COLLAPSING EXCHANGE-RATE REGIMES

Some linear examples*

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We construct a pair of linear examples to study the collapse time of a fixed exchange-rate regime. The first example is a perfect-foresight, continuous-time model which allows calculation of the exact collapse time and the tracking of reserves. The second example is a discrete time, stochastic model which yields an endogenous probability distribution over the collapse time and produces a forward discount of the exchange rate.

1. Introduction

In this paper we construct a pair of simple linear examples to study the collapse of a fixed exchange-rate regime. Introduced by Salant and Henderson (1978), the concepts we employ were extended to the foreign exchange market by Krugman (1979) and to gold standards by Flood and Garber (1984).

Our first example, a continuous-time, perfect-foresight model, allows explicit calculation of the collapse time while preserving essential elements of Krugman's non-linear analysis. However, to perform the analysis we develop the concept of a 'shadow floating exchange rate' which allows easy extensions to stochastic environments. We examine the timing of regime collapses either based entirely on market fundamentals or based in part on arbitrary speculative behavior.

A discrete time model, our second example, contains stochastic market fundamentals which force the regime collapse. Hence, agents lack perfect foresight about the collapse time. The stochastic framework removes an unsatisfactory aspect of the perfect foresight example: under perfect foresight, a fixed exchange rate regime can collapse without ever producing a forward.

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discount on the collapsing currency.\(^1\) In a stochastic model with a random collapse time, agents' behavior will produce a forward discount on a weak currency even when the exchange rate remains fixed.\(^2\)

In section 2 we develop the continuous-time, perfect-foresight example; in section 3 we develop the discrete-time, stochastic model. Section 4 contains some concluding remarks.

2. The continuous-time model with perfect foresight

Our first example employs a small country model with purchasing power parity. We assume that agents have perfect foresight and that the assets available to domestic residents are domestic money, domestic bonds, foreign money, and foreign bonds. The domestic government holds a stock of foreign currency for use in fixing the exchange rate. Domestic money and domestic and foreign bonds dominate foreign money which yields no monetary services to domestic residents; therefore, private domestic residents will hold no foreign money. Domestic and foreign bonds are perfect substitutes.

We build the model around five equations:

\[
\frac{M(t)}{P(t)} = a_0 - a_1 i(t), \quad a_1 > 0, \tag{1}
\]

\[
M(t) = R(t) + D(t), \tag{2}
\]

\[
\dot{D}(t) = \mu, \quad \mu > 0, \tag{3}
\]

\[
P(t) = P^*(t)S(t), \tag{4}
\]

\[
i(t) = i^*(t) + \left[\frac{\dot{S}(t)}{S(t)}\right], \tag{5}
\]

where \(M(t), P(t),\) and \(i(t)\) are the domestic money stock, price level, and interest rate, respectively. \(R(t)\) and \(D(t)\) represent the domestic government book value of foreign money holdings and domestic credit, respectively. \(S(t)\) is the spot exchange rate, i.e. the domestic money price of foreign money. An asterisk (*) attached to a variable indicates 'foreign', and a dot over a variable ('\(\cdot\)') indicates the time derivative.

Eq. (1) reflects a money market equilibrium condition, the right-hand side representing the demand for real balances.\(^3\) Eq. (2) states that the money

\(^1\)We are referring to the instantaneous forward discount, as in Krugman (1979). For longer maturities of forward contracts, the discount may become positive, even in the certainty case.

\(^2\)We employ a discrete-time model purely as an analytical convenience. Our results can be duplicated for some continuous-time processes.

\(^3\)We interpret eq. (1) as the linear terms of a Taylor series expansion of some non-linear function \(M^d/P(t) = i(t)\). Our linearization is appropriate for values of \(a_0, a_1,\) and \(i(t)\) such that \(a_0 - a_1 i(t) > 0\). We have adopted the present linearization rather than the more standard semi-log-linearization to exploit the inherent linearity of our money supply definition.
supply equals the book value of international reserves plus domestic credit. Eq. (3) states that domestic credit always grows at the positive, constant rate $\mu$. Eqs. (4) and (5) impose purchasing power parity and uncovered interest parity, respectively.

We use (4) and (5) in (1) to obtain:

$$M(t) = \beta S(t) - \alpha \dot{S}(t), \quad (6)$$

where $\beta \equiv (a_0 P^* - a_1, P^* i^*)$, which we assume is positive, and $\alpha = a_1 P^*$. Both $\beta$ and $\alpha$ are constants since we assume that $P^*$ and $i^*$ are constant.

