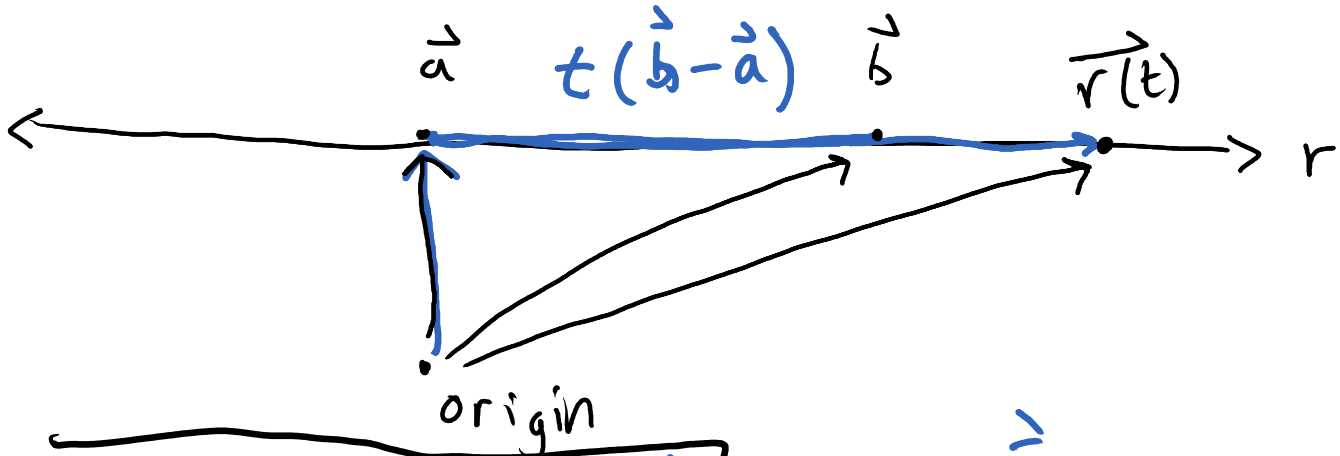
Problem 1 Find an equation of the line passing through the points $(-1,-1,-1)$ and $(1,-1,2)$.

- Every two distinct points determine a line.



So $\boxed{r(t) = \vec{a} + t\vec{v}}$, $\vec{v} = \vec{b} - \vec{a}$

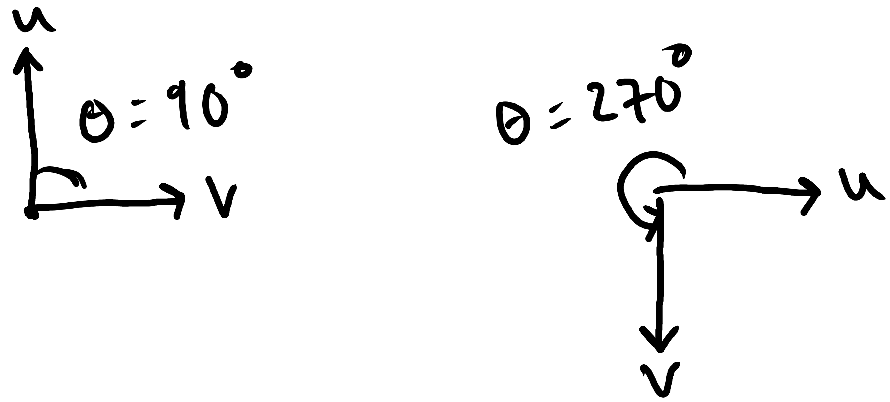
- $\vec{a}$ is a point on the line
- $\vec{v}$ is a vector parallel to the line

Solution: $\vec{a} = (-1,-1,-1)$ is a point in the line. A vector parallel to the line is

$$\vec{b} - \vec{a} = (1,-1,2) - (-1,-1,-1)$$
$$= (2,0,3)$$

So, $r(t) = (-1,-1,-1) + t(2,0,3)$ ∎

$\boxed{\text{Problem 2}}$ Find all values of x such that $(x,1,x)$ and $(x,-6,1)$ are orthogonal.



Recall: The dot product $u \cdot v$ is defined as

$$u \cdot v = |u|\,|v| \cos \theta$$

- If $u \perp v$, then $\theta = 90°, 270°$, so $\cos\theta = 0$. So $u \cdot v = 0$.

- If $u \cdot v = 0$, then $|u|\,|v| \cos\theta = 0$ so $\cos\theta = 0$. So $\theta = 90°$ or $\theta = 270°$ since $0 \le \theta < 360°$

Thm $u \cdot v = 0$ if and only if $u \perp v$.

Sol'n Solve $0 = (x,1,x) \cdot (x,-6,1) = x^2 - 6 + x$

So $x^2 + x - 6 = (x+3)(x-2) = 0$
$\Rightarrow x = -3$ or $x = 2$ ∎

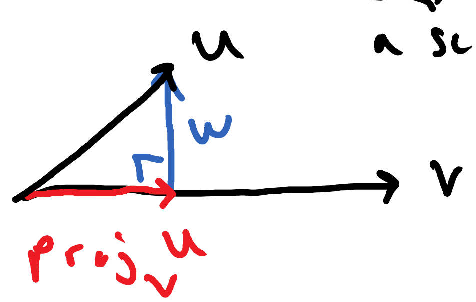
$\boxed{\text{Problem 3}}$ Find the orthogonal projection of $u = (-1, 1, 1)$ onto $v = (2, 1, -3)$.

