Collapsing exchange rate regimes:
Another linear example

Robert P. Flood\textsuperscript{a,\textdagger}, Peter M. Garber\textsuperscript{b}, Charles Kramer\textsuperscript{a}

\textsuperscript{a}Research Department, International Monetary Fund, 700 19th St. NW, Washington, DC 20431, USA
\textsuperscript{b}Brown University, Providence, RI, USA

Received 4 December 1995; revised 8 January 1996

---

Abstract

In the literature on speculative attacks on a fixed exchange rate it is usually assumed that the monetary authority responsible for fixing the rate reacts passively to the monetary disruption caused by the attack. This assumption is grossly at odds with actual experience where the monetary base implications of the attacks are usually sterilized. Such sterilization means that the attack is no longer exclusively a monetary event and requires that the standard attack model be broadened to include the effects of sterilization on other markets. These issues are motivated by reference to the December 1994 collapse of the Mexican peso.

\textit{Key words:} Exchange rates; Exchange rate regimes; Speculative attacks

\textit{JEL classification:} F31; F32; G15

---

1. Introduction

When a country pursues a policy of fixing its exchange rate or controlling the rate of currency depreciation relative to one or more trading partners, market participants understand that the exchange rate policy will be maintained as long as it does not conflict with other, more important, economic policies or political

\textsuperscript{\textdagger}Corresponding author: E-mail: rflood@imf.org.
constraints. Believing that the exchange rate policy will be altered eventually, inventors can precipitate a series of events, a speculative attack, that tests the credibility of the commitment and the ability to maintain the exchange rate policy. Speculative attacks on the currency bands in the European Monetary System in 1992 and the collapse of the Mexican Peso in December 1994 underscore the relevance of studying such attacks.¹

The literature modeling speculative attacks as the market response to unsustainable government price fixing policy began with Salant and Henderson (1978), who developed the main ideas in an application to a government policy to fix the price of gold. The modeling then moved to the foreign exchange market where Krugman (1979) and Flood and Garber (1984b) adapted the Salant and Henderson model for collapsing exchange rate regimes. In the exchange rate model, the policy eventually to be attacked is an announced fixed exchange rate. The country responsible for the fixed rate also adopts other high-priority expansionary policies inconsistent with the long-term maintenance of the fixed rate. Because of the expansionary, high-priority policy, the modeled country slowly depletes its international reserves, which are exhausted in a final discrete speculative attack. Following the attack, the high-priority policies continue unaltered and the exchange rate policy is switched to a sustainable alternative.²

In this attack scenario base money and its components, international reserves plus domestic credit, play the leading roles. In addition to fixing the exchange rate, it is assumed that the government finances a real fiscal deficit by expanding domestic credit. Real and nominal money demand are fixed in these models while the exchange rate is fixed so that domestic credit expansion results in exactly equal reserve losses, or to give an opposite spin, reserve losses are perfectly sterilized during the fixed rate regime.³ Money financing of the fiscal deficit is the high-priority policy that is inconsistent with the fixed rate. Eventually, the international reserves that serve as a buffer between the fixed exchange rate policy and the overly expansionary domestic credit policy are exhausted and the fixed exchange rate policy is abandoned.

The distinctive feature of the attack models is that foreseen speculative attacks are consistent with private equilibrium. The exchange rate need not jump at the time of the attack. Instead of having prices jump, the Salant and Henderson insight was the prediction that the attack would take place at precisely the time prices need not jump. In the context of a speculative attack on a fixed exchange rate, this is when the change in money supply due to the attack is exactly balanced by the

¹The European experience is studied in Goldstein et al. (1993) and the Mexican experience is studied in International Monetary Fund (1995).
²Flood and Garber (1984a) and Obstfeld (1986) investigate the possibility that the attack causes changes in the high-priority policy, which can result in multiple equilibria.
³In empirical applications, such as Blanco and Garber (1986), money demand is not fixed. It responds to output changes, terms-of-trade changes and to a disturbance term. In the applications, money demand shifts assume as important a role as does money supply.
change in money demand due to the interest rate effect of the policy change to a sustainable regime. In the current paper we challenge the applicability of the standard attack scenario. The standard requires the supply of base money in the country being attacked to plunge downward at the instant of the attack, reflecting the reserve loss during the attack. In recent experiences, that assumption does not fit the facts.

