The need for international policy coordination:
what’s old, what’s new, what’s yet to come?

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Abstract

The first generation of policy coordination models was introduced some 25 years ago; it provided a rationale for policy coordination, but the gains from coordination were generally thought to be small. Now, a new generation of policy coordination models is emerging, incorporating monopolistic competition and nominal inertia. Here, we examine macroeconomic interdependence and the scope for policy coordination in a tractable second generation model with two countries and multiple sectors. Initial calibration of the model suggests that second generation models may have more scope for policy coordination than did the first.

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1. Introduction

Fifty years ago, during the Bretton Woods era, the Chicago School saw a flexible exchange rate as a way of insulating domestic employment from foreign economic

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disturbances, including foreign monetary policy.\footnote{Milton Friedman, Harry Johnson and others described the ability of flexible exchange rates to absorb foreign disturbances. Sohmen (1961) provides a discussion of some of these arguments. Mussa (1979) provides a rigorous derivation of the extreme version of these arguments that we have in mind.} There is no need, they argued, for central banks to intervene in foreign exchange markets or coordinate their monetary policies; all that is needed is flexible exchange rates. Twenty five years later, the Bretton Woods system of fixed exchange rates was gone, and Hamada (1974, 1979), Oudiz and Sachs (1984) and Canzoneri and Gray (1985) introduced the first generation of game-theoretic models to describe monetary policy coordination.\footnote{Goodfriend and King (1997) provide an excellent introduction to the New-Keynesian models, or what they call the New Neoclassical Synthesis. This “synthesis” has engendered a new debate on the role of monetary policy. Major contributions to this debate include (but are hardly limited to): Ireland (1996), Rotemberg and Woodford (1997, 1999), King and Wolman (1999) and Erceg et al. (2000). The papers cited in the main text extend the debate to the question of international policy coordination. Other contributions to this emerging literature include: Benigno (2001), Benigno and Benigno (2001, 2003), Cooley and Quadrini (2002), Corsetti and Pesenti (2002), Clarida et al. (2002), Kollmann (2002), Pappa (2002), Sutherland (2002), Tchakarov (2002) and Tille (2002a).} These Old-Keynesian models did provide a theoretical rationale for policy coordination, but found that the gains from coordination were quantitatively small. Now, Obstfeld and Rogoff (2002) (O&Rb) and Corsetti and Pesenti (2001b) (C&Pb) and others have introduced a new generation of policy coordination models. These New-Keynesian models incorporate optimizing households, monopolistic competition, and some form of nominal inertia.\footnote{These characteristics are not independent: Eqs. (2) and (3) would imply Eq. (1) is an intertemporal version of the model.} The need for policy coordination in second generation models is still an open question, but O&Rb’s initial results made them rather skeptical. They asked: “Are the stabilization gains from having separate currencies largely squandered in the absence of effective international monetary coordination?”, and they concluded that: “...under plausible assumptions the answer is no”.

In this paper, we examine macroeconomic interdependence and the need for policy coordination in a prototypical second generation model. We relate our findings to the old arguments of the Chicago School, to earlier analyses of first generation models, and to recent empirical work on productivity differences across sectors. We conclude—based on theoretical considerations and a “first pass” calibration of our model—that second generation models may have more scope for policy coordination than did the first.

Obstfeld and Rogoff (2000) (O&Ra) and Corsetti and Pesenti (2001a) (C&Pa) developed an early benchmark model for the study of macroeconomic interdependence. In accordance with the recent literature on monetary policy, the model includes optimizing households in each country, monopolistic competition, nominal inertia, and a stochastic environment. Four strategically chosen characteristics make the benchmark model amazingly tractable:\footnote{These characteristics are not independent: Eqs. (2) and (3) would imply Eq. (1) is an intertemporal version of the model.} (1) a balanced current account, (2) log utility of consumption, (3) constant expenditure shares (on components of the composite consumption good), and...
(4) a log specification for the utility of money. The benchmark model readily admits exact solutions, even in this inherently complex framework.

Our discussion of the macroeconomic interdependence exhibited by this benchmark model will be reminiscent of the old Chicago School assertions: flexible exchange rates insulate domestic employment and output from all foreign disturbances, including foreign monetary policy. The three characteristics that account for the truly amazing tractability of the benchmark model carry with them a certain amount of collateral damage: the theory of exchange rate determination embodied in the benchmark model is extremely simple, and as a result the macroeconomic interdependence is rather limited.