If the exchange rate is fixed at $\bar{S}$, reserves adjust to maintain money market equilibrium. The quantity of reserves at any time $t$ is

$$R(t) = \beta \bar{S} - D(t). \quad (7)$$

The rate of change of reserves, i.e. the balance of payments deficit is

$$\dot{R}(t) = - \dot{D}(t) = - \mu. \quad (8)$$

With a lower bound on net reserves and $\mu > 0$, the fixed-rate regime cannot survive forever; any finite reserve stock earmarked to support the fixed rate would be exhausted in finite time. We assume that the government will support the fixed rate while its net reserves remain positive. After the fixed-rate regime's collapse the exchange rate floats freely forever.

The central problem in finding the collapse time lies in connecting the fixed-rate regime to the post-collapse floating regime. As our strategy we first determine the floating exchange rate conditional on a collapse at an arbitrary time $z$, referring to it as the 'shadow floating exchange rate'. Next we investigate the transition from fixed rates to the post-collapse flexible-rate system.

If the fixed-rate regime collapses at any time $z$, the government will have exhausted its reserve stock at $z$. In general, agents extinguish the reserve stock in a final speculative attack, yielding a discrete downward jump in the money stock.

Immediately following the attack, money market equilibrium requires, from eq. (6):

$$M(z_+) = \beta S(z_+) - \alpha \dot{S}(z_+) \quad (9)$$

*While we assume that the government exhausts its reserve stock during a collapse, our analysis would change very little if we assumed that the government maintains some reserve stock, $R$, after the collapse. On this point, see Krugman (1979). Furthermore, the analysis would remain unchanged if the lower bound on reserves were some finite negative number representing a borrowing limit.*
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where \( z_+ \) indicates the instant after the attack and \( M(z_+) = D(z_+) \), since \( R(z_+) = 0 \). Since the government does not intervene in exchange markets under the new regime, the exchange rate floats. To find a floating exchange rate solution, we use the method of undetermined coefficients, conjecturing the solution form \( S(t) = \lambda_0 + \lambda_1 M(t) \). Remembering that \( M(t) = D(t) = \mu \) and substituting this trial solution into eq. (6), we find \( \lambda_0 = \alpha \mu / \beta^2 \) and \( \lambda_1 = 1 / \beta \). Therefore,

\[
S(t) = \left[ \frac{\alpha \mu}{\beta^2} \right] + \frac{M(t)}{\beta}, \quad t \geq z.
\]  

(10)

Since agents foresee the collapse, predictable exchange-rate jumps at time \( z \) are precluded. Since this point is crucial to the analysis, we will discuss it in some detail. Suppose first that agents expect a collapse at \( z \) and anticipate \( S(z_+) > S \). At time \( z \) speculators who attack government reserves will profit by an amount \( [S(z_+) - S] R(z_-) \), where \( z_- \) indicates the moment before the collapse. While finite in magnitude, these profits accrue at an infinite rate; and speculators compete to capture them. An individual speculator, expecting an attack at \( z \), has incentives to pre-empt his competitors by purchasing all the reserves an instant before \( z \). Therefore, the attack will occur prior to \( z \). Indeed, whenever agents expect a discrete exchange rate increase they will precipitate an attack on reserves prior to the increase. We conclude that our supposition of an attack at \( z \) and a discrete exchange rate increase at \( z \) involves a contradiction.

Now suppose that agents expect both an attack at time \( z \) and a discrete currency appreciation, \( S(z_+) < S \). The profits accruing to speculators will then be \( [S(z_+) - S] R(z_-) < 0 \). Agents would have no incentive to attack the government's reserves; and the fixed exchange-rate regime would survive.

We conclude that \( S(z_+) = S \) at the moment of an anticipated attack on foreign exchange reserves. We now use this condition to determine both the timing of the attack and the extent of government reserve holdings at the time of the attack. Substituting \( D(t) = D(0) + \mu t \) for \( M(t) \) in (10) produces the shadow floating exchange rate, the floating rate which would materialize if the fixed exchange rate collapsed at any given time \( t \). Plotting the shadow floating rate and \( S \) against time in fig. 1, we note that the time \( z \) of the collapse occurs when the two curves intersect. A little algebra yields:

\[
z = \left[ \frac{\beta S - D(0)}{\mu} - \frac{\alpha}{\beta} \right] R(0) - \frac{\alpha}{\beta}.
\]  

(11)

Eq. (11) is intuitively reasonable because an increase in initial reserves delays the collapse, while an increase in \( \mu \), the rate of domestic credit growth, hastens the collapse. As \( \mu \to 0 \), the collapse is delayed indefinitely.

