Testing for Bubbles in Exchange Markets: A Case of Sparkling Rates?

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This paper investigates the possibility that the observed deviations of major bilateral exchange rates from values implied by market fundamentals are a consequence of rational asset market bubbles. When a new econometric methodology for detecting asset market bubbles is used, the joint hypothesis of no bubbles and stable autoregressive processes for relative money supplies and real incomes is rejected for the dollar/deutsche mark and dollar/pound rates using monthly data over the period 1973–82. Additional tests for coefficient stability and for lack of cointegration between exchange rates and market fundamentals suggest that the bubble findings must be interpreted with care.

I. Introduction

In sharp contrast to the 1970s, the 1980s have witnessed a dramatic dollar appreciation. As calculated by Morgan Guaranty Trust, the dollar's real effective exchange rate appreciated by roughly 30 percent against the deutsche mark and sterling and 15 percent against the yen over the period of early 1980 to late 1982. The strong dollar has frustrated forecasters (Levich 1983) and lent credence to the view that exchange rate changes are governed by more than simple market fundamentals (relative money supply growth rates, interest differen-

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tials, deviations from purchasing power parity [PPP], etc.). Some economists have argued (McKinnon 1976) that frequent and large exchange rate fluctuations can be explained by speculative runs that may reflect self-fulfilling expectations on the part of market participants. There exists, however, very little academic empirical evidence on which we might assess the validity of this "bubbles" hypothesis. In important early work Dooley and Shafer (1976) document the transactions cost adjusted profitability of simple filter rules applied to daily exchange rate data. This 15–20 percent annualized profitability remains in more recent studies (Dooley and Shafer 1983; Sweeney 1984*a*, 1984*b*). The latter author finds that risk-adjusted filter rule profits are significant.[1]

It is well known that time differences of spot and forward exchange rates are well approximated by random walks (Poole 1967; Mussa 1979; Meese and Rogoff 1983*a*, 1983*b*; among others). While serial correlation in exchange rate changes has proved unstable and difficult to predict, the filter rule studies highlight the fact that the first-order autocorrelation coefficient of daily exchange rate changes has remained positive over the floating rate period. Given the apparent profitability of filter rules on daily data, it would appear that a major omission of empirical academic exchange rate research has been to ignore analysis of intraday exchange rate movements, since banks frequently maintain zero net open positions in each currency overnight.[2] What evidence that exists for daily, weekly, monthly, and quarterly exchange rates suggests that the filter rule profitability disappears as the time horizon increases from daily to less finely sampled data.

Additional evidence of bubbles in exchange markets is suggested by (1) leptokurtic (fat-tailed) distributions of daily exchange rate changes and (2) the ratio of return variance of trading hours to nontrading hours. Friedman and Vandersteel (1982) find that the coefficient of kurtosis for daily dollar/deutsche mark ($/DM) exchange rate changes over the period June 1, 1973, to September 14, 1979, is 19.2. In contrast, monthly $/DM changes over the period March 1973–December 1982 give rise to a kurtosis coefficient of 4.2. (The kurtosis coefficient of a normal distribution is 3; a lower bound for the statistic is 1.) Just a few large changes ("crashes") in a sample of daily exchange rate differences can explain the observed leptokurtosis. Sec-

[1] Sweeney (1984*a*) finds that the filter rule profitability, in excess of a buy and hold strategy, is significant; his results constitute a violation of a single-period capital asset pricing model.

[2] See Fieleke (1981) or Shapiro (1982). In the later part of this sample, banks became more aggressive in attempting to profit from exchange rate movements. Over the early part of the sample, banks' foreign exchange departments were more service oriented.

ond, when the methodology of French and Roll (1984) is used, it can be shown that the ratio of return variance during trading hours to nontrading hours is about 3 for a sample of daily dollar/yen exchange rates over the period December 6, 1983, to December 6, 1984.[3] If "news" is uniformly distributed over the week, then trader overreaction is a plausible explanation of the 3 : 1 variance ratio.

While suggestive of self-fulfilling expectations, the profitability of filter rules, the evidence of leptokurtosis, and the difference between return variances on daily exchange rate data do not necessarily imply asset market bubbles in the sense to be defined below. Arbitrage does not by itself prevent bubbles (Blanchard and Watson 1982), and bubbles need not be associated with irrationality of market participants. Rational asset market bubbles can exist in a world of risk-averse agents with heterogeneous information sets; again see the discussion in Blanchard and Watson (1982). There are, however, theoretical models in which bubble paths can be ruled out on economic grounds, as demonstrated by Gray (1982) and Obstfeld and Rogoff (1983), among others.

Flood and Garber (1980) were the first to attempt empirical tests of bubbles in the context of a rational expectations model of the German hyperinflation. Their methodology is appropriate for a deterministic bubble. In the study of exchange markets a deterministic bubble is unrealistic, for to be rational the bubble (and hence the value of a currency in terms of another) must increase indefinitely. A second problem with empirical studies of deterministic bubbles is that conventional asymptotic distribution theory precludes exponentially growing regressors.

The literature on solution indeterminacies in rational expectations models suggests that self-fulfilling expectations are equally interpretable as omitted variables. Under the latter interpretation, these variables are observed and acted on by market participants but unobserved by econometricians; see the discussions in Flood and Garber (1980, p. 749, n. 5), Burmeistér, Flood, and Garber (1983), and Hamilton and Whiteman (in press). Whatever the interpretation, a test for uniqueness of rational expectations solutions (the no-bubble hypothesis) is still of considerable interest. The possibility of bubbles or extraneous variables in rational expectations models suggests that full-information procedures for estimating forward-looking asset price models will generally be inconsistent, as demonstrated below.

[3] The calculations involving the dollar/yen rate were based on open and close quotes from the New York branch of a Japanese financial institution. Sweeney (1984*a*) type filter rules (net of transactions costs) did not yield significant profits in this short data set.

In this paper I provide econometric evidence against the *joint* null hypothesis that there are no bubbles in exchange markets and that the driving processes for relative money supplies and real incomes are stable over the current floating rate period. The econometric methodology is suggested by West (1984*a*, 1984*b*), who applies it to equity markets. The methodology admits stochastic bubbles and gives rise to a condition that validates standard asymptotic distribution theory for parameter estimates under the alternative hypothesis of exchange rate bubbles. The test for bubbles in exchange markets is conditioned on a hybrid monetary exchange rate model, a model consistent with the observed long-term deviations of exchange rates from PPP values. The model and the testing methodology are described in Section II, while empirical results are presented in Section III.

To investigate the robustness of the findings of the model-based empirical work, I test for stability of the model's parameters. In addition, tests for cointegration of exchange rates, relative money supplies, and real incomes are conducted in Section IV. The methodology, suggested by Granger and Engle (1984), gives rise to a testable condition that precludes exchange market bubbles. Tests of cointegration can be conducted while imposing a minimum of structure on the dynamic interactions of the exchange rate and the market fundamentals. Last, Section V presents conclusions.

II. Testing for Bubbles in a Hybrid Monetary Exchange Rate Model

Following Dornbusch (1976), Frenkel (1976), Bilson (1978, 1979), Frankel (1979, 1981), or Mussa (1982), assume a transactions-type money demand equation of the following form:[4]

$$m_t - p_t = a_1 y_t - a_2(i_t - i_t^*), \tag{1}$$

where m_t, p_t, and y_t are the logs of relative (U.S. to foreign) money supplies, price levels, and real incomes and $i_t - i_t^*$ is the short-term interest differential (U.S. minus foreign). Woo (1985) provides evidence for the equality of income elasticities (a_1) and interest rate

[4] For simplicity I will suppress deterministic terms—a constant and seasonal dummy variables—throughout the theoretical analysis. Such terms are included in the empirical work reported in the next section. Since I assume that the same deterministic terms are present in all estimating equations (see [12] and [15] below), the inclusion of these terms does not introduce additional cross-equation constraints. A stochastic disturbance term could be appended to the money demand equation as well. While it is customary to omit money demand disturbances in derivations of monetary exchange rate models, this practice runs counter to the rational expectations literature where the source of disturbances is deemed to be crucial.

