Make sure to include the units.

(b) (3 points) Explain the meaning of the partial derivative that you computed in part (a) in the context of the problem. You can answer this part of the problem even if you don't know how to estimate $O_T(15, 10)$.

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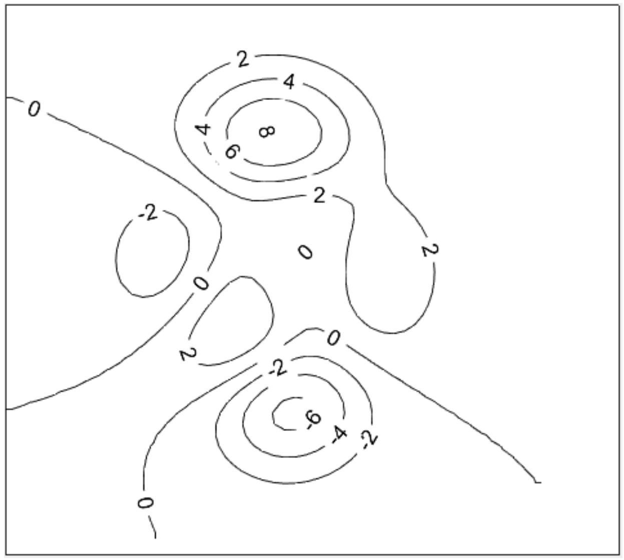
Name:	**Student ID:**

8. (6 points) Consider the following contour diagram of a continuously differentiable function $f(x, y)$:



Complete the following, each of which is worth **2 points**.

(a) Find a point A where $f_x(A) < 0$ and $f_y(A) < 0$. Mark your point in the diagram with a dot and the letter A. Explain your reasoning in the box.

(b) Find a point B at which $f_x(B) = 0$ and $f_y(B) > 0$. Mark your point in the diagram with a dot and the letter B. Explain your reasoning in the box.

(c) Find a point C at which $f_x(C) = 0$ and $f_y(C) = 0$. Mark your point in the diagram with a dot and the letter C. Explain your reasoning in the box.

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[1] John Biggs. Enhancing teaching through constructive alignment. *Higher education*, 32(3):347–364, 1996.

[2] Daniel Coyle. *The Talent Code: Greatness Isn't Born. It's Grown. Here's How.* Bantam Books, New York, 2009.

[3] Carol S Dweck. *Mindset: The new psychology of success*. Random House, 2006.

[4] Dana C. Ernst. Setting the stage. https://danaernst.com/setting-the-stage/. Accessed: 2025-11-18.

[5] Dana C Ernst, Angie Hodge, and Andrew Schultz. Enhancing proof writing via cross-institutional peer review. *Primus*, 25(2):121–130, 2015.

[6] Scott Freeman, Sarah L Eddy, Miles McDonough, Michelle K Smith, Nnadozie Okoroafor, Hannah Jordt, and Mary Pat Wenderoth. Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, 111(23):8410–8415, 2014.

[7] Steven Schlicker, David Austin, and Matthew Boelkins. *Active Calculus Multivariable: 2018 Edition*. Grand Valley State University Libraries, 2018.