Prior to the collapse, eq. (7) governs reserves, and it implies:

\[
S = \left[ R(z_-) + D(z_-) \right] / \beta.
\]  

(12)
Given the expression for $z$ from (11) and our knowledge that $D(z_-) = D(0) + \mu z$, we determine from (12) that

$$R(z_-) = \frac{\alpha \mu}{\beta}. \quad (13)$$

In fig. 2 we have portrayed the time paths of reserves, domestic credit, and the money supply during the period surrounding the collapse. Prior to the collapse at $z$, money remains constant, but its components vary. $D(t)$ rises at the rate $\mu$ and reserves decline at the same rate. At time $z$, both money and reserves fall by $\alpha \mu/\beta$. Since reserves fall to zero, money equals domestic credit after $z$. On the horizontal axis in fig. 2 we have recorded the time $R(0)/\mu$, the time when reserves would be exhausted in the absence of a speculative attack. The time length $\alpha/\beta$ represents a correction which hastens the collapse to ensure that the exchange rate will not jump.

We have based our results on an assumption that the solution for the floating exchange rate depends only on market fundamentals. In general, however, the floating rate obeys the dynamic law:

$$s(t) = A \exp \left[ (t-z) \beta/\alpha \right] + \alpha \mu/\beta^2 + M(t)/\beta, \quad (14)$$

where $A$, previously set to zero, is an arbitrary constant determined at time $z$. The shadow floating exchange rate is now:

$$s(t) = A + \alpha \mu/\beta^2 + D(t)/\beta. \quad (15)$$

---

5 $D(t)$ is a continuous variable, so $D(z_+) = D(z) = D(z_-)$.
6 As an artifact of our linearization, we must restrict $A \geq 0$. If $A < 0$, then $S(t) < 0$ for some $t$. 

To find \( z \), we again equate the shadow rate to \( \bar{S} \):

\[
z = \left[ \frac{R(0)}{\mu} - \alpha/\beta \right] - A/\alpha.
\]  

(16)

From (12) and (16) we determine the reserves at the time of the attack:

\[
R(z-) = \beta A + \alpha \mu/\beta.
\]  

(17)

Eq. (16) reveals that the collapse time depends both on market fundamentals, \( [R(0)/\mu - \alpha/\beta] \), and on the arbitrary constant \( A \). The constant \( A \) captures the behavior which may cause an indeterminacy in the path of the post-collapse floating rate.\(^7\) An increase in \( A \) will hasten the collapse, causing it to occur at a higher value of \( R(z-) \), which magnifies the amplitude of the attack on the currency.

As a special case, we suppose that \( \mu = 0 \), a situation in which the fixed-rate regime need never collapse in the absence of arbitrary speculative behavior. In this case, eq. (17) yields \( R(z-) = \beta A \). If \( \mu = 0 \), then \( R(t) \) remains constant at \( R(0) \). Therefore, a collapse may occur at any time that agents' speculative behavior sets \( A \geq R(0)/\beta \).

The possibility that arbitrary speculative behavior can cause the collapse of a fixed exchange-rate regime bears on a traditional argument favoring fixed exchange rates. The argument is that since a flexible exchange rate may be subject to arbitrary speculative fluctuations, the exchange rate should be fixed in order to protect the real sectors of an economy. Our analysis

\(^7\)Notice that if a collapse occurs due to the arbitrary element \( A \), then the post-collapse exchange rate must be expected to follow a bubble. Such a bubble could be distinguished in the data using tests like those in Flood and Garber (1980).
indicates that arbitrary speculative behavior identical in nature to that which may manifest itself under floating rates can also render arbitrary and indeterminate the time of a fixed exchange-rate regime collapse. Hence, arbitrary speculative behavior, if present, is an economic force which is masked, not purged, by the fixing of exchange rates.

3. The discrete-time model with uncertainty

Our analysis has required that agents know the collapse time of the fixed-rate regime. While the regime operates, the instantaneous expected rate of change of the exchange rate, $S(t)$, must always equal zero. However, that a weak currency's forward exchange rate may exceed the fixed rate for long periods of time, a generally encountered empirical phenomenon, does not fit into the previous analysis. In this section we incorporate uncertainty into a discrete time model to study the forward exchange rate for a currency which may experience a regime collapse.