The insularity of employment in the benchmark model might be expected to limit the scope for policy coordination. However, we shall see that welfare maximization in the benchmark model focuses on the stabilization of consumption, rather than employment. And oddly enough, in the benchmark model, the foreign money supply is the only policy variable that affects domestic consumption of imported goods, so, policy coordination might play an important role even in the benchmark model.

It turns out however that O&Rb, C&Pb and Devereux and Engel (2003) all found no need for coordination in the benchmark model. Each extended the benchmark model in a different way. O&Rb’s extension generalized the representative household’s utility function. O&Rb calibrated their model and found that the welfare gains from cooperation were second order when compared to the gains from simply reacting to shocks in a sensible way at the national level (as in a Nash solution). Their finding is reminiscent of McKibbin’s (1997) survey of the first generation models; in fact, McKibbin reports that this conclusion has attained the status of a folk theorem in the literature on first generation models.

We will not pursue either of these extensions here. Instead, we will return to the basic benchmark model and ask why there is no need for policy coordination in it, despite the policy spillovers in consumption. The reason is straightforward: the four characteristics of the benchmark model imply that there is no terms of trade externality, that the flexible price solution is the constrained optimum, and that central banks achieve the flexible price outcome in the Nash solution. Since the flexible price outcome is the (constrained) optimum, the Nash and Cooperative solutions coincide.

One way of introducing a coordination problem into the benchmark model is suggested by recent empirical work on the Balassa–Samuelson hypothesis. Both O&Rb and C&Pb assume that productivity shocks are perfectly correlated across traded and non-traded good sectors. Empirical work on the Balassa–Samuelson hypothesis suggests that the stochastic

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5 Actually, O&Ra postulated a constant elasticity function for the utility of money. O&Rb, C&Pa and C&Pb used the log specification, as have many others in the literature.

6 This too is a result of the simple theory of exchange rate determination in the benchmark model.

7 C&Pb’s extension allowed exporters to partially index prices to exchange rate movements, and showed that this provides a role for policy coordination. Devereux and Engel’s extension looked at the desirability of fixed and flexible exchange rates under producer currency pricing and consumer currency pricing.

8 Benigno and Benigno (2003) show that with more general preferences, there is a terms of trade externality. The expected disutility of work would depend on terms of trade movements and the volatility of the terms of trade could be used strategically to reduce the expected disutility of work. Therefore there would be gains from coordination. See also Obstfeld and Rogoff (2002) and Sutherland (2002).
processes generating productivity shocks in the two sectors are quite different.\(^9\) If we relax the assumption that productivity shocks are perfectly correlated, then the Nash solution no longer achieves the flexible price solution in the benchmark model, and there are potential gains from coordination.

In this paper, we show that, depending on the size and correlation of various shocks, the gains from coordination can in theory be second order, first order, or quite large in comparison with simply reacting to the shocks in a sensible way at a national level (as in a Nash solution). Canzoneri and Minford’s (1988) analysis of the structure of first generation models suggested that the gains from coordination in those models must necessarily be small. Our theoretical examples illustrate that this is not the case with second generation models. We also present a calibration of the benchmark model that suggests the gains from coordination are first order.\(^{10}\) However, as our discussion makes clear, the benchmark model is probably too simple to render a credible verdict on this question.

The rest of the paper is organized as follows: In Section 2, we outline our version of the benchmark model; the only essential difference with O&Ra and C&Pa is that we do not assume productivity shocks are perfectly correlated across sectors in each country. In Section 3, we analyze the macroeconomic interdependence exhibited by the benchmark model. In Section 4, we assess the scope for policy coordination in the benchmark model. In Section 5, we conclude by identifying the empirical work that is needed to guide future modeling of second generation models and to provide more definitive answer to Obstfeld and Rogoff’s question.

