

DERIVATION RULES OF SD

Reiteration (R)

P	
▷ P	

Conjunction Introduction (&I)

P	
Q	
▷ P & Q	

Conjunction Elimination (&E)

P & Q	or	P & Q	
▷ P		▷ Q	

Conditional Elimination (\supset E)

P \supset Q	
P	
▷ Q	

Conditional Introduction (\supset I)

	P	
	Q	
▷ P \supset Q		

Negation Introduction (\sim I)

	P	
	Q	
	\simQ	
▷ \simP		

Negation Elimination (\sim E)

	\simP	
	Q	
	\simQ	
▷ P		

Disjunction Introduction (\vee I)

P	or	P	
▷ P \vee Q		▷ Q \vee P	

Disjunction Elimination (\vee E)

P \vee Q		
	P	
	R	
	Q	
	R	
▷ R		

Biconditional Introduction (\equiv I)

	P	
	Q	
	Q	
	P	
▷ P \equiv Q		

Biconditional Elimination (\equiv E)

P \equiv Q	or	P \equiv Q	
▷ P		▷ Q	
▷ Q		▷ P	

DERIVATION RULES OF SD+

Modus Tollens (MT)

	P ⊃ Q
	~Q
▷	~P

Hypothetical Syllogism (HS)

	P ⊃ Q
	Q ⊃ R
▷	P ⊃ R

Disjunctive Syllogism (DS)

	P ∨ Q	or		P ∨ Q
	~P			~Q
▷	Q		▷	P

Commutation (Com)

$$P \& Q \leftrightarrow Q \& P$$

$$P \vee Q \leftrightarrow Q \vee P$$

Association (Assoc)

$$P \& (Q \& R) \leftrightarrow (P \& Q) \& R$$

$$P \vee (Q \vee R) \leftrightarrow (P \vee Q) \vee R$$

Implication (Impl)

$$P \supset Q \leftrightarrow \sim P \vee Q$$

Double Negation (DN)

$$P \leftrightarrow \sim \sim P$$

De Morgan (DeM)

$$\sim(P \& Q) \leftrightarrow \sim P \vee \sim Q$$

$$\sim(P \vee Q) \leftrightarrow \sim P \& \sim Q$$

Idempotence (Idem)

$$P \leftrightarrow P \& P$$

$$P \leftrightarrow P \vee P$$

Transposition (Trans)

$$P \supset Q \leftrightarrow \sim Q \supset \sim P$$

Exportation (Exp)

$$P \supset (Q \supset R) \leftrightarrow (P \& Q) \supset R$$

Distribution (Dist)

$$P \& (Q \vee R) \leftrightarrow (P \& Q) \vee (P \& R)$$

$$P \vee (Q \& R) \leftrightarrow (P \vee Q) \& (P \vee R)$$

Equivalence (Equiv)

$$P \equiv Q \leftrightarrow (P \supset Q) \& (Q \supset P)$$

$$P \equiv Q \leftrightarrow (P \& Q) \vee (\sim P \& \sim Q)$$

Primary Proof Strategies

Clue

main connective of sentence to be proved is a \supset

main connective of sentence to be proved is a \equiv

main connective of sentence to be proved is a \sim

main connective of one of the premises is a \vee

main connective of one of the premises is a \sim

Strategy

use $\supset I$

use $\equiv I$

use $\sim I$

use $\vee E$

use $\sim E$ or $\sim I$ and try to construct an un-negated version of the negated premise

Fallback Strategies

Use $\sim E$ or $\sim I$. ($\sim E$ and $\sim I$ will always work but sometimes takes longer.)

In SD, try to spot applications of SD+ rules and use workarounds for the SD+ rule (See reverse. This usually takes longer.).

WORKAROUND FOR MODUS TOLLENS

1	A \supset B	Assumption		
2	\sim B	Assumption		
3	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">A</td> </tr> </table>		A	A/ \sim I
	A			
4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">B</td> </tr> </table>		B	1,3 \supset E
	B			
5	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">\simB</td> </tr> </table>		\sim B	2 R
	\sim B			
6	\sim A	3-5 \sim I		

WORKAROUND FOR HYPOTHETICAL SYLLOGISM

1	A \supset B	Assumption		
2	B \supset C	Assumption		
3	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">A</td> </tr> </table>		A	A/ \supset I
	A			
4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">B</td> </tr> </table>		B	1,3 \supset E
	B			
5	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">C</td> </tr> </table>		C	2,4 \supset E
	C			
6	A \supset C	3-5 \supset I		

WORKAROUND FOR DISJUNCTIVE SYLLOGISM

1	A \vee B	Assumption				
2	\sim A	Assumption				
3	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">A</td> </tr> </table>		A	A/ \vee E		
	A					
4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">\simB</td> </tr> </table> </td> </tr> </table>		<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">\simB</td> </tr> </table>		\sim B	A/ \sim E
	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">\simB</td> </tr> </table>		\sim B			
	\sim B					
5	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">\simA</td> </tr> </table> </td> </tr> </table>		<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">\simA</td> </tr> </table>		\sim A	2 R
	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">\simA</td> </tr> </table>		\sim A			
	\sim A					
6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">A</td> </tr> </table>		A	3 R		
	A					
7	B	4-6 \sim E				
8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">B</td> </tr> </table>		B	A/ \vee E		
	B					
9	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-left: 5px;">B</td> </tr> </table>		B	8 R		
	B					
10	B	1, 3-7, 8-9 \vee E				