semielasticities ($-a_2$) for the United States and Germany when a transactions money demand equation is appended with a stock adjustment mechanism. We shall assume uncovered interest parity (UCIP),

$$i_t - i_t^* = E(s_{t+1}|\Phi_t) - s_t, \tag{2}$$

where $E(s_{t+1}|\Phi_t)$ denotes the linear least-squares projection of the time $(t + 1)$ spot exchange rate s_{t+1} (natural logarithm of dollars per foreign currency units) based on information dated t. The information set Φ_t contains at least the current and past values of all variables introduced thus far (see below). While econometric techniques have proved sufficiently powerful to reject (2) (Geweke and Feige 1979; Hansen and Hodrick 1980), no one has demonstrated that mean deviations from UCIP are large or explainable in the context of a stable financial model of risk (Frankel 1982; Hodrick and Srivastava 1984). We take (2) to be a reasonable first-order approximation. Finally, I shall impose the condition that deviations from PPP follow a random walk:

$$s_t - p_t = u_t, \quad u_t = u_{t-1} + \epsilon_t, \tag{3}$$

where ϵ_t is a white noise with variance σ_ϵ^2.[5] Equation (3) is consistent with evidence reported in Adler and Lehmann (1983) and Hakkio (1984) and can be viewed as an approximation to a sticky price monetary exchange rate model (Dornbusch 1976; Frankel 1979, 1981), where the goods market speed of adjustment is very slow. Substituting (2) and (3) into (1) yields

$$s_t = m_t - a_1 y_t + a_2[E(s_{t+1}|\Phi_t) - s_t] + u_t. \tag{4}$$

Define $b = a_2/(1 + a_2)$, $0 < b < 1$, where a_2 is minus the interest semielasticity of money demand. Equation (4) can be written more usefully as

$$s_t = (1 - b)(m_t - a_1 y_t) + bE(s_{t+1}|\Phi_t) + (1 - b)u_t. \tag{5}$$

There exists overwhelming evidence that the level of s_t follows a borderline nonstationary process (Meese and Singleton 1982; Meese and Rogoff 1983*a*, 1983*b*; among others). As a consequence, we shall rely on the first difference of (5) for empirical applications:

$$\begin{aligned}\Delta s_t = (1 - b)(\Delta m_t - a_1 \Delta y_t) + b[E(s_{t+1}|\Phi_t) \\ - E(s_t|\Phi_{t-1})] + (1 - b)\epsilon_t,\end{aligned} \tag{6}$$

[5] In what follows, I shall require the further assumption that ϵ_t is uncorrelated with m_t and y_t. Meese and Rogoff (1985) provide evidence that ϵ_t is orthogonal to m_t and y_t, as well as $(i_t - i_t^*)$, expected inflation differentials, and cumulated trade balances.

where Δ denotes the first-difference operator. To promote notational simplicity, define the "market fundamentals" process Δx_t as

$$\Delta x_t \equiv (\Delta m_t - a_1 \Delta y_t) = c\Delta x_{t-1} + \delta_t, \quad |c| < 1, \tag{7}$$

where δ_t is a white noise with variance σ_δ^2. We are implicitly assuming that a_1 can be treated as known for expositional purposes. Sample determination of the order of the Δx_t process is discussed below. For given value(s) of a_1, (6) can be estimated by McCallum's (1976) technique: the unobservable expectation $E(s_{t+1}|\Phi_t)$ is replaced by its actual value s_{t+1} minus a forecast error, n_{t+1}, uncorrelated with Φ_t. This substitution creates a first-order moving average, MA(1), composite disturbance process for (6). Nevertheless, an instrumental variables estimator of b is consistent when instruments are chosen appropriately, as shown below. Equation (6) may also be solved recursively forward to obtain

$$\Delta s_t = (1 - b) \sum_{i=0}^{\tau-1} b^i [E(x_{t+i}|\Phi_t) - E(x_{t-1+i}|\Phi_{t-1})] + b^\tau [E(s_{t+\tau}|\Phi_t) - E(s_{t+\tau-1}|\Phi_{t-1})] + (1 - b) \sum_{i=0}^{\tau-1} b^i \epsilon_t. \tag{8}$$

If the transversality condition

$$\lim_{\tau \to \infty} b^\tau [E(s_{t+\tau}|\Phi_t) - E(s_{t+\tau-1}|\Phi_{t-1})] = 0 \tag{9}$$

holds, then the unique, no-bubbles solution to (6) is

$$\Delta s_t = (1 - b) \sum_{i=0}^{\infty} b^i [E(x_{t+i}|\Phi_t) - E(x_{t-1+i}|\Phi_{t-1})] + \epsilon_t. \tag{10}$$

From (7), the optimal prediction formula for x_{t+i} conditional on the information set Φ_t is

$$E(x_{t+i}|\Phi_t) = x_t + \sum_{j=1}^{i} c^j \Delta x_t. \tag{11}$$

The rational expectations, no-bubbles solution to (6) can be shown to be

$$\Delta s_t^* \equiv \Delta s_t = \Delta x_t + \frac{bc}{1 - bc} (\Delta x_t - \Delta x_{t-1}) + \epsilon_t, \tag{12a}$$

$$\Delta x_t = c\Delta x_{t-1} + \delta_t, \tag{12b}$$

where Δs_t^* will be used to distinguish the so-called market fundamentals solution. Under the null hypothesis of no bubbles the estimate of b that can be extracted from the system (12) is asymptotically efficient. The estimate of b obtained using McCallum's technique on (6) is inefficient, as demonstrated below. If the transversality condition (9) is violated, then any solution of the form

$$s_t = s_t^* + d_t, \tag{13}$$

where $E(d_{t+1}|\Phi_t) = (1/b)d_t$,[6] also satisfies (5) (see Blanchard and Watson 1982). The bubble d_t need not be restricted to a deterministic process like the one considered by Flood and Garber (1980); Blanchard and Watson (1982) provide examples of plausible stochastic bubbles that continually grow and break. West's (1984*a*) insight is to notice that, under the bubble alternative, McCallum's technique applied to (6) provides a consistent estimate of b, while maximum likelihood estimation of (12) produces an inconsistent estimate of b if a bubble (omitted variable) is correlated with the regressors in (12a). The null hypothesis of no bubbles (eq. [9]) can thus be tested using Hausman's (1978) specification error test. The test is by construction a joint test of (9) and a stable driving process (12b), as the forward-looking solution (10) is only estimable after substitution for the expected future values of the exogenous process x.

To see why solutions of the form (13) invalidate maximum likelihood estimation of (12), note that ordinary least squares (OLS) estimation of (12a) is asymptotically efficient if Δx_t is exogenous with respect to Δs_t. Under this assumption b and c are just identified, the system (12) is recursive, and equation-by-equation OLS is the efficient estimation strategy. Define $\gamma = bc/(1 - bc)$. Using (13) we can show that

$$\operatorname*{plim}_{T \to \infty} \hat{\gamma} = \gamma + \operatorname{plim} \frac{1}{T} \sum_{t=1}^{T} \frac{\Delta d_t(\Delta x_t - \Delta x_{t-1})}{2\sigma_\delta^2/(1 + c)}. \tag{14}$$

The probability limit of $\hat{\gamma}$ is not necessarily γ since there is no presumption that

$$\operatorname*{plim}_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \Delta d_t(\Delta x_t - \Delta x_{t-1}) = 0,$$

that is, that the bubble is uncorrelated with the fundamentals process $\{\Delta x_t\}$.

It is useful to rewrite equation (6) in a more convenient form for applying McCallum's technique:

$$(\Delta s_t - \Delta x_t) = b(\Delta s_{t+1} - \Delta x_t) + [(1 - b)\epsilon_t - b(\eta_{t+1} - \eta_t)], \tag{15}$$

[6] At this point it is important to distinguish between the agents' information set Φ_t and the econometricians' information set, call it Λ_t, a subset of Φ_t. Agents, e.g., observe d_t while econometricians do not. The empirical work reported below requires only $\Lambda_t = \{x_{t-i}, s_{t-i}; i \geqq 0\}$.

where the composite disturbance, call it θ_t, can be expressed as

$$\begin{aligned}\theta_t &= (1 - b)\epsilon_t - b(\eta_{t+1} - \eta_t) \\ &= (1 - b)\epsilon_t - b\left[\frac{1}{1 - bc}(\delta_{t+1} - \delta_t) + (\epsilon_{t+1} - \epsilon_t)\right]\end{aligned} \tag{16}$$

under the null hypothesis of no bubbles. It is clear from (15) and (16) that only Δx_s, for $s \leq (t - 1)$, are legitimate instruments of $(\Delta s_{t+1} - \Delta x_t)$, as the composite disturbance contains δ_t.[7] Under our assumptions and using just Δx_{t-1} as an instrument of $(\Delta s_{t+1} - \Delta x_t)$, we can express the limiting distribution of the instrumental variables estimator as

$$\sqrt{T}\,(\hat{b}_{\mathrm{IV}} - b) \xrightarrow{\mathrm{L}} N(0, \sigma_\theta^2 Q_{\mathrm{IV}}), \tag{17}$$

where $\hat{b}_{\mathrm{IV}}$ is the instrumental variables estimator of b, $\sigma_\theta^2 = \mathrm{var}(\theta_t)$,

$$Q_{\mathrm{IV}} = \frac{(1 + c)^2(1 - bc)^2}{\sigma_\delta^4 c^2}\left(\frac{\sigma_\delta^2}{1 - c^2} + \frac{2c\sigma_\delta^2\phi}{1 - c^2}\right),$$

and $\phi = \mathrm{cov}(\theta_t, \theta_{t-1})/\mathrm{var}(\theta_t)$.[8] The limiting distribution of $\hat{b}$ extracted from the estimators of δ and c in (12) can be shown to be

$$\sqrt{T}(\hat{b} - b) \xrightarrow{\mathrm{L}} N\left\{0, \left[\frac{(1 - c^2)b^2}{c^2} + \frac{\sigma_\epsilon^2(1 - bc)^4(1 + c)}{\sigma_\delta^2 2c^2}\right]\right\}.^{9} \tag{18}$$

[7] Note that Δs_{t-i}, $i \geqq 1$, are also legitimate instruments. Our concern here is *not* with the choice of an optimal instrument set.