The model's principal equations are:

\begin{align}
M(t)/P(t) &= a_0 - a_1 i(t), \quad (18) \\
M(t) &= R(t) + D(t), \quad (19) \\
D(t) &= D(t - 1) + \mu + \varepsilon(t), \quad (20) \\
P(t) &= P^*(t)S(t), \quad (21) \\
i(t) &= i^*(t) + \left[ E(S(t + 1) | I(t)) - S(t) \right]/S(t). \quad (22)
\end{align}

Variables common to eqs. (18)–(22) and (2)–(6) are defined as before, except that we now interpret $t$ as an integer. In eq. (2), $\varepsilon(t)$ represents a random disturbance with zero mean which obeys:

\begin{equation}
\varepsilon(t) = -1/\lambda + v(t). \quad (23)
\end{equation}

The random variable $v(t)$ is distributed exponentially with an unconditional probability density function.\footnote{Note that $E[\varepsilon(t + 1)|I(t)] = 1/\lambda$. We choose the exponential distribution to work this example because it permits arbitrarily large realizations while allowing relatively easy analytical manipulations. We require the large realizations so that there always will exist a non-zero probability of speculative attack in the next period, which automatically produces the forward premium. Alternatively, we may assume that only two possible realizations of the domestic credit disturbance may materialize: zero or a positive number with given probabilities. While this process will provide an even simpler analysis, there will be no forward premium for small enough levels of domestic credit.}
We adopt this distribution to allow easy manipulations. To ensure that domestic credit remains positive, we specialize the distribution with the assumption $\mu > 1/\lambda$. This assumption ensures non-negative growth of $D(t)$ and permits a simple stochastic extension of the certainty model of the previous section. Eq. (22) introduces the notation $E(\cdot | I(t))$, the mathematical expectation operator conditional on the information set $I(t)$. $I(t)$ includes complete information about all variables dated $t$ or earlier and the structure of the model.

Suppose that $S$ is the fixed exchange rate and define $\tilde{S}(t)$ as the shadow floating rate, the flexible exchange rate which would prevail if agents were to purchase all the government's reserves at $t$. Alternatively, $\tilde{S}(t)$ is the flexible rate conditional on a collapse of the fixed-rate regime at $t$. A fixed-rate regime will collapse at $t$ if and only if $S(t) \geq \tilde{S}$. If $S(t) \geq \tilde{S}$, agents may purchase reserves from the central bank at a price $\tilde{S}$, reselling immediately on the post-collapse market at price $S(t)$ and earning a profit per unit of reserves of $[S(t) - \tilde{S}] > 0$. If such a profit were available, agents would purchase all the government's reserves. If $\tilde{S}(t) < S$, agents certainly would not purchase the government's reserves at $\tilde{S}$ for resale at $S(t)$. Thus, for an attack to occur at any $t$ we require $S(t) \geq \tilde{S}$, i.e. the shadow floating rate must equal or exceed the fixed rate.

Our analysis of the discrete-time uncertainty model will proceed first by examining the unconditional expected exchange rate, $E[S(t + 1) | I(t)]$. Next, we study the stochastic path of reserves. We may express $E[S(t + 1) | I(t)]$ as:

$$E[S(t + 1) | I(t)] = (1 - \pi(t))S + \pi(t) E[\tilde{S}(t + 1) | I(t)].$$

In (25), $\pi(t)$ is the probability evaluated at $t$ that a collapse will occur at time $t + 1$. The unconditional expected future exchange rate is a probability weighted average of the fixed rate, $S$, which will prevail if there is no collapse at $t + 1$, and the rate expected to prevail if there is a collapse at $t + 1$.

If a collapse occurs at $t + 1$, then $R(t + 1) = 0$; and the exchange rate conditional on a collapse is:

$$\tilde{S}(t + 1) = \alpha \mu/\beta^2 + D(t + 1)/\beta,$$

with $\alpha$ and $\beta$ given in the previous section. The probability of a collapse at $t + 1$ is the probability that $\tilde{S}(t + 1) \geq \tilde{S}$. Formally:

$^9$Owing to the linear nature of the model, the shadow spot exchange rate could be negative if negative realizations of domestic credit growth were permitted. To avoid this difficulty, we assume only positive growth. For a log-linear model of money demand, negative realizations of $\Delta D(t)$ would be permitted.
\[
\pi(t) = \Pr[\alpha \mu / \beta^2 + D(t + 1)/\beta - \bar{S} > 0]. \quad (27)
\]