### 2. The benchmark model

The benchmark model is characterized by: (1) a balanced current account, (2) log utility of consumption, (3) constant expenditure shares on the sectoral goods that make up the consumption bundle in each country, and (4) log utility of money (or a cash in advance constraint). A few strategic modeling choices account for these characteristics. A Cobb–Douglas consumption aggregator fixes the expenditure shares. In this paper, we simply impose current account balance by assuming that there is no capital mobility, but the current account would always be balanced in an intertemporal version of our model.\(^{11}\) This is due to the Cobb–Douglas consumption aggregator, the log utility of consumption, and an assumption of no initial debt.

We begin by defining the sectors in our benchmark model. Following C&Pa and O&Rb, we have a tradable goods sector and a non-tradable goods sector. We also distinguish between tradable goods that are sold domestically and tradable goods that are

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\(^9\) Canzoneri et al. (1999) provide a recent example, and cites to the literature.

\(^{10}\) More recent papers—following a variety of approaches—have also argued that the gains from coordination may be significant: Benigno (2001), Cooley and Quadrini (2002), Kollmann (2002), Pappa (2002), Sutherland (2002), Tchakarov (2002) and Tille (2002a).

\(^{11}\) See Canzoneri et al. (2003a) for a proof of this assertion in a closely related model. The basic result, which draws on Cole and Obstfeld (1991), is well known: see C&Pa and Benigno (2004).
sold abroad. Standard trade theory suggests that only a subset of tradable goods are exported by any given country; moreover, following Corsetti and Dedola (2002), this allows for the possibility that transportation costs or distribution networks may differ in the domestic and export markets.

More specifically, the production technology is linear and stochastic. One unit of labor produces $Z_N$ non-tradable goods. One unit of labor produces $Z_D$ tradable goods that are sold domestically or $Z_E$ tradable goods that are exported. Similarly, in the foreign country, one unit of labor produces $Z_{N*}$ non-tradeable goods, $Z_{D*}$ tradeable goods that are sold domestically, or $Z_{E*}$ tradeable goods that are exported. Our general framework encompasses two interesting special cases. One lets productivity shocks be perfectly correlated across tradeable goods sold at home and abroad: $Z_D=Z_E$ and $Z_{D*}=Z_{E*}$. The other lets productivity shocks be perfectly correlated across all sectors in each country: $Z_N=Z_D=Z_E$ and $Z_{N*}=Z_{D*}=Z_{E*}$. The models of C&Pa,b and O&Ra,b fall into this last case.

The new generation of policy coordination models assumes utility maximizing households, monopolistic competition, and some form of nominal inertia. The simplest way of capturing these elements is to adopt a Yeoman–Farmer framework. In the home country, there is a continuum of households indexed by $h \in [0,1]$. Each household produces tradeable goods for the domestic and export markets, $Y_D(h)$ and $Y_E(h)$, and a non-tradeable good, $Y_N(h)$. Utility of home household-$h$ is:

$$U(h) = \log(C(h)) - \frac{Y_D(h)}{Z_D} + \frac{Y_E(h)}{Z_E} + \frac{Y_N(h)}{Z_N} + \log(M(h)/P)$$

(1)

where $C(h)=C_D(h)^{1/3}C_{E*}(h)^{1/3}C_N(h)^{1/3}$ is a composite consumption good consisting of domestically produced tradeable goods, $C_D$, imported tradeable goods, $C_{E*}$, and non-tradeable goods, $C_N$; $Y_j(h)/Z_j$ is the work effort of household $h$ in sector $j$ (where $j=D, E, N$); and $M(h)/P$ are money balances. Foreign households are indexed by $h^* \in [0,1]$; they are modeled in an analogous way.

The treatment of monopolistic competition in these models is now standard. Following Chari et al. (2000), we assume the artifice of a bundler who assembles the output of households into the sectoral goods that make up the composite consumption goods:

$$Y_j = \left[ \int_0^1 Y_j(h)^{(\theta - 1)/\theta} dh \right]^{\theta/(\theta - 1)}$$

$$j = D, E, N,$$

$$Y_j = \left[ \int_0^1 Y_j(h^*)^{(\theta - 1)/\theta} dh^* \right]^{\theta/(\theta - 1)}$$

$$j = D^*, E^*, N^*$$

(2)