[8] Define the $(T \times T)$ vectors $\mathbf{A}' = (\Delta x_0, \ldots, \Delta x_{T-1})$ and $\mathbf{B}' = (\Delta s_2 - \Delta x_1, \ldots, \Delta s_{T+1} - \Delta x_T)$. Let $\mathbf{V}$ be a symmetric banded matrix for an MA(1) error process with ϕ as the off-diagonal entries and ones down the diagonal. Then

$$Q_{\mathrm{IV}} = \operatorname*{plim}_{T\to\infty}\left(\frac{1}{T}\mathbf{B}'\mathbf{A}\right)^{-1}\left(\frac{1}{T}\mathbf{A}'\mathbf{V}\mathbf{A}\right)\left(\frac{1}{T}\mathbf{A}'\mathbf{B}\right)^{-1}.$$

Under the null hypothesis of no bubbles, this expression has the limiting form given in the text where $\sigma_\theta^2 = \sigma_\epsilon^2(1 + b^2) + \sigma_\delta^2[2b^2/(1 - bc)^2]$ and $\mathrm{cov}(\theta_t, \theta_{t-1}) = \sigma_\epsilon^2(-b) + \sigma_\delta^2[-b^2/(1 - bc)^2]$.

[9] Define the function $b = g(\gamma, c) = \gamma/c(1 + \gamma)$, $c \neq 0$, $\gamma \neq -1$. Under our assumptions,

$$\sqrt{T}(\hat{\gamma} - \gamma) \xrightarrow{\mathrm{L}} N\left[0, \frac{\sigma_\epsilon^2(1 + c)}{2(1 - b)^2\sigma_\delta^2}\right]$$

and $\sqrt{T}(\hat{c} - c) \xrightarrow{\mathrm{L}} N[0, (1 - c^2)]$. Using theorem 12 of Rao (1973, p. 124) and the delta rule, we see that

$$\begin{aligned}\sqrt{T}[g(\hat{\gamma}, \hat{c}) - b] \xrightarrow{\mathrm{L}} N(0, \Omega)\colon \Omega &= \left(\frac{dg}{d\gamma}\right)^2\frac{\sigma_\epsilon^2(1 + c)}{2\sigma_\delta^2} + \left(\frac{dg}{dc}\right)^2(1 - c^2) \\ &= \frac{(1 - c^2)b^2}{c^2} + \frac{\sigma_\epsilon^2(1 - bc)^4(1 + c)}{\sigma_\delta^2 2c^2}.\end{aligned}$$

In deriving this expression we have used the fact that $\mathrm{cov}(\hat{\gamma}, \hat{c}) = 0$.

The Hausman specification error test statistic, distributed as χ^2 with one degree of freedom, has the following form:

$$\frac{T(\hat{b}_{IV} - \hat{b})^2}{\left\{\dfrac{b^2(1 + c)^2}{c^2} + \sigma_\epsilon^2 \dfrac{(1 + c)^3(1 - bc)^2[(1 - bc)^2 + 2b^2(1 - c)]}{\sigma_\delta^2 2(1 - c^2)c^2}\right\}}. \tag{19}$$

While it is not explicitly necessary to derive the population expression for $\text{var}(\hat{b}_{IV}) - \text{var}(\hat{b})$, direct application of the Hausman test using the estimated variance of $\hat{b}_{IV}$, $S^2(\hat{b}_{IV})$, the McCallum procedure, and the estimated variance of $\hat{b}$, $S^2(\hat{b})$, from (12) does not necessarily produce an estimate of the variance of the difference of $(\hat{b}_{IV} - \hat{b})$ that is positive.[10] To avoid this difficulty, I have explicitly derived the test statistic (19), whose denominator must be positive.

It is possible to test for the existence of bubbles in a model where a_1, the income elasticity of money demand, is explicitly estimated (see below). There does not, however, appear to be any general form for the test statistic (19), as the derivation of (19) relied heavily on the assumed form of the relationship between current spot rates and expected future spot rates, the assumptions on the driving process x_t, and the behavior of the structural disturbances.

In the context of our model, Diba and Grossman (1984) and Hamilton and Whiteman (in press) have argued that the only observable implication of no exchange market bubbles is that both the driving process x_t and the logarithm of the exchange rate s_t are stationary after first differencing. In other words, if exchange market expectations are self-fulfilling, then the first difference of s_t should still exhibit nonstationary behavior, even if the first difference of x_t is stationary. The problem underlying this quasi test procedure—comparing the autocorrelation functions (ACF) of Δs_t and Δx_t—is that the bubble term d_t may not exhibit nonstationary behavior that is readily discernible from the ACF of Δs_t. A clever example of a stochastic bubble d_t is provided by Blanchard and Watson (1982):

$$\begin{aligned} d_t &= (b\pi)^{-1}d_{t-1} + q_t \quad \text{with probability } \pi, \\ d_t &= q_t \quad \text{with probability } (1 - \pi), \end{aligned} \tag{20}$$

where $E(q_t|\Phi_{t-1}) = 0$. Six examples of artificially generated bubbles of the form (20) are provided in table 1. Their ACFs and partial auto-

[10] For the experiments reported below, the difference of $S^2(\hat{b}_{IV})$ and $S^2(\hat{b})$ was always negative; hence the need for deriving $\text{var}(\hat{b}_{IV}) - \text{var}(\hat{b})$ explicitly. West (1984*a*) makes use of a general computational scheme that guarantees a positive test statistic.

TABLE 1

AUTOCORRELATIONS AND PARTIAL AUTOCORRELATIONS FOR ARTIFICIAL BUBBLES EQUATION (20)

μ [σ]	π	b	STATISTIC	LAG 1	2	3	4	5	6	7	8	9	10	11	12
−188.0	.1	.9	ACF	.66	.49	.38	.29	.23	.18	.15	.13	.13	.14	.16	.01
[599.3]			PAF	.66	.10	.03	.01	.01	.01	.01	.02	.03	.04	. . .	. . .
−265.0	.5	.9	ACF	.69	.37	.28	.20	.16	.10	.06	.04	.03	.03	.01	.00
[1,007.9]			PAF	.69	−.21	.24	−.13	.14	−.13	.09	−.07	.07	−.05	. . .	. . .
−42.6	.9	.9	ACF	.81	.65	.52	.42	.33	.27	.21	.16	.13	.10	.08	.05
[124.8]			PAF	.81	−.00	−.00	−.00	−.00	−.00	−.00	−.00	−.00	−.00	. . .	. . .
−54.7	.1	.95	ACF	.54	.41	.33	.27	.23	.20	.18	.16	.16	.17	.18	.02
[159.5]			PAF	.54	.16	.09	.04	.03	.03	.02	.03	.03	.04	. . .	. . .
−73.9	.5	.95	ACF	.68	.39	.30	.22	.18	.12	.08	.06	.05	.05	.02	.01
[239.1]			PAF	.68	−.15	.19	−.09	.11	−.10	.06	−.04	.04	−.01	. . .	. . .
−13.6	.9	.95	ACF	.85	.71	.60	.50	.43	.36	.30	.25	.21	.17	.14	.11
[32.4]			PAF	.85	−.00	−.01	.00	.00	−.01	−.01	.00	.00	−.01	. . .	. . .