Since from (20) and (23), \(D(t + 1) = D(t) + \mu - 1/\lambda + \nu(t + 1)\), we write (27) as:

\[
\pi(t) = \Pr[\nu(t + 1) > K(t)], \quad (28)
\]

with \(K(t) = \beta \bar{S} - \alpha \mu / \beta - D(t) - \mu + 1/\lambda\). Use the probability density function (24) to obtain:

\[
\pi(t) = \int_{K(t)}^{\infty} \lambda \exp[-\lambda \nu(t + 1)] \, d\nu(t + 1), \quad K(t) \geq 0. \quad (29)
\]

Integrating (29) yields:

\[
\pi(t) = \begin{cases} 
\exp[-\lambda K(t)], & K(t) \geq 0, \\
1, & K(t) < 0.
\end{cases} \quad (30)
\]

To obtain an analytic expression for \(E[S(t + 1) \mid I(t)]\), we must use (26) to find:

\[
E[S(t + 1) \mid I(t)] = \alpha \mu / \beta^2 + E[D(t + 1) \mid I(t), C(t + 1)] / \beta, \quad (31)
\]

where \(C(t + 1)\) indicates that the expectation is conditional on a collapse at \(t + 1\). The conditional expectation of \(t + 1\) domestic credit is:

\[
E[D(t + 1) \mid I(t), C(t + 1)] = D(t) + \mu - 1/\lambda + E[\nu(t + 1) \mid I(t), C(t + 1)]. \quad (32)
\]

To find \(E[\nu(t + 1) \mid I(t), C(t + 1)]\), we must form the conditional p.d.f. over \(\nu(t + 1)\), where the conditioning information is \(\nu(t + 1) > K(t)\). This conditional p.d.f. is:

\[
g[\nu(t + 1)] = \begin{cases} 
\lambda \exp[\lambda(K(t) - \nu(t + 1))], & K(t) \geq 0, \\
\lambda \exp[-\lambda \nu(t + 1)], & K(t) < 0.
\end{cases} \quad (33)
\]

Hence, for \(K(t) \geq 0:\)

\[
E[\nu(t + 1) \mid I(t), C(t + 1)] = \int_{K(t)}^{\infty} \nu(t + 1) \exp[\lambda(K(t) - \nu(t + 1))] \, d\nu(t + 1). \quad (34)
\]

\(^{10}\)The conditional p.d.f. is derived in the appendix.
Carrying out the integration in (34), we obtain:

\[ E[u(t+1) | I(t), C(t+1)] = K(t) + 1/\lambda. \quad (35) \]

Substituting from (32) into (31) and the result into (30) yields:

\[ E[S(t+1) | I(t)] = \frac{1}{\alpha \mu / \beta^2 + [D(t) + \mu + K(t)]/\beta}. \quad (36) \]

Using the definition of \( K(t) \) in (36), we find:

\[ E[S(t+1) | I(t)] = \bar{S} + 1/\lambda. \quad (37) \]

Combining (37) and (25) yields:

\[ E[S(t+1) | I(t)] = S + \pi(t)/\beta \lambda, \quad (38) \]

which implies

\[ E[S(t+1) | I(t)] - S = \pi(t)/\beta \lambda, \quad K(t) \geq 0. \quad (39) \]

When \( K(t) < 0 \), we find from (32) that \( E[u(t+1) | I(t), C(t+1)] = 1/\lambda \). For this case, we obtain:

\[ E[S(t+1) | I(t)] - \bar{S} = \alpha \mu / \beta^2 - [D(t) + \mu]/\alpha - \bar{S}. \quad (40) \]

In fig. 3 we plot the forward discount, \( E[S(t+1) | I(t)] - \bar{S} \), against the level of \( D(t) \). In this figure, \( D^* \) is the level of \( D(t) \) for which \( S(t) = \bar{S} \). Hence, \( D^* = \beta \bar{S} - [\alpha \mu / \beta] \). In this figure, we assume that \( D^* - \mu + 1/\lambda > 0 \). Then, when \( D(t) = 0 \), the forward discount is \( \exp[-\lambda (D^* - \mu + 1/\lambda)] \). As \( D(t) \) rises from zero, the forward discount rises exponentially in accord with (38) until \( D(t) \) reaches \( D^* - \mu + 1/\lambda \), where the discount is \( 1/\beta \lambda \). When \( D(t) \) exceeds \( D^* - \mu + 1/\lambda \), \( K(t) = D^* - D(t) - \mu + 1/\lambda < 0 \). Eq. (40) then governs the discount, which rises linearly with \( D(t) \) at the rate \( 1/\beta \). When \( D(t) = D^* \), the premium reaches \( \mu / \beta \). Further increases in \( D(t) \) collapse the fixed-rate regime and under floating rates the discount \( \mu / \beta \).