A more detailed discussion can be found in the work of Canzoneri et al. (2003b).
(where $\theta>1$) and sells them for the prices:

$$P_j = \left[ \int_0^1 P_j(h)(1-\theta) \, dh \right]^{1/(1-\theta)}, \quad j = D, E, N$$

and

$$P_j^* = \left[ \int_0^1 P_j(h^*)(1-\theta) \, dh^* \right]^{1/(1-\theta)}, \quad j = D^*, E^*, N^*.$$  \hspace{1cm} (3)

The bundler’s demand for the products of household $h$ and household $h^*$ is:

$$Y_j(h) = (P_j(h)/P_j)^{-\theta} Y_j \quad j = D, E, N,$$

and

$$Y_j(h^*) = (P_j(h^*)/P_j)^{-\theta} Y_j \quad j = D^*, E^*, N^*.$$  \hspace{1cm} (4)

Another bundler assembles the sectoral goods, $Y_j$, into the composite consumption goods:

$$C = C_{D}^{1/3} C_{E}^{1/3} C_{N}^{1/3} \text{ and } C^* = C_{D^*}^{1/3} C_{E^*}^{1/3} C_{N^*}^{1/3}$$  \hspace{1cm} (5)

and sells them to the home and foreign households at the prices:

$$P = 3P_{D}^{1/3} P_{E}^{1/3} P_{N}^{1/3} \text{ and } P^* = 3P_{D^*}^{1/3} P_{E^*}^{1/3} P_{N^*}^{1/3}.$$  \hspace{1cm} (6)

These are national consumer price indices.\textsuperscript{13} The bundler’s cost minimization implies that:

$$P_D C_D = P_{E^*} C_{E^*} = P_{N^*} C_N = 1/3PC$$

and

$$P_{D^*} C_{D^*} = P_{E^*} C_{E^*} = P_{N^*} C_{N^*} = 1/3P^* C^*.$$  \hspace{1cm} (7)

One third of household expenditure is spent on each sectoral good. The budget constraint for household $h$ is given by:

$$M(h) + PC(h) + T = M_0(h) + P_D(h) Y_D(h) + P_E(h) Y_E(h) + P_N(h) Y_N(h)$$  \hspace{1cm} (8)

where $T$ is a tax, and $M_0(h)$ are initial money holdings.\textsuperscript{14} The household chooses $C(h)$, $M(h)$, $P_N(h)$, $P_D(h)$ and $P_E(h)$ to maximize Eq. (1) subject to Eqs. (4) and (8). We consider two different pricing policies: When prices are “flexible”, the household sets its prices knowing the values of the productivity shocks. When prices are “fixed”, the household sets its prices before the shocks are known; we assume that home households set their prices in terms of the home currency.\textsuperscript{15} Foreign households face an analogous problem.

\textsuperscript{13} A word on notation may be helpful at this point. An “*” in a subscript denotes a good produced in the foreign country. An “*” superscript on a price signifies that the price is in terms of the foreign currency.

\textsuperscript{14} Governments in each country balance their budgets: $M - M_0 = -T$ and $M^* - M_0^* = -T^*$.

\textsuperscript{15} This is sometimes referred to as “producer currency pricing”. Alternatively, under “customer currency pricing”, the household sets prices in terms of the currency that its customers use. Our working paper, Canzoneri et al. (2002), discusses both. With complete information (the flexible price case), the distinction does not matter; with incomplete information (the fixed price case), it may. C and Pb allow partial indexation to the exchange rate. Ideally, we would like to follow Devereux and Engel (2001) and Corsetti and Pesenti (2002) and endogenize the household’s choice of a pricing policy, but that is beyond the scope of the present paper.
The first order conditions are identical for the households in each country. Thus, we can focus on a symmetric equilibrium in which household variables are equal to aggregate variables:

\[ C_j(h) = Y_j = Y_j(h) \quad \text{for} \quad j = N, D; \quad C_E(h) = Y_E = Y_E(h); \quad C_j(h*) = Y_j = Y_j(h*) \quad \text{for} \quad j = N^*, D^*; \quad \text{and} \quad C_E(h*) = Y_E = Y_E(h). \]

Also, \( P_j(h) = P_j \) for \( j = N, D, E \) and \( P(h*) = P \) for \( j = N^*, D^*, E^* \). Moreover, home currency prices translate directly to foreign currency prices: \( P_j = SP \), where \( S \) is the home currency price of foreign exchange. Finally, trade must be balanced in equilibrium:

\[ P_E C_E = P_E C_E^* = P_E Y_E = Y_E(h). \]