NOTE.—$d_t = (b\pi)^{-1}d_{t-1} + q_t$ with probability π and $= q_t$ with probability $(1-\pi)$; $d_0 = 0$. Drawings from a continuous uniform distribution on [0, 1] determine whether the bubble grows or pops. q_t is independently and identically distributed $N(0, 1)$. $1/\sqrt{T} = .0945$, so $\pm .189$ provides $2-\sigma$ confidence bands for the estimated correlation coefficients. The same random number sequence was used for all six experiments. μ = sample mean; σ = sample standard deviation.

correlation functions (PAFs) are based on 112 pseudo-observations. While nonstationary by construction,[11] the artificial bubbles might all be identified as low-order stationary autoregressive processes. The values of b (.90 and .95) correspond closely to $\hat{a}_2/(1 + \hat{a}_2)$ from tables 2–6 below. Clearly, it will be difficult to implement the Diba-Grossman (1984) and Hamilton-Whiteman (in press) observation in practice. In an attempt to find an alternative statistical test for ruling out exchange market bubbles, we appeal in Section IV to tests of cointegration. The methodology is less model based in the sense that assumptions (1), (2), and (3) are not imposed a priori. The results of these tests tend to corroborate the findings of the model-based approach described in the following section.

III. Uncorking Evidence of Bubbles in Bilateral Deutsche Mark and Sterling Rates with the U.S. Dollar

Tests of the no-bubbles hypothesis were conducted for a grid of reasonable parameter values on a_1, the income elasticity of money demand. The interval [.2, 1.0] was explored in steps of .1. This encompasses values from theoretical transactions demand for money models, roughly .3–1.0 depending on integer constraints and the assumptions regarding the size versus the frequency of transactions as income rises.[12] The range also includes Goldfeld's (1973) short-run (roughly .2) and long-run (approximately .7) income elasticity estimates. While this coefficient is estimated directly in a more elaborate version of the model than the one described in Section II, point estimates of the other focus parameters, c and b or a_2, are qualitatively unaffected by the use of this more complicated procedure. Therefore, detailed results are given only for the model of Section II over a range of plausible a_1 values. The implicit constraint imposed on the driving processes Δm_t and Δy_t in the theoretical development of the previous section is that both variables are adequately represented by univariate autoregressions with the same lag (L) polynomial. In other words, in the bivariate autoregression

[11] Suppose that the initial d_0 has zero expectation and variance $\sigma^2_{d_0}$. Let n_t be independent and identically distributed with zero mean and variance σ^2_n. Then

$$\mathrm{var}(d_t) = \frac{\sigma^2_{d_0}}{(\pi b^2)^t} + \left(\frac{\sigma^2_n}{1 - \pi}\right)\left[\frac{1 - (1/\pi b^2)^t}{1 - (1/\pi b)}\right].$$

[12] See the discussions in Miller and Orr (1966), Whalen (1966), and Barro (1976).

$$\Delta m_t\left(1 - \sum_{i=1}^{N} a_i L^i\right) + \Delta y_t \sum_{i=1}^{N} b_i L^i = \Delta m_t^*,$$
$$\Delta y_t\left(1 - \sum_{i=1}^{N} e_i L^i\right) + \Delta m_t \sum_{i=1}^{N} f_i L^i = \Delta y_t^*, \tag{21}$$

where $(\Delta m_t^*, \Delta y_t^*)$ is a bivariate white noise orthogonal to Φ_t, it is the case that $N = 1$, $b_1 = 0 = f_1$, and $a_1 = e_1$. These constraints cannot be rejected for bilateral U.S.-German and U.S.-U.K. data sets. In addition, the condition that Δs_t not Granger cause $\Delta x_t = (\Delta m_t - a_1 \Delta y_t)$ is accepted for the \$/yen, \$/DM, and \$/£ rates for all values of a_1 considered.[13] These Granger causality tests contained a constant, 11 seasonal dummies, and eight lags of Δs_t and Δx_t. All regressions discussed thus far are based on 110 observations of the dependent variables, October 1973–November 1982 inclusive. Variables at the beginning and end of the sample period (January 1973–December 1982) are required for leads and lags in the various regressions. The data are described in the appendix of Meese and Rogoff (1983*b*).

The results are presented by country, but first the form of the test statistic (19) merits further comment. The denominator of the test statistic can be computed using values of b, c, σ_ϵ^2, and σ_δ^2 extracted entirely from the estimation of the system (12), entirely from the estimation of (15) and (12b), or from a mixture of the two. Only the estimates based on (15) and (12b) need be consistent for their population values under the alternative hypothesis of bubbles. This distinction turns out to be critical. In terms of economic significance the estimates of b derived from (15) are reasonable and imply values of interest rate semielasticities consistent with those reported in the literature. The estimates of b implied by the system (12) are, however, always too large ($\hat{b} > 1$), suggesting a positive interest rate semielasticity of money demand from (1). Since the derivation of the test statistic (19) requires the assumption $b < 1$, it makes little sense to evaluate the statistic using the implausible (no-bubbles) estimate of b. For comparative purposes two values of the test statistic are reported in table 1: the first is based on parameter estimates of (15) and (12b), while the second is based on a mixture of (15) and (12b) with (12a).[14]

The nice feature of McCallum's technique in this context is that,

[13] The condition that Δs_t not Granger cause Δx_t is necessary but not sufficient for Δx_t to be exogenous with respect to Δs_t. I choose to report Granger tests of Granger causality since Geweke, Meese, and Dent (1983) report strong evidence for preferring these tests to other variants proposed in the literature.

[14] The first test statistic for bubbles reported in table 1 relies on parameter estimates of b, c, σ_ϵ^2, and σ_δ^2 from (15) and (12b). An estimate of σ_ϵ^2 can be extracted from σ_θ^2 (see n. 8). The second test statistic is calculated from estimates of b from (15), c and σ_δ^2 from (12b), and σ_ϵ^2 from (12a).

under the alternative hypothesis of bubbles, (i) parameter estimates are consistent for their population counterparts, (ii) conventional asymptotic distribution theory still applies to (15) when $|c| < |b|$, and (iii) the model (15) should pass conventional diagnostic checks for goodness of fit if the hybrid monetary exchange rate model of Section II provides a reasonable in-sample description of the U.S.-German and U.S.-U.K. data sets under the alternative hypothesis of bubbles. We have already discussed point i above. To confirm the validity of point ii, note that condition (13) is satisfied by

$$d_t = \frac{1}{b} d_{t-1} + \xi_t, \quad 0 < b < 1, \tag{22}$$

where ξ_t is orthogonal to Φ_t. Assuming that the process (ξ_t, δ_t) is a serially independent vector white noise with a (2×2) covariance matrix $\mathbf{\Sigma}$, we see that Q_{IV} (defined in n. 4) is finite and nonzero provided $|c| < |b|$. This condition stems from the interaction of the driving process Δx_t with $\Delta s_t = (\Delta s_t^* + \Delta d_t)$ and is a condition for a convergent sum of a geometric series.[15] This condition is satisfied for all models reported below.

[15] Since conventional asymptotic distribution theory precludes exponentially growing regressors, it is important to demonstrate that Q_{IV} is finite and nonzero under the alternative of bubbles. In this case

$$Q_{\mathrm{IV}} = \frac{\sigma_\delta^2 \dfrac{1 + 2\phi c}{1 - c^2}}{\left\{\operatorname*{plim}_{T\to\infty}\left[\dfrac{1}{T}\sum_{t=2}^{T-1} (\Delta s_{t+1} - \Delta x_t)\Delta x_{t-1}\right]\right\}^2}.$$

Substituting $\Delta s_t = (\Delta s_t^* + \Delta d_t)$ into the denominator introduces the term

$$\operatorname*{plim}_{T\to\infty} \frac{1}{T}\sum_{t=2}^{T-1} (\Delta d_{t+1} - \Delta x_t)\Delta x_{t-1},$$

which must be finite if Q_{IV} is to be nonzero. The expression

$$\operatorname*{plim}_{T\to\infty} \frac{1}{T}\sum_{t=2}^{T-1} \Delta d_{t+1}\Delta x_{t-1} = \operatorname*{plim}_{T\to\infty} \frac{1}{T}\sum_{t=2}^{T-1}\left[\sum_{i=0}^{\infty}\left(\frac{1}{b}\right)^i \zeta_{t+1-i}\right] \cdot \left(\sum_{j=0}^{\infty} c^j \delta_{t+1-j}\right).$$

Since the (ζ_t, δ_t) process is contemporaneously but not serially correlated, the probability limit of the expression above is finite when $|c/b| < 1$. In this case, the limit is $\sigma_{12}/b^2[1 - (c/b)]$, where σ_{12} is the off-diagonal element of $\mathbf{\Sigma}$. Consistency of the test procedure (as opposed to parameter estimates of [15]) may not be ensured for all possible circumstances (see West 1984*a*, p. 16, n. 11). There are circumstances where the bubble alternative is true, yet the asymptotic power of the Hausman test statistic need not be unity.