In addition to explaining the forward-rate, spot-rate spread, our model implies that reserves are lower under fixed rates than the reserve level implied in our non-stochastic model. This results from the positive forward discount which reduces money demand relative to the certainty case.\(^{11}\) The country's reserve stock absorbs the entire reduction.

\(^{11}\)An interesting aspect of our model is that the 'offset coefficient' — the fraction of any domestic credit expansion reversed by central bank foreign reserve losses — exceeds unity. If the exchange rate were permanently fixed, our assumptions that domestic and foreign bonds are perfect substitutes and that the foreign interest is exogenous would produce a coefficient of unity. However, in addition, a domestic credit expansion will raise the domestic interest rate through its effect on the forward premium, thus lowering domestic money demand.
In our analysis we have ignored the possibility that the country may devalue its currency when a crisis seems imminent. While our model does not directly address this issue, it does imply a lower bound on the forward discount for a currency which will be devalued in a crisis. The variable $S(t)$ always serves as a lower bound for a viable fixed exchange rate. If a monetary authority decides to devalue when $S(t)$ approaches $S$, it cannot support a fixed rate which lies below $S(t)$. Hence, with devaluations possible, the expected rate of exchange-rate change must always equal or exceed that predicted by our model. A more complete treatment of expected devaluations would require a model of the government's decision-making process, which is not pursued here.\textsuperscript{12}

4. Concluding remarks

We can imagine two alternative situations in which a fixed exchange regime can collapse. In the first, an unpredictable and cataclysmic disturbance may so change the environment that maintenance of the fixed exchange rate is no longer possible.

\textsuperscript{12}Blanco and Garber (1982) extend a log-linear version of our model to study the problem of recurrent devaluation.
exchange regime is not possible. Such a viewpoint has an attraction since it associates a large event — the collapse of an exchange system — with a large cause. Since the disturbance is unpredictable and its origin is outside the bounds of our theories, the only problem remaining for the economist is to determine what its minimum size must be to have such a drastic effect. In the second situation, a predictable, cumulating sequence of small events culminates in the predictable collapse of the system. Since the exchange system dies from a thousand cuts, economists need not attribute the collapse to unusually large disturbances. While this may violate some inbred intuition about the proper relationship between the magnitudes of cause and effect, it opens the attractive possibility of applying the usual economists' tools to study both the question of timing and other phenomena attendant upon the collapse.

In this paper we have devised two workable examples of predictable collapses. We view the first, the perfect foresight case, primarily as a simple realization of Krugman's (1979) model in which an analytical solution for the collapse's timing can be readily derived. We have also shown that the fixed exchange system is subject to exactly the same type of dynamic instability problem which may affect a floating system. In the second example we have considered the problem of a foreseeable collapse in a model subject to stochastic monetary disturbances. The model produces the forward discount during the fixed-rate system which is known as the 'peso problem'. However, the result is not attributable to the possibility that unusual, large random disturbances may impinge on the system as in Krasker (1980). Indeed, the shocks in our model are always drawn from the same distribution; and even a small disturbance may collapse the system.

Appendix: Conditional P.D.F. for \( \nu(t+1) \)

The random variable \( \nu(t+1) \) is drawn from the exponential distribution. However, if we know \( \nu(t+1) > K(t) \), then we know that \( \nu(t+1) \) must come from the right-hand tail of the \( \nu(t+1) \) distribution. To reflect this knowledge, we must form a conditional p.d.f. Our p.d.f. is of the form \( B \lambda \exp \left[ -\lambda \nu(t+1) \right] \) and must obey

\[
1 = \int_{K(t)}^{\infty} B \lambda \exp \left[ -\lambda \nu(t+1) \right] d\nu(t+1),
\]

which implies:

\[
B = \exp \left[ \lambda K(t) \right].
\]

The p.d.f. we seek is then:
which is the p.d.f. recorded in the text, eq. (32). If \( K(t) < 0 \), then \( \exp[\lambda K(t)] \) is replaced with \( \exp[\lambda 0] = 1 \).

References