Reserve losses at the time of the attacks were usually sterilized fully in recent attacks. With such sterilization, the standard attack model is a non-starter. In Section 2 we develop a version of the standard model, demonstrate its main results and illustrate its problems with an example drawn from the Mexican experience in 1994. In Section 3 we present a twist on the standard model that allows us to follow the trail of the attack's discrete portfolio adjustment from the money market, where it usually resides, into the bond markets, where it is driven by the sterilization-connected open-market operations.

2. The simplest model

The Flood and Garber (1984b) speculative attack model combines a money market equilibrium condition, Eq. (1), with uncovered interest rate parity, Eq. (2):\(^4\)

\[\frac{M}{P} = a_0 - a_1 i, \quad a_1 > 0,\]  
\[i = i^* + (\dot{S}/S).\]  
(1)

These equations are presented for a certainty example in continuous time, but the main points carry over to more realistic settings. In Eq. (1), \(M\) is the level of high-powered money, where \(M = R + D\), with \(R\) the domestic currency book value of international reserves and \(D\) the domestic credit held by the domestic monetary authority.\(^5\) \(P\) is the domestic price level. The domestic currency interest rate is \(i\). The foreign currency interest rate is \(i^*\), which is assumed constant. \(S\) is the exchange rate quoted as the domestic currency price of foreign exchange and \(\dot{S}\) is the time rate of change of \(S\).\(^6\) We also assume purchasing power parity so that \(P = P^* S\) with \(P^*\) the foreign price level, which is assumed to be constant. Domestic credit grows at the linear rate \(D = \mu > 0\), which is used to finance government expenditure.

\(^4\)In Section 2 we copy the Flood and Garber (F&G) model and notation except for the attack time, for which F&G used the notation \(z\) and we use \(T\).

\(^5\)For simplicity we assume that interest payments on government holdings of foreign securities either are rebated to the foreign government, roughly cancel with domestic interest payments to foreign governments or are part of the fiscal deficit.

\(^6\)A dot over a variable, e.g. \(x\) vs. \(\dot{x}\), signifies the right-hand time derivative.
While the exchange rate is fixed, $S = \bar{S}$ and $\dot{S} = 0$. From Eq. (1), the demand for domestic nominal money balances is, therefore, constant during the fixed rate regime. Reserve losses by the monetary authority exactly match domestic credit expansion. Money is constant but reserves are lost unit for unit as domestic credit expands. This process ends in a final discrete reserve loss, a speculative attack, that is foreseen by the private sector and is, in equilibrium, devoid of profit opportunities. We now turn to characterizing the final attack.

Following the final speculative attack, reserves are exhausted so that $M = D$, and the exchange rate is allowed to float freely taking on the value consistent with

$$D/P^* \hat{S} = a_0 - a_i(i^* + (\hat{S}/\bar{S})), \quad (3)$$

where $\hat{S}$ is the post-attack flexible exchange rate termed the shadow flexible exchange rate. We simplify notation by rewriting Eq. (3) as

$$D = \beta \hat{S} - \alpha \hat{\dot{S}}, \quad (3a)$$

where $\beta = P^*(a_0 - a_i i^*) > 0$ and $\alpha = a_i P^*$. The solution to this equation is

$$\hat{S} = \lambda_0 + \lambda_1 D, \quad (4)$$

where $\lambda_0 = (\alpha \mu / \beta^2)$ and $\lambda_1 = 1 / \beta$. Notice that the shadow freely floating rate does not depend on reserves, which we assume would have hit their minimum level, set at zero for simplicity.