Def The orthogonal projection of u onto v is the vector

$$\text{proj}_v u = \underbrace{\left(\frac{u \cdot v}{u \cdot u}\right)}_{\text{a scalar}} u \leftarrow$$

Picture

$\mathbb{R}^2$ or $\mathbb{R}^3$



- $\text{proj}_v u = cv$ for some $c \in \mathbb{R}$
- There is a vector w such that $\underline{u = w + cv}$ and $w \perp v \iff w \cdot v = 0$.

To compute c, consider $u \cdot v$

$$u \cdot v = v \cdot (w + cv) = v \cdot w + v \cdot (cv)$$
$$= 0 + c(v \cdot v)$$

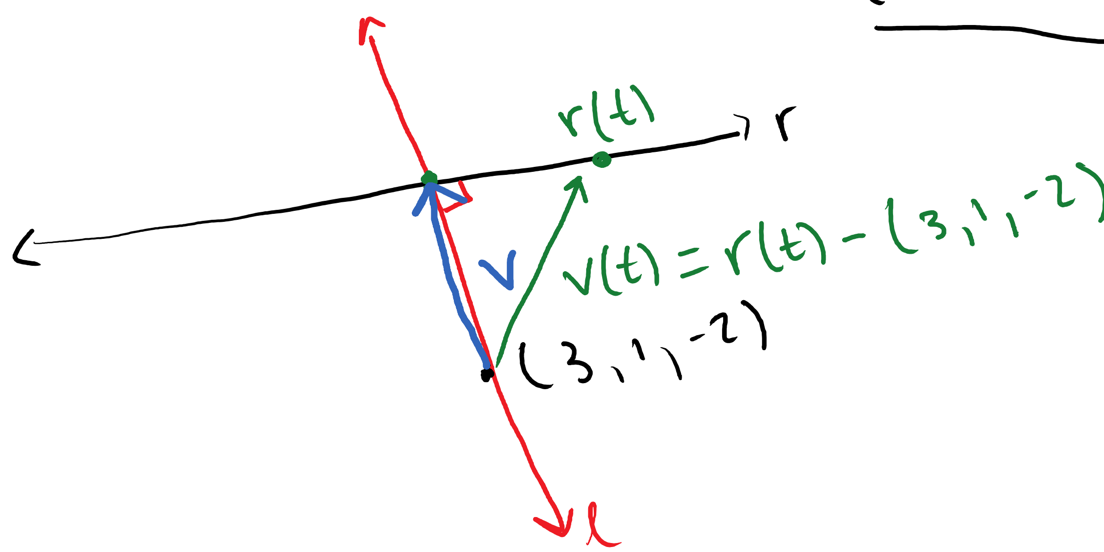
So $c = \dfrac{u \cdot v}{u \cdot u}$. $\blacksquare$

<u>Solution</u>: $\text{proj}_v u = \dfrac{(-1,1,1) \cdot (2,1,-3)}{(-1,1,1) \cdot (-1,1,1)} (-1,1,1)$

$$= \frac{-2 + 1 + -3}{1 + 1 + 1} (-1, 1, 1) = -\frac{4}{3}(-1, 1, 1) = \left(\frac{4}{3}, \frac{-4}{3}, \frac{-4}{3}\right) \blacksquare$$

Chapter 11.2

Problem 4 Find the line through $(3,1,-2)$ that intersects and is perpindicular to the line $r(t) = \underline{(t-1, t-2, t-1)}$.



- A point on l is $(3,1,-2)$
- We need a vector parallel to l, i.e. perpindicular to r.

Consider $v(t) = r(t) - (3,1,-2) = (t-4, t-3, t+1)$

A vector parallel to r is $(1,1,1)$. So we need to solve for t:

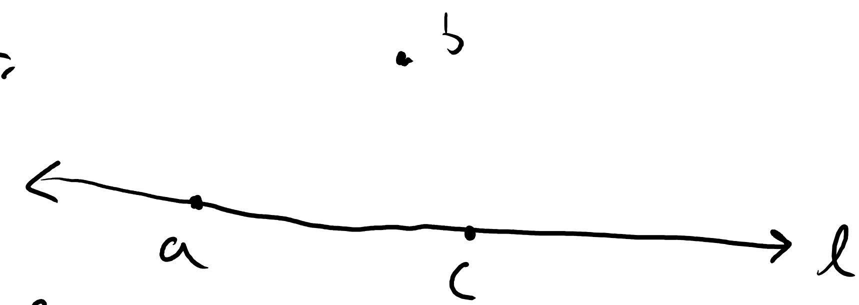
$$\begin{aligned} 0 &= (t-4, t-3, t+1)\cdot(1,1,1) \\ &= t-4+t-3+t+1 \\ &= 3t-6 \end{aligned}$$

So $t=2$. So an equation of the line is
$l(s) = (3,1,-2) + s\,v(2) = (3,1,-2) + s(-2,-1,3)$ ▨

Anthony's Q

Problem 5 Determine if $(1,1,1)$, $(3,2,3)$ and $(-3,-1,-3)$ are collinear.

Consider the picture:



Does b lie on l?

<u>Solution</u> Paramaterize the line containing $(1,1,1)$ and $(3,2,3)$:

$$l(t) = (1,1,1) + t(2,1,2)$$
$$= (2t+1,\ t+1,\ 2t+1)$$

Does $(-3,-1,-3)$ lie on $l(t)$? Solve for t:

$$(-3,-1,-3) = (2t+1,\ t+1,\ 2t+1).$$

We have
$$\boxed{\begin{aligned} -3 &= 2t+1 \\ -1 &= t+1 \end{aligned}} \begin{aligned} &\Rightarrow t = -2 \\ &\Rightarrow t = -2 \end{aligned}$$

The system of equations has a solution. So the points are collinear!

$\square$