The exact form of σ_θ^2, the disturbance variance of (15), in terms of b, c, σ_δ^2, and σ_ϵ^2 is not known under the alternative hypothesis of bubbles. Nevertheless, it is still the case that, a priori, we expect the disturbances of (15), based on an instrumental variables estimator of b, to exhibit MA(1) behavior, as there is no reason to rule out contemporaneous correlation of η_t and ϵ_t. Before correction for the MA(1) error process (see the estimator described in n. 8), the autocorrelation function of the residuals of (15) exhibits precisely this behavior. As such, examination of the autocorrelation function of (15) provides a diagnostic check of the adequacy of the model under the alternative, point iii above.

Tables 2–6 are organized as follows. The value of the income elasticity of money demand and the bilateral data set are indicated by the table title. For each value of a_1 a test statistic—an $F(8, 82)$—for the test that Δs_t does not Granger cause Δx_t is reported, two versions of the bubble test (19), both $\chi^2(1)$, are given, and the individual regression results for (12a), (12b), and (15) are displayed. Results of equation (15) include the residual sample autocorrelation function, lags 1–13. For the U.S.-German data set, regressions based on $a_1 = .3$, .4, and .5 are reported: they bracket the estimate of a_1 (.39) obtained from an unconstrained version of the model of Section II.[16] The income elasticity range of the U.S.-U.K. data set, $a_1 = .3$ and .4, also brackets the unrestricted estimate of $a_1 = .35$. For convenience, in the row labeled average interest elasticity, I have converted the implied estimates of a_2, the interest semielasticity, into elasticities by dividing by the sample mean of i_t.

All versions of the test (19) indicate rejection of the no-bubbles hypothesis at very small significance levels. Parameter estimates from

[16] The more general model of Sec. II would have the form

$$\Delta s_t - \Delta m_t = -a_1(1 - b)\Delta y_t + b(\Delta s_{t+1} - \Delta m_t) + \theta_t, \tag{15'}$$

$$\Delta s_t - \Delta m_t = -a_1\Delta y_t + \frac{bc}{1 - bc}(\Delta m_t - \Delta m_{t-1}) - \frac{a_1 be}{1 - be}(\Delta y_t - \Delta y_{t-1}) + \epsilon_t, \tag{12a'}$$

$$\Delta y_t = e\Delta y_{t-1} + \delta_{1,t}, \quad \Delta m_t = c\Delta m_{t-1} + \delta_{2,t}, \tag{12b'}$$

where Δy_{t-1} and Δm_{t-1} are used as instruments for the two regressors of (15′). Calculation of the bubbles test statistic for this model (a statistic comparable to [19]) is more difficult. Since we accept $e = c$, there is no loss of generality in exploiting the simple form of the model presented in Sec. II. Estimates of b from (15′) are still reasonable, while those of (12a′) and (12b′) are again greater than one. However, residual diagnostic checks of (15′) indicate more than an MA(1) disturbance.

TABLE 2

U.S.-German Data: Income Elasticity $a_1 = .3$

Parameter	Equation (12a)	Equation (12b)	Equation (15)
$-a_2$	1.49	...	−26.2
	(.828)		(31.6)
c	...	−.251*	...
		(.982-1)	
Average interest elasticity	.18	...	−3.09
σ^2_ϵ	.103-2	...	...
σ^2_δ	...	.343-3	...
σ^2_θ	...	...	.139-2
January	−.337-1	−.120	−.441-1
	(.184-1)	(.110-1)	(.669-1)
February	.587-2	.223-1*	.396-2
	(.170-1)	(.953-2)	(.430-1)
March	−.296-1	.213-3	.369-1
	(.159-1)	(.942-2)	(.364-1)
April	−.200-1	.135-2	−.424-2
	(.171-1)	(.105-1)	(.253-1)
May	−.959-2	−.427-1*	−.259-1
	(.193-1)	(.102-1)	(.800-1)
June	.699-2	−.368-1*	−.176-1
	(.161-1)	(.881-2)	(.594-1)
July	.117-1	−.560-1*	−.180-1
	(.181-1)	(.916-2)	(.859-1)
August	.123-1	−.455-1*	−.343-1
	(.161-1)	(.874-2)	(.856-1)
September	−.198-1	.309-1*	−.194-1
	(.152-1)	(.898-2)	(.167-1)
October	−.350-1	.145-1	−.127-1
	(.184-1)	(.124-1)	(.306-1)
November	.489-2	−.520-1	−.281-1
	(.201-1)	(.103-1)	(.108)
Constant	.934-2	.174-1*	.193-1
	(.130-1)	(.734-2)	(.494-1)
$\bar{R}^2$	.37	.68	.156
D-W	1.55	2.02	N.A.
$Q(13)$	22.7	14.5	39.0*

Residual Autocorrelations of (15): Lag	Rho
1	−.423*
	(.0953-1)
2	−.171-1
	(.111)
3	.203-2
	(.111)
4	.125
	(.111)
5	.121
	(.112)
6	−.182
	(.114)
7	.185
	(.116)
8	−.483-1
	(.119)
9	.335-1
	(.119)
10	.698-1
	(.119)
11	.806-1
	(.120)
12	−.187
	(.120)
13	−.212-1
	(.123)

Note.—Granger test: $F(8, 82) = .640$. Bubble test 1: $\chi^2(1) = 56.6$*; bubble test 2: $\chi^2(1) = 6.52$*. Standard errors in parentheses. N.A. means statistic not calculated.

* Indicates significance at the 5 percent level.

TABLE 3

U.S.-German Data: Income Elasticity a_1 = .4

Parameter	Equation (12a)	Equation (12b)	Equation (15)
$-a_2$	1.87	. . .	−8.26
	(.786)		(7.16)
c	. . .	−.270*	. . .
		(.976-1)	
Average interest elasticity	.22	. . .	−.97
σ^2_ϵ	.103-2	. . .	. . .
σ^2_δ	. . .	.393-3	. . .
σ^2_θ	. . .	. . .	.129-2
January	−.257-1	−.205-1	−.395-1
	(.178-1)	(.109-1)	(.511-1)
February	.101-1	−.208-1*	.578-2
	(.159-1)	(.958-2)	(.272-1)
March	−.294-1	.272-2	−.352-1
	(.156-1)	(.982-2)	(.248-1)
April	−.198-1	.534-2	−.361-2
	(.163-1)	(.106-1)	(.183-1)
May	−.833-2	−.420-1*	−.213-1
	(.183-1)	(.105-1)	(.531-1)
June	.108-1	−.392-1*	−.142-1
	(.157-1)	(.935-2)	(.409-1)
July	.217-1	−.687-1*	−.118-1
	(.177-1)	(.950-2)	(.655-1)
August	.239-1	−.560-1*	−.287-1
	(.156-1)	(.948-2)	(.613-1)
September	−.225-1	.439-1*	−.213-1
	(.156-1)	(.936-2)	(.216-1)
October	−.404-1*	.218-1	−.115-1
	(.180-1)	(.132-1)	(.210-1)
November	.134-2	−.423-1*	−.227-1
	(.183-1)	(.105-1)	(.650-1)
Constant	.709-2	.170-1*	.166-1
	(.121-1)	(.730-2)	(.322-1)
$\bar{R}^2$	.44	.71	.30
D-W	1.57	2.03	N.A.
$Q(13)$	22.5	15.7	39.0*

Residual Autocorrelations of (15): Lag	Rho
1	−.429*
	(.953-1)
2	−.111-1
	(.111)
3	.595-3
	(.111)
4	−.121
	(.111)
5	.121
	(.112)
6	−.188
	(.113)
7	.192
	(.116)
8	−.520-1
	(.119)
9	−.331-1
	(.119)
10	.676-1
	(.119)
11	.890-1
	(.120)
12	−.184
	(.120)
13	−.208-1
	(.123)

Note.—Granger test: $F(8, 82)$ = .656. Bubble test 1: $\chi^2(1)$ = 71.5*; bubble test 2: $\chi^2(1)$ = 9.13*. Standard errors in parentheses. N.A. means statistic not calculated.
* Indicates significance at the 5 percent level.