Speculators profit in an attack by purchasing foreign exchange from the monetary authority at the fixed price $\bar{S}$ and reselling at the market-determined price $\hat{S}$. While $\bar{S} \lessdot \hat{S}$ and speculators would not profit by attacking the fixed rate so the regime survives. If speculators wait until $\bar{S} \gtrdot \hat{S}$, then the lucky speculators who purchase the monetary authority's reserves will reap a capital gain. In this perfect foresight example, however, luck has no role. Competition ensures that the foreseen attack takes place at the instant when there are no profits or losses, which requires $\bar{S} = \hat{S}$. To determine the date of the attack, set $\bar{S} = \hat{S}$. At this instant the decrease in money supply from the reserve loss is precisely equal to the decrease in money demand from the increase in the domestic interest rate. It follows that the attack takes place at time $T$, where

$$T = \frac{R_0 - (\alpha \mu / \beta)}{\mu}, \quad (5)$$

with $R_0$ the level of reserves at some arbitrary time $0$, which is the time at which the model is started. These results duplicate those presented in Flood and Garber (1984b).

Crucial to the above logic is the assumption that the monetary authority does not sterilize the reserve loss at the time of the attack. The implication of this assumption is displayed in Fig. 1, which tracks base money and its components before and after the attack. Prior to the attack, base money is constant with reserve
losses sterilized exactly by increases in domestic credit. At the time of the attack base money plunges downward by the size of the reserve loss at the attack. This is a striking aspect of this model, which is not always a good match for the data in actual attack episodes.

Fig. 2 plots Mexican base money and its components from January 1992 through September 1995. The (sliding) Mexican peso peg to the US dollar collapsed in December 1994. From the figure, it is clear that domestic credit is the near mirror image of international reserves, meaning that during the period large reserve movements were normally sterilized so that base money growth is
smoother than either of its components. This agrees with the model presented above and with Fig. 1. What does not agree with the model, however, is the behavior of the components of the base during the attack period. At the attack, reserve losses were sterilized fully so that the attack did not accompany a discrete reduction in base money. Such sterilization is clearly a relevant policy option and is probably the option most relevant to current policy modeling and discussions.\(^7\) When reserve losses at the time of the attack are sterilized fully, the standard model is inconsistent with a foreseen attack. In this model, when the authorities fully sterilize the reserve losses during an attack, the attack takes place at the inception of the fixed rate regime and speculators reap a capital gain.

If the authorities fully sterilize reserve losses, 3a becomes

\[
\bar{S} = \lambda_0 + \lambda_1 D^+, \tag{6}
\]

where \(\lambda_0\) and \(\lambda_1\) are as given above and the forcing variable is now denoted \(D^+\) to indicate that it measures domestic credit just after the attack. This distinction is significant because \(D\) jumps upward at the time of the attack to exactly sterilize reserve losses in the attack. It follows that \(D^+\) is always equal to the pre-attack base money supply. It follows further that \(\bar{S}\) is always greater than \(\bar{S}\). In other words, a fixed rate with full sterilization of reserve losses in an attack would be attacked immediately. Subtracting \(\bar{S}\) from \(\bar{S}\) in Eq. (5) results in

\[
\bar{S} - \bar{S} = \alpha \mu / \beta^2. \tag{7}
\]

The shadow flexible rate is always above the fixed rate when the attack reserve loss is sterilized fully.\(^8\) This model is, therefore, incapable of producing an anticipated future attack on a fixed exchange rate regime when the attack reserve loss is sterilized.\(^9\) Fully sterilizing reserve losses in the attack removes the discrete adjustment from the base money supply, the reserve loss, but does not remove the discrete adjustment from money demand due to the shift in the domestic currency interest rate. It is impossible to balance the jumps and the fixed rate cannot survive, even temporarily.

---

\(^7\)See International Monetary Fund (1995).

\(^8\)The shadow rate would always be above the fixed rate for stochastic growth in \(D\) or if the minimum level of \(R\) were not known for certain by the private sector in this simple model with full sterilization. We thus ignore these complications.

\(^9\)This is not a multiple equilibrium issue. There is one equilibrium and it is to attack at inception of the fixed rate.
3. A bond-market modification

Since attacks have been observed on fixed rate regimes that fully sterilize final reserve losses, with the attacks occurring long after the inception of the fixed rate regime, it is clear that the model needs modification. The policy action of reserve loss sterilization is a conscious effort to remove the discrete adjustments attached to an attack from the money market. When the monetary authority sterilizes reserve losses, it expands domestic credit by the size of the reserve loss so that the attack does not influence the base money supply. Domestic credit is normally expanded by the monetary authority using base money to purchase domestic government securities from the private sector. Attack sterilization, therefore, involves discrete bond-market shifts rather than money-market shifts.