The fixed and flexible price equilibria can be calculated from the following equations:

\[ PC = 3P_D C_D = 3P_E C_E^* = 3P_N C_N \quad \text{and} \]

\[ P_D^* C_D^* = P_E^* C_E = P_N^* C_N^* = 1/3P^*C^* \]

\[ M = PC \quad \text{and} \quad M^* = P^*C^* \]

\[ Y_j/Z_j = 1/3\mu \quad \text{for} \quad j = N, D, E \quad \text{and} \quad N^*, D^*, E^* \]

\[ E[Y_j/Z_j] = 1/3\mu \quad \text{for} \quad j = N, D, E \quad \text{and} \quad N^*, D^*, E^* \]

where \( \mu = \theta(\theta - 1) \) is the markup factor. Eq. (10) displays the constant expenditure shares discussed above. Eq. (11) follows from the households’ first order condition for money balances; in the benchmark model, money market equilibrium is equivalent to binding cash in advance constraints.\(^{16}\) Eq. (12)\(_{\text{flexible}}\) follows from the households’ first order conditions for price setting under flexible prices; Eq. (12)\(_{\text{fixed}}\) holds in the case of fixed prices.

In the flexible price solution, employment levels \( (Y_j/Z_j) \) are fixed, and productivity shocks pass directly to output and consumption.\(^{17}\) The flexible price solution is an interesting benchmark: as we shall see, it is the “constrained” welfare optimum in each country. In the fixed price solution, expected employment levels are equal to their flexible price values, but actual employment and consumption levels are determined by sectoral demand.\(^{18}\)

With constant expenditure shares, sectoral demand in each country is proportional to aggregate expenditure. So, aggregate demand policy affects each sector proportionately. Aggregate demand policy can make the fixed price solution mimic the flexible price solution in each country if the sectoral shocks are perfectly correlated. If they are not, then policymakers will face sectoral tradeoffs.

\(^{16}\) In a dynamic version of the model, interest rates would appear in Eq. (11). In what follows, we take national nominal expenditures—\( PC \) and \( P^*C^* \)—to be the effective instruments of monetary policy. We discuss this more fully in Section 3.

\(^{17}\) This result is due to the log utility of consumption. If, for example, we replaced \( \log(C) \) with \((1-\gamma)C^{-(1-\gamma)}\) where \( \gamma > 1 \), then households would want to respond to a positive productivity shock by working less; see Canzoneri et al. (2003b) for a fuller discussion of this.

\(^{18}\) The levels at which the prices are fixed can be found by substituting Eqs. (10) and (11) into Eq. (12)\(_{\text{fixed}}\). The details of this are discussed by Canzoneri et al. (2003b).
3. Macroeconomic interdependence in the benchmark model

First, we show how the exchange rate is determined in the benchmark model. The theory of exchange rate determination embodied in the benchmark model is quite simple, whether prices are fixed or flexible. Also, this has strong implications for the macroeconomic interdependence exhibited by the benchmark model under fixed prices.

3.1. Exchange rate determination in the benchmark model

Three of the characteristics that define the benchmark model—current account balance (C1), constant expenditure shares (C2), and log utility of money (C3)—have strong implications for the theory of exchange rate determination:

**Proposition 1. Exchange rate determination**

(i) $C_1$ and $C_2$ imply $S = PC/P^*C^*$.
(ii) $C_1$, $C_2$ and $C_3$ imply $S = M/M^*$.

The proof is straightforward.

Any model that exhibits characteristics $C_1$ and $C_2$ will imply that the exchange rate fluctuates with the ratio of national nominal expenditures. In models that exhibit $C_2$ and fixed prices, it may seem natural to take total nominal expenditure (or aggregate demand) as the instrument of monetary policy, since total nominal expenditure controls demand—and therefore output and employment—in each sector. In the benchmark model, $C_3$ implies that total nominal expenditure is equal to the money supply, and we take money to be the instrument of monetary policy in each country. The fact that the nominal exchange rate fluctuates with the ratio of the instruments of monetary policy plays a large role in what follows.