TABLE 4

U.S.-GERMAN DATA: INCOME ELASTICITY a_1 = .5

Parameter	Equation (12a)	Equation (12b)	Equation (15)	Residual Autocorrelations of (15): Lag	Residual Autocorrelations of (15): Rho
$-a_2$	1.56*	...	−6.37	1	−.415*
	(.740)		(4.29)		(.953-1)
c	...	−.287*	...	2	−.990-2
		(.971-1)			(.111)
Average interest elasticity	.18	...	−.75	3	−.177-2
					(.111)
				4	−.119
					(.111)
σ^2_ϵ	.104-2	...	...	5	.120
					(.112)
σ^2_δ	...	.456-3	...	6	−.189
					(.113)
σ^2_θ	...	...	.126-2	7	.194
					(.116)
January	−.177-1	−.294-1*	−.369-1	8	−.529-1
	(.173-1)	(.111-1)	(.440-1)		(.119)
February	.145-1	−.200-1	.582-2	9	.322-1
	(.154-1)	(.101-1)	(.207-1)		(.119)
March	−.291-1	.495-2	−.350-1	10	.664-1
	(.154-1)	(.104-1)	(.201-1)		(.119)
April	−.196-1	.894-2	−.395-2	11	.937-1
	(.158-1)	(.110-1)	(.162-1)		(.119)
May	−.706-2	−.416-1*	−.196-1	12	−.183
	(.176-1)	(.109-1)	(.407-1)		(.120)
June	.148-1	−.419-1*	−.128-1	13	−.189-1
	(.155-1)	(.101-1)	(.319-1)		(.122)
July	.316-1	−.820-1*	−.809-2		
	(.173-1)	(.101-1)	(.326-1)		
August	.357-1*	−.671-1*	−.259-1		
	(.153-1)	(.107-1)	(.563-1)		
September	−.248-1	.564-1*	−.241-1		
	(.165-1)	(.101-1)	(.501-1)		
October	−.459-1*	.291-1*	−.116-1		
	(.177-1)	(.141-1)	(.260-1)		
November	−.209-2	−.330-1*	−.219-1		
	(.170-1)	(.108-1)	(.448-1)		
Constant	.475-2	.169-1*	.158-1		
	(.116-1)	(.747-2)	(.243-1)		
$\bar{R}^2$	.52	.74	.41		
D-W	1.59	2.03	N.A.		
$Q(13)$	22.4	16.9	38.7*		

NOTE.—Granger test: $F(8, 82)$ = .693. Bubble test 1: $\chi^2(1)$ = 90.40*; bubble test 2: $\chi^2(1)$ = 14.17*. Standard errors in parentheses. N.A. means statistic not calculated.

* Indicates significance at the 5 percent level.

TABLE 5

U.S.-U.K. DATA: INCOME ELASTICITY a_1 = .3

PARAMETER	EQUATION (12a)	EQUATION (12b)	EQUATION (15)	RESIDUAL AUTO-CORRELATIONS OF (15): Lag	Rho
$-a_2$	1.22	...	−5.99	1	−.401*
	(.802)		(3.10)		(.953-1)
c	...	−.174	...	2	−.497-1
		(.101)			(.110)
Average interest elasticity	.14	...	−.71	3	.358-1
					(.110)
				4	−.121
					(.110)
σ^2_ϵ	.347-3	...	...	5	.414-1
					(.111)
σ^2_δ	...	.286-3	...	6	.335-2
					(.111)
σ^2_θ	...	...	.873-3	7	.512-1
					(.111)
January	.783-2	.545-1*	.142-1	8	−.995-1
	(.172-1)	(.963-2)	(.371-1)		(.111)
February	−.135-1	.211-1*	.219-1	9	.808-1
	(.124-1)	(.803-2)	(.246-1)		(.112)
March	−.139-1	.276-1*	.363-2	10	.206-1
	(.142-1)	(.869-2)	(.297-1)		(.113)
April	−.795-2	.613-2	−.132-2	11	.765-1
	(.127-1)	(.822-2)	(.172-1)		(.113)
May	.206-1	.958-2	.170-1	12	−.156
	(.131-1)	(.913-2)	(.159-1)		(.113)
June	−.476-2	.370-1*	−.695-2	13	.772-1
	(.162-1)	(.974-2)	(.285-1)		(.115)
July	.302-2	.562-2	.196-1		
	(.125-1)	(.800-2)	(.180-1)		
August	.176-1	−.117-1	.124-1		
	(.131-1)	(.926-2)	(.156-1)		
September	−.219-1	.715-1*	−.636-2		
	(.188-1)	(.984-2)	(.458-1)		
October	−.347-1*	.392-1*	.889-2		
	(.121-1)	(.830-2)	(.272-1)		
November	−.251-1	.529-1*	−.229-2		
	(.143-1)	(.801-2)	(.348-1)		
Constant	.659-2	−.294-1*	−.650-2		
	(.130-1)	(.601-2)	(.227-1)		
$\bar{R}^2$	.74	.72	.44		
D-W	1.93	1.93	N.A.		
$Q(13)$	9.92	13.0	28.1*		

NOTE.—Granger test: $F(8, 82)$ = 1.615. Bubble test 1: $\chi^2(1)$ = 148.05*; bubble test 2: $\chi^2(1)$ = 23.52*. Standard errors in parentheses. N.A. means statistic not calculated.
* Indicates significance at the 5 percent level.

TABLE 6

U.S.-U.K. DATA: INCOME ELASTICITY $a_1 = .4$

PARAMETER	EQUATION (12a)	EQUATION (12b)	EQUATION (15)	RESIDUAL AUTO-CORRELATIONS OF (15): Lag	Rho
$-a_2$	1.25	...	−6.74*	1	−.403*
	(.723)		(3.06)		(.953-1)
c	...	−.200*	...	2	−.509-1
		(.100)			(.110)
Average				3	.375-1
interest					(.110)
elasticity	.15	...	−.80	4	−.122
					(.110)
σ^2_ϵ	.700-3	...	...	5	.413-1
					(.111)
σ^2_δ	...	.347-3	...	6	.136-2
					(.111)
σ^2_θ	...	...	.885-3	7	.526-1
					(.111)
January	.108-1	.574-1*	.144-1	8	−.101
	(.176-1)	(.110-1)	(.349-1)		(.112)
February	−.166-1	.319-1*	.211-1	9	.810-1
	(.127-1)	(.879-2)	(.362-1)		(.112)
March	−.172-1	.340-1*	.351-2	10	.207-1
	(.143-1)	(.944-2)	(.297-1)		(.113)
April	−.504-2	.343-2	−.821-3	11	.749-1
	(.128-1)	(.912-2)	(.162-1)		(.113)
May	.239-1	−.652-2	.164-1	12	−.155
	(.137-1)	(.105-1)	(.170-1)		(.113)
June	−.399-2	.387-1*	−.670-2	13	.772-1
	(.161-1)	(.107-1)	(.275-1)		(.115)
July	.563-2	.643-2	.197-1		
	(.127-1)	(.890-2)	(.178-1)		
August	.240-1	−.179-1	.131-1		
	(.133-1)	(.104-1)	(.146-1)		
September	−.239-1	.871-1*	−.759-2		
	(.202-1)	(.114-1)	(.488-1)		
October	−.439-1*	.519-1*	.830-2		
	(.122-1)	(.933-2)	(.285-1)		
November	−.300-1*	.639-1*	−.284-2		
	(.146-1)	(.881-2)	(.360-1)		
Constant	.698-2	−.343-1*	−.631-2		
	(.107-1)	(.673-2)	(.230-1)		
$\bar{R}^2$	.63	.74	.53		
D-W	1.63	1.93	N.A.		
$Q(13)$	20.8	9.92	28.4*		

NOTE.—Granger test: $F(8, 82) = 1.664$. Bubble test 1: $\chi^2(1) = 151.09$*; bubble test 2: $\chi^2(1) = 26.47$*. Standard errors in parentheses. N.A. means statistic not calculated.

* Indicates significance at the 5 percent level.

the model (15) are reasonable for both data sets except the U.S.-German model with $a_1 = .3$. For this model the $\bar{R}^2$ is low and the interest elasticity is quite large by comparison with extant empirical work. Strong evidence of bubbles also emerges from an analysis of the DM/£ cross rate (not reported), although the residuals of (15) indicate the presence of a complicated serial correlation pattern. This suggests that the hybrid monetary model may be misspecified on the German-U.K. data set. Tests for bubbles were also performed using the U.S.-Japanese exchange rate. While test statistics here also provide evidence of bubbles, the adequacy of (15) as a description of the U.S.-Japanese data set is seriously in doubt. The $\bar{R}^2$ of (15) is low, the focus parameters are imprecisely estimated, and the disturbance term appears to possess a complicated serial correlation pattern. For these reasons the U.S.-Japanese bubble tests (not reported) are inconclusive.

It is frequently the case that empirical models of asset prices exhibit conditionally heteroscedastic disturbances, as these markets experience episodes of relative calm and turbulence (see White [1980] or Engle [1982] for econometric methodology, and Cumby and Obstfeld [1984] for an example using exchange rate data). For the U.S.-U.K. model I found no evidence of autoregressive conditional heteroscedastic (ARCH) disturbances in (15) or (12b) using either four or 12 lags of squared residuals. There is strong evidence of first-order ARCH disturbances in the U.S.-German exchange rate model (15), but not in the driving process (12b). Reestimation of (15) using Engle's (1982) one-step scoring algorithm to fit a first-order ARCH process to the U.S.-German model leaves the first test statistic (bubble test 1) in tables 2–4 qualitatively unaffected.