Our approach to modeling anticipated sterilized attacks is to follow Blackburn (1988) and Willman (1988) by adding an explicit bond market to our monetary-approach model. Modeling so far required only an explicit money market. Bond-market considerations were handled by two assumptions: (1) perfect substitutability between domestic and foreign bonds, which is the certainty version of risk neutrality, and produces Eq. (2), and (2) the exogeneity of \( i^* \), the foreign interest rate. With \( i^* \) exogenous and Eq. (2) determining \( i \), the model did not need to confront domestic bond-market repercussions of reserve loss sterilization or the foreign bond-market implications of the interest-paying reserves released by the domestic monetary authority. We relax the model by dropping the perfect substitutes assumption, but we retain the exogeneity of \( i^* \). This is the certainty version of adding a risk premium to Eq. (2).

The perfect substitutes assumption is replaced by a bond-market model based on a convenient version of the portfolio balance model of Tobin (1971). We retain Eq. (1) but we drop Eq. (2) replacing it with

\[
B - SB^* = \theta^{-1}(i - i^* - (\dot{S}/S)SP^*w, \quad \theta > 0, \tag{8}
\]

where \( B \) is the stock of domestic currency denominated interest-paying debt issued by the domestic government and held in the private sector at home and abroad, and \( B^* \) is the analogous foreign magnitude; \( w \) is worldwide liquid real wealth, which is constant, so \( SP^*w \) is the domestic currency value of world wealth. Eq. (8) may be a bit unfamiliar; it results from having identical bond demand functions at home and abroad all proportional in wealth. Rearranging Eq. (8) gives a revised version of Eq. (2) augmented by the bond-market model, Eq. (8):

\[
i = i^* + (\dot{S}/S) + \frac{\theta(B - SB^*)}{SP^*w}, \quad \theta \geq 0, \tag{9}
\]
where $\theta(B - SB^*)/SP^*w$ is compensation required due to imperfect substitutability in perfect foresight.\(^1\)

We retain the assumption that domestic credit rises at the rate $\mu$ in order to finance government expenditure, but now we need to be explicit about the bond-market implications of interventions to fix the exchange rate and to sterilize reserve losses during an attack, when applicable. When the exchange rate is fixed, we rewrite Eq. (1) using other assumptions as

$$R + D = P^*S[a_0 - a_1[i^* + (\theta(B - SB^*)/\bar{SP}^*w)]].$$  \hspace{1cm} (10)

$P^*$ and $i^*$ are constant. $S$ is constant while the exchange rate is fixed, so $\bar{S} = 0$. For now it is also assumed that $B$ is constant. $B^*$, however, is not constant. The reserves that are drained from the monetary authority are interest-paying foreign government foreign-currency assets and the sum of all reserve losses is added to the initial $B^*$ available to the private sector. We assume that $B^*$ is constant at $B_0^*$ except for reserve movements, so that $B_i^* = B_0^* + (R_0 - R_i)/\bar{S}$. \hspace{1cm} (11)

We find the rate of reserve loss during the fixed rate regime by taking the time derivative of each side of Eq. (10):

$$\ddot{R} + \mu = -(a\theta_1/w)\dot{R},$$  \hspace{1cm} (12)

implying $\dot{R} = -\mu/[1 + (a_i\theta/w)] \equiv -\rho$ during the period exchange rates are

---

\(^1\) The perfect substitutes model in the previous section ran into trouble because there was no foreseen future jump in base money that could be matched up with a foreseen jump in the domestic interest rate due to foreseen exchange rate changes. The currently adopted modification allows the attack to be timed so that the jump in $\theta(B - SB^*)/SP^*w$ can be matched up with the jump in the expected rate of exchange rate change. Any modification of the model that allows something to jump in addition to the expected rate of exchange rate change can serve in theory as the relevant modification. In a stochastic model, the last term in Eq. (9) would be a model of the risk premium in which the quantities of domestic and foreign government bonds outstanding in the private sector are crucial to the interest rate differential. For a risk premium interpretation of the compensation required by foreigners (in the US) for holding (Mexican) domestic currency securities. In this view, $\theta$ would depend on the conditional moments of the returns on the available assets and would not be constant as a collapse approaches. To ensure that the 'risk premium' has the correct sign a constant term may be inserted into Eq. (8).

As in Calvo (1995), domestic and foreign bonds play an important role although in Calvo that role is due to bond-financing of fiscal deficits. Flood and Marion (1996) revisit the present model with bond-financing of the deficit and with $\theta$ time varying in accord with a model of risk and return preferences.

\(^{11}\) $R$ is denominated in domestic currency units so it must be converted to foreign currency units by dividing by the exchange rate.
fixed.\textsuperscript{12} This rate of reserve loss applies regardless of reserve sterilization during the inevitable attack. We next analyze the entire dynamic attack process using the new asset market structure under two options: (1) reserve losses in the attack are not sterilized, and (2) reserve losses in the attack are sterilized fully.\textsuperscript{13}

3.1. Option 1: no sterilization

This option replicates the policy studied in Section 2, but now the bond-market implications must be studied also. Following a successful attack, asset market equilibrium requires

\[ D = c_1 + \beta_1 \tilde{S} - \alpha_1 \tilde{S}, \tag{13} \]

where \( c_1 = -w^{-1} a_i \theta B_0, \) \( \beta_1 = P^*(\alpha_0 - a_i i^*) + w^{-1} a_i \theta (B_0^* + (R_0^*/\tilde{S})) \), \( \alpha_1 = \alpha = a_i P^* \) and \( B_0 \) and \( B_0^* \) are initial levels of \( B \) and \( B^* \) at the time the model is started. In the event of an attack, all remaining international reserves are purchased from the monetary authority, so the money supply falls at the attack from \( M = R + D \) to \( M = D \). These reserves reappear now in the bond market, increasing the aggregate privately held stock of foreign bonds by the size of the remaining reserve stock. The solution to Eq. (6) is:

\[ \tilde{S} = \lambda_{20} + \lambda_{21} D, \tag{14} \]

where \( \lambda_{21} = 1/\beta_1, \) \( \lambda_{20} = (\alpha_1 \lambda_{21} \mu - c_1)/\beta_1 \). Until the time of the attack, \( \tilde{S} \) is below \( \tilde{S} \). The attack occurs when \( \tilde{S} \) reaches \( \tilde{S} \) at time \( T_1 \), where

\[ T_1 = \frac{R_0 - \frac{\alpha_1 \lambda_{21} \mu}{[1 + (\theta a_i /w)]\beta_1}}{\rho}. \tag{15} \]

Notice that for \( \theta \to 0, \) \( \rho \to \mu, \) \( \lambda_{21} \to 1 \) and \( T_1 \to T \), as expected.

3.2. Option 2: sterilization at attack

When the monetary authority sterilizes the attack on reserves, domestic credit expands by precisely the size of the attack. This increase is accomplished by the monetary authority’s open-market purchase of \( B \) from the private sector in an amount equal to the size of the final attack. The sterilization operation, therefore, has additional bond-market consequences.

\textsuperscript{12} Eq. (12) is a special case that results from our assumption of identical bond demand functions at home and abroad. In general the time derivative of Eq. (10) would involve the time-dependent wealth transfer involved in current account imbalance.

\textsuperscript{13} In Appendix A we present the results for the model modified for partial sterilization.
Following a successful attack that is sterilized the asset markets are in equilibrium when

$$D^+ = P^*\hat{S}\left\{a_0 - a_1 \left[i^* + \frac{\hat{S}}{\hat{S}} + \frac{\theta[B_0 + R^* - \hat{S}(B_0^* + (R_0^*/\hat{S}))]}{P^*\hat{S}w}\right]\right\}, \quad (16)$$

where $D^+$ is the level of domestic credit immediately following the attack and is increased over pre-attack $D$ by precisely the amount of reserve losses. $D^+$ is equal to the pre-attack monetary base. $R^*$ is the level of international reserves held by the domestic monetary authority at time $t$. We rewrite Eq. (13) as

$$D^+ = c_{2i} + \beta_2\hat{S} - \alpha_2\hat{S}, \quad (17)$$

where $c_{2i} = c_1 + (R_0 - R_i)/\mu$, $\beta_2 = \beta_1$, and $\alpha_2 = \alpha_1$. Note that $c_{2i}$ is fixed from the attack time forward, but is variable until a successful attack.