West (1984*b*) has suggested an additional test for bubbles that does not depend on a comparison of two different estimates of b from models (12) and (15). This variance bound procedure is based on the result that for a random variable q_t

$$E[E(q_t|Q_t) - E(q_t|Q_{t-1})]^2 \geqq E[E(q_t|\Phi_t) - E(q_t|\Phi_{t-1})]^2, \qquad (23)$$

where the information set Q_t is contained in Φ_t. To make use of (23) in our context, let $q_t = (1 - b)\sum_{i=0}^{\infty} b^i x_{t+i}$ and $Q_t = \{x_{t-i}, i \geqq 0\}$. Then the left-hand side (LHS) of (23) is $\sigma_\delta^2/(1 - bc)^2$. The right-hand side (RHS) can be shown to be $E(\eta_t - \epsilon_t)^2$. Under the null hypothesis of no bubble, both sides are equal since x_t is assumed exogenous. Under the bubble alternative, West (1984*b*) shows that the RHS of (23) may exceed the LHS; the direction of the variance inequality will depend on the covariance of the bubble with the market fundamental process x_t. Nevertheless, if (23) is violated, it is possible that bubbles are present. Under assumptions (2) and (3), we can substitute for ϵ_t and $\eta_t = s_t$

$- E(s_t|\Phi_{t-1})$ to get $E(\eta_t - \epsilon_t)^2 = E(i^*_{t-1} - i_{t-1} + p_t - p_{t-1})^2$, the ex post real interest differential. Using the logarithm of the ratio of U.S. to foreign consumer price indexes for p_t, 3-month interbank rates for i_t and i^*_t (expressed at monthly rates), and the values of $\hat{b}$ (from [15]), $\hat{c}$, and $\hat{\sigma}^2_\delta$ (from [12b]) in tables 2–6, we see that the variance inequality (23) is not violated when the RHS is estimated by the sample variance of $(i^*_{t-1} - i_{t-1} + p_t - p_{t-1})$, but is violated when the RHS is estimated by the sample second moment of $(i^*_{t-1} - i_{t-1} + p_t - p_{t-1})$ around zero. I did not conduct a test for the significance of the latter result.

The no-bubbles results for (23) based on the sample variance of $(i^*_{t-1} - i_{t-1} + p_t - p_{t-1})$ suggest that the earlier bubble findings may be attributable to the maintained hypothesis of a stable driving process for x_t. Indeed, the most menacing empirical regularity that confronts exchange rate modelers is the failure of the current generation of empirical exchange rate models to provide stable results across subperiods of the modern floating rate period. The results of this paper provide no exception to the rule. Tests for bubbles were conducted for two subperiods of the sample: the pre- (October 1973–September 1979) and post- (October 1979–November 1982) change in Federal Reserve operating procedures. For the U.S.-German data set, the first subperiod estimation results of (12a), (12b), and (15) are qualitatively the same as those reported in table 1, when the income elasticity of money demand is greater than or equal to .40. In the second subperiod, an AR(1) process is no longer appropriate for Δx_t. Nevertheless, when AR(4) is used to approximate Δx_t, these results again provide evidence of bubbles for $a_1 \geqq .40$. Smaller values of the income elasticity result in the failure of equation (15) to provide an adequate description of the data, as the estimated b exceeds one for small values of a_1.

Split-sample results from the U.S.-U.K. data bear little resemblance to the full-sample results reported in table 1. In this case, equation (15) fails to provide sensible estimates of the interest semielasticity of money demand for all values of a_1 in either subperiod.

IV. Additional Analysis of the No-Bubbles Hypothesis for Exchange Markets

The test results of Section III constitute joint tests of the transversality condition and the assumption of a time-stable autoregressive representation of x_t. There are other maintained hypotheses as well. However, the assumptions of UCIP, identical transaction-type money demand equations, and deviations from PPP following a random walk are common to both empirical models (12) and (15). Since the model (15) appears to fit adequately in sample, these assumptions are

TABLE 7

TESTS FOR INTEGRATION OF ORDER 1 USING EQUATION (24)

Dependent Variable	$\hat{f}_1$	t-Ratio for f_1	$\hat{f}_2$	Roots of the Lag Polynomial*	
United States–Germany:					
s_t	−.026	−1.227	.206	1.034	4.703
m_t	−.041	−1.695	−.189	1.036	−5.096
y_t	−.146	−2.471	−.284	1.124	−3.128
x_t: a_1 = .3	−.043	−1.754	−.242	1.035	−3.984
x_t: a_1 = .4	−.044	−1.771	−.260	1.036	−3.706
x_t: a_1 = .5	−.046	−1.786	−.276	1.037	−3.498
United States–United Kingdom:					
s_t	−.021	−1.047	.260	1.029	3.733
m_t	−.012	−1.472	−.056	1.012	−17.506
y_t	−.071	−1.816	−.194	1.315	−4.854
x_t: a_1 = .3	−.014	−1.529	−.178	1.012	−5.541
x_t: a_1 = .4	−.015	−1.515	−.203	1.013	−4.863
United States–Japan:					
s_t	−.030	−1.288	.116	1.035	8.324
m_t	−.075	−2.214	−.224	1.065	−4.195
y_t	.020	−.900	.235	.984	−4.330
x_t: a_1 = .3	−.057	−1.753	−.228	1.048	−4.190
x_t: a_1 = .4	−.049	−1.532	−.228	1.041	−4.213

NOTE.—All regressions include a constant and 11 seasonal dummy variables. Observations on the dependent variable cover October 1973–May 1982; T = 110.

* The roots are the solutions of $1 - (1 + \hat{f}_1 + \hat{f}_2)L + \hat{f}_2 L^2$, where L is the lag operator.

roughly consistent with the data. Nonetheless, there exists considerable skepticism regarding the adequacy of monetary models as explanations of exchange rate behavior over the current floating rate period (see Meese and Rogoff 1983*a*, 1983*b*, 1985).

To check robustness of the findings of the previous section, table 7 contains the results of tests for the order of integration of exchange rates, relative money supplies, and real incomes. A time series is integrated of order d, denoted I(d), if it must be differenced d times to achieve stationarity. Regressions of the following form are reported in table 7, where s_t is used for expository convenience only:

$$\Delta s_t = \text{(constant and seasonal dummies)} + f_1 s_{t-1} + f_2 \Delta s_{t-1} + \text{error}. \tag{24}$$

If (24) contains a unit root, the t-ratio for f_1 should be consistent with the hypothesis $f_1 = 0$. Conventional t-tables are inappropriate for this hypothesis test, and we use the results of Dickey and Fuller (1979) and the tabulated distribution in Fuller (1976) to interpret the t-statistic. The critical value for the t-ratio using a 5 percent significance level is approximately −2.89. With this test all three spot rates ($/DM, $/yen, and $/£) appear to be I(1). For all values of a_1 reported in tables 2–6,

x_t appears to be I(1). When m_t and y_t are treated separately, the conclusion that the market fundamentals are I(1) remains intact.

It is always the case that point estimates of the roots of the s_t process from (24), the last two columns of table 7, indicate that the \$/DM spot rate is "less stationary" than its market fundamentals. This accords with the Diba-Grossman (1984) and Hamilton-Whiteman (in press) observation that if the asset price and market fundamentals are integrated of the same order, bubbles cannot exist. Unfortunately, the unit root test employed above has low power against a plausible range of borderline stationary alternatives and a smaller range of borderline nonstationary alternatives.[17]

Point estimates of the roots of (24) for the \$/£ and \$/yen rates are not as sanguine for the bubbles hypothesis. The U.S.-U.K. x_t process has a smaller root than the \$/£ rate, as the smallest root of the relative money supply variable m_t is 1.012. The U.S.-Japanese x_t process has larger (more stable) roots than the \$/yen rate even though the relative real income variable y_t appears to have an explosive root.[18] At best, we find corroborative evidence for the bubbles hypothesis for two of the three currencies considered. In the case of the U.S.-U.K. data set, the unit root tests suggest that the large test statistics of Section III might be a consequence of the assumed time-stable autoregressive representation (12b). This observation is consistent with the split-sample results discussed at the end of that section.