The solution to Eq. (17) is:

$$\hat{S} = \lambda_{30i} + \lambda_{31}D^+, \quad (18)$$

where $\lambda_{31} = 1/\beta_2$, and $\lambda_{30i} = (c_{2i} - \alpha_{2i}\lambda_{31}\mu)/\beta_2$. In Eq. (17), the variable $R^*$ enters the solution through $c_{2i}$, but does not enter the forward-looking derivative of the solution with respect to time. $R^*$ is the magnitude of the one-and-for-all open-market pure base of domestic currency securities that is required to sterilize the attack. This variable now enters the expression for $\hat{S}$ because of the bond-market implications of the sterilization operation. The time derivative of $\hat{S}$ that is relevant to asset holders is the partial derivative with respect to $D^+$. Although $D$ jumps at the time of attack, its forward-looking derivative remains $\mu$.

If the fixed exchange rate regime is viable at its inception, $\hat{S}$ approaches $\hat{S}$ from below, reaching $\hat{S}$ at time $T_2$, where

$$T_2 = \frac{R_0 - (\mu P^*w/2\theta\beta_2)}{\rho} \quad (19)$$

With full sterilization, the fixed rate is viable temporarily if reserves are sufficiently high. In general, reserve losses in the final attack may be sterilized in any percentage, with 0 and 100% analyzed in the text.\(^{14}\)

4. Summary and tentative conclusions

The speculative attack literature has always made clear that government nominal price fixing will be successful indefinitely only when that policy is the govern-

\(^{14}\)The other possibilities are studied in Appendix A, where it is shown that partial sterilization delays an attack relative to full sterilization.
ment's highest monetary priority. When the monetary authority is asked to do other things, such as partially finance an extraordinary fiscal deficit or keep base money growth on a particular path then these multiple goals inevitably conflict. In the previous literature, fixing the exchange rate was taken as a secondary priority to financing the fiscal deficit. Too large a deficit would eventually destroy the fixed rate regime. When a base money target is added to the monetary authority's duties and is also given higher priority than the fixed rate, then the demise of the fixed rate is hastened. In the realistic case of full sterilization of a speculative attack we find that the usual monetary model is incapable of producing even a temporarily viable fixed rate regime. Because many fixed rate regimes survive, at least temporarily, we have adopted a simple modification of the pure monetary model. We dropped the covered interest parity assumption, replacing it with a portfolio balance model of the uncovered interest rate spread.

The main contribution of this paper is to point out the increased fragility of a fixed exchange rate policy when the monetary authorities also target the base money supply. A base money target removes the inherent 'braking mechanism' provided by reserve losses, and for full sterilization requires that the monetary model of a speculative attack be expanded to include explicit bond markets as well.

Acknowledgments

This paper is based in part on chapter 4 of International Monetary Fund (1995). We thank David Folkerts-Landau, Garry Schinasi, John Montgomery and other members of the IMF Capital Markets Division for comments. We also thank members of the Western Hemisphere and Policy Development and Review Departments for both data and comments. The views expressed here are those of the authors and do not necessarily reflect the views of Directors of the International Monetary Fund or other IMF employees.

Appendix A

Partial sterilization

The monetary authority may choose any degree of sterilization of the final attack. To reflect this possibility we rewrite the relevant text equations as

\[ D^* = D_0 + \mu t + \gamma R_t, \]  

(A.1)

where \( \gamma \) is the degree of sterilization. For \( \gamma = 0 \) there is no sterilization. For \( \gamma = 1 \)
sterilization exactly offsets reserve losses in the attack. In principle \( \gamma \) can be any real number. For sterilization governed by A.1 the attack time is

\[
T_g = \frac{R_0 - \frac{\alpha_1 \mu}{\beta_2[1 - \gamma + [\alpha, \theta(\gamma + 1)/P^* w]]}}{\rho},
\]

(A.2)

where \( T_g \) is the generalized attack time. The cases studied in the text are replicated by setting \( \gamma \) and \( \theta \) at 0 and 1 as required.

References