Continuing in this vein of descriptive data analysis, table 8 contains the results of tests for cointegration of exchange rates and market fundamentals. The concept of cointegration was introduced by Granger (1983) and is refined in Granger and Engle (1984) and Stock (1984). Succinctly, if each element of a vector time series must be differenced d times to induce stationarity, yet a linear combination of

[17] Dickey and Fuller (1979, p. 430) tabulate the results of Monte Carlo power calculations for regressions of the form of (24) when s_t is generated by an AR(1) process with a constant c_0, AR coefficient ρ, and variance of disturbance σ^2. For the sample size used in this study, the powers of the unit root test for size .05 tests for alternatives $\rho = .9, .95, .99, 1.02$, and 1.05 are, respectively, .55, .17, .04, .59, and .97. Evans and Savin (1981, p. 771) construct exact power functions for $\rho = 1$ against $\rho \neq 1$ for size .05 tests assuming normally distributed errors and using several ratios of c_0/σ. When $T = 100$ and $c_0/\sigma = 0$ are used, the powers of the unit root test for alternatives .9, .95, .99, 1.01, 1.025, and 1.05 are, respectively, .56, .18, .05, .13, .70, and 1.00. In asset price models deterministic bubbles grow at rate $1/b$, where b is the "discount factor." Reported b's are usually quite close to one. Thus, without a larger sample size, it will be difficult to delineate between a deterministic bubble and an I(1) process. Finally, given the results of table 1 for a stochastic bubble, it may be impossible to distinguish these types of bubbles from I(1) or borderline stationary processes using conventional time-series analyses.

[18] The sign of $\hat{f}_1$ for U.S.-Japanese y_t is wrong for the variable to be stationary in levels. If a time trend is included in this regression, it is significant (t-ratio of -2.636) and the roots become stationary (1.030 and -3.881).

TABLE 8

TESTS FOR COINTEGRATION OF EXCHANGE RATES AND MARKET FUNDAMENTALS

Country and Regression	a_1	$\hat{g}_1$	$\hat{g}_2$	$\hat{g}_3$	$\hat{g}_4$	D-W	D-F
United States–Germany:							
1	.3	−1.348				.16	−1.96
1	.4	−1.296				.17	−2.04
1	.5	−1.243				.18	−2.10
2	.3	−.150	−1.219			.14	−2.11
2	.4	−.191	−1.126			.14	−2.19
2	.5	−.228	−1.038			.14	−2.26
3	N.A.	−.102	−1.232	.135	.351	.14	−2.16
United States–United Kingdom:							
1	.3	.248				.05	−1.07
1	.4	.237				.05	−1.06
2	.3	.290	−.042			.05	−1.07
2	.4	.182	.055			.05	−1.06
3	N.A.	.873	−.421	.637	−.109	.08	−1.57
United States–Japan:							
1	.3	−.754				.08	−1.31
1	.4	−.607				.08	−1.26
2	.3	.254	−1.059			.09	−1.41
2	.4	.373	−1.032			.09	−1.35
3	N.A.	−.541	−.791	.244	−.792	.13	−1.85

NOTE.—Regression 1: $s_t = g_1x_t$ + error; regression 2: $s_t = g_1x_t + g_2x_{t-1}$ + error; regression 3: $s_t = g_1m_t + g_2m_{t-1} + g_3y_t + g_4y_{t-1}$ + error. All regressions include a constant and 11 seasonal dummies. Observations on the dependent variable cover October 1973–May 1982; $T = 110$. Durbin-Watson (D-W) statistic: 5 percent critical value = .282; Dickey-Fuller (D-F) residual statistic: 5 percent critical value = −3.17. The D-W and D-F critical values for regressions 2 and 3 must be interpreted with care since Granger and Engle (1984) provide simulation results only for the bivariate (simple regression) case.

the vector time series need be differenced only $(d - b)$ times, then the vector time series is said to be cointegrated of order (d, b).

Suppose for the moment that exchange rates, relative money supplies, and real incomes are all I(1). Call the vector time series with these three components $\mathbf{q}_t$. If $\mathbf{q}_t$ is cointegrated, there exists a possibly nonunique vector $\boldsymbol{\alpha}$ such that $z_t = \boldsymbol{\alpha}'\mathbf{q}_t$ is I(0) (z_t is covariance stationary in levels). Granger and Engle (1984) label z_t the "equilibrium error." In this example, cointegration means that if the components of $\mathbf{q}_t$ are I(1), then the equilibrium error is I(0), and z_t will rarely drift far from zero (if it has zero mean) and will often cross the zero line. In other words, equilibrium periodically occurs (see Granger and Engle 1984, p. 4). Cointegration places restrictions on the low-frequency components of $\mathbf{q}_t$ as the levels of these series have infinite variance, yet a linear combination of levels has finite variance. Clearly, if the exchange rate and its market fundamentals are cointegrated, then bubbles cannot be present as the equilibrium error is I(0).[19]

[19] As pointed out earlier, it cannot be literally true that exchange rates and market fundamentals are integrated of the same order if bubbles are present. However, estab-

Given that s_t and x_t (or s_t, m_t, and y_t) appear to be I(1), the test of cointegration proceeds as follows: the equilibrium regression s_t on x_t (or s_t on m_t and y_t) is calculated. The null hypothesis of no cointegration of s_t and x_t (or s_t, m_t, and y_t) is rejected if the Durbin-Watson statistic is larger than the critical values tabulated by Granger and Engle (1984, tables 2, 3). The 5 percent critical value is approximately .282. The preferred test of cointegration by Granger and Engle (1984) is an augmented Dickey-Fuller (1979) test of the form (24) using the residuals of the equilibrium regression of s_t on x_t. The critical value for the t-test on f_1 is, for a 5 percent significance level, approximately -3.17 (see Granger and Engle 1984, tables 2, 3).

Two types of cointegration tests are reported in table 8. The first ignores the structure of the model presented in Section II. The maintained hypothesis here is that the appropriate market fundamentals for the exchange rate are contemporaneous relative money supplies and real incomes. The second test of cointegration exploits more of the structure of equation (12). In the equilibrium regression the spot rate is regressed on both x_t and lagged x_t. Both sets of tests on the $/DM, $/yen, and $/£ exchange rates indicate lack of cointegration at standard significance levels, even if x_t is broken into its separate m_t and y_t components. These results are consistent with the possible existence of bubbles in exchange markets.

V. Conclusion

While many economists believe that asset prices reflect the values of the underlying market fundamentals, asset market participants often express the view that fundamentals are just part of the story. Characterizations of asset price movements from the latter group often include discussions of "extraneous events"; see, for example, the frequent explanations of exchange rate or other asset price movements given in the "What's News" column of the *Wall Street Journal*. As Blanchard and Watson (1982, p. 1) point out, "economists have overstated their case. Rationality of both behavior and of expectations often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the price from this value, rational bubbles."

lishing that a time series is I(1) is a "judgment call" given the low power of unit root tests to detect borderline alternatives. Nonetheless, it would appear that a finding of cointegration is clearly inconsistent with bubbles, while acceptance of the hypothesis of no cointegration may or may not be consistent with bubbles. The problem with our descriptive analysis is clearly manifest in table 1. If bubbles are present, then the class of ARIMA models is not likely to provide an adequate representation of the time series. If a test for cointegration is performed, assuming both s_t and x_t are I(1) when a bubble is present—i.e., s_t is not actually I(1)—then I would expect not to find cointegration if sample size is large enough.

This paper provides mixed evidence of asset market bubbles or extraneous variables in exchange markets, using a monthly monetary model of the dollar/deutsche mark and the dollar/pound exchange rates. The model appears to be better suited to the dollar/deutsche mark rate, as split-sample tests highlight the inability of the monetary model to explain the dollar/pound rate across two different U.S. monetary policy regimes. Since the statistical test developed in this paper is a joint test of no bubbles, stable driving processes for the exogenous variables, and other maintained hypotheses of standard monetary exchange rate models, additional tests of the no-bubbles hypothesis were conducted using statistical procedures with less maintained assumptions.

Last, it is striking that any evidence of bubbles emerges from a sample of monthly observations, 1973–82, thought not to be characterized by extraordinary events such as hyperinflations or "tulip mania." What circumstantial evidence of bubbles that exists comes from daily exchange rate data: (1) filter rule profits net of transaction costs seem excessive, (2) the distribution of daily exchange rate changes is leptokurtic, and (3) rates move much more when markets are open than between ending and opening quotes overnight or over market holidays. If the distribution of "news" is uniform over the week, the latter phenomenon seems puzzling. Unfortunately, the finest sampled data for exchange rates that include most of the relevant market fundamentals are monthly. Since most of the leptokurtosis in daily exchange rate changes disappears as the time difference increases, it is indeed surprising to detect bubbles using monthly exchange rate data. As such, future work will attempt to exploit more finely sampled data, and the power of the test statistics will be examined in a simulation experiment where the sensitivity of the test statistics to various forms of model misspecification can be studied in great detail.

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