3.2. Consumption, output and employment determination under fixed prices

Home households set $P_N$, $P_D$, and $P_E$, and foreign households set $P_{N^*}$, $P_{D^*}$ and $P_{E^*}$. Combining Eqs. (10) and (11):

$$M = 3P_N C_N = 3P_D C_D = 3SP_{E^*} C_{E^*} \text{ and}$$

$$M^* = 3P_{N^*} C_{N^*} = 3P_{D^*} C_{D^*} = 3(P_E/S)C_E.$$  

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19 C&Pb assume that the central banks set interest rates to control the level of national nominal expenditures (via the Euler equation). In our setup, this is equivalent to assuming that the central banks set national money supplies.

20 The choice of instruments may not be an “innocent assumption” in a game theoretic context, even when central banks have full information about the shocks. In the duopoly literature, for example, price setting behavior and quantity setting behavior can result in very different equilibria.
and from Proposition 1:

\[ S = \frac{M}{M^*}. \]  

(14)

These equations determine consumption in the benchmark model; output and employment \((Y_j/Z_j)\) follow immediately.

How does monetary policy work in the fixed price solution? An increase in \(M\) induces an increase in aggregate demand, \(PC\), and a proportionate increase in expenditure on each of the sectoral goods that home households consume. Since \(P_N\) and \(P_D\) are fixed, \(C_N\) and \(C_D\) have to rise. However, the nominal exchange rate depreciates in proportion with \(M\); so, \(P_E *(= S P_E **)\) increases in proportion with \(M\), and there is no effect on \(C_E^*\). \(C_E^*\) is instead determined by the foreign money supply, \(M^*\), via its effect on the exchange rate. The model is symmetric; a change in \(M^*\) has analogous effects.

These results are summarized in Proposition 2:

**Proposition 2.** Macroeconomic interdependence in the benchmark model\(^{21}\)

(A) The Insulated Supply Side:

\[
\begin{align*}
Y_N &= \frac{1}{3}(M/P_N), \\
Y_D &= \frac{1}{3}(M/P_D), \\
Y_E &= \frac{1}{3}(M/P_E);
\end{align*}
\]

\[
\begin{align*}
Y_{N*} &= \frac{1}{3}(M^*/P_{N*}), \\
Y_{D*} &= \frac{1}{3}(M^*/P_{D*}), \\
Y_{E*} &= \frac{1}{3}(M^*/P_{E*});
\end{align*}
\]

(B) The Interdependence of Consumption:

\[
\begin{align*}
C_N &= \frac{1}{3}(M/P_N), \\
C_D &= \frac{1}{3}(M/P_D), \\
C_{E*} &= \frac{1}{3}(M^*/P_{E*});
\end{align*}
\]

\[
\begin{align*}
C_{N*} &= \frac{1}{3}(M^*/P_{N*}), \\
C_{D*} &= \frac{1}{3}(M^*/P_{D*}), \\
C_E &= \frac{1}{3}(M/P_E).
\end{align*}
\]

The proof follows directly from Eqs. (13) and (14).

The three assumptions that characterize the benchmark model—a balanced current account, constant expenditure shares, and log utility of money—lead to an extremely tractable model, but they also lead to a model in which macroeconomic interdependence is rather limited and highly stylized. Domestic monetary policy controls consumption and output of the domestically produced goods, while foreign monetary policy controls consumption of the imported tradeable good. The exchange rate completely insulates the supply side of the economy—employment and output—from any shock emanating from abroad; neither foreign productivity shocks nor foreign monetary policy matter.

During the Bretton Woods era, the old Chicago School made similar claims when it was extolling the merits of a flexible exchange rate regime. Mussa (1979) outlined the conditions (in terms of first generation models) under which the Chicago School’s arguments would imply the complete insulation of domestic employment from foreign conditions. Interestingly enough, they included the absence of capital mobility. The builders

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\(^{21}\) With Consumer Currency Pricing, there is a complete inversion of these results: each country’s consumption is insulated from foreign shocks and policy, and output of the exported good is determined by the foreign money supply. With Consumer Currency Pricing, there can be no gains from coordination in the benchmark model; see Canzoneri et al. (2002).
of the benchmark model were interested in identifying the assumptions that would lead to closed form solutions in a rather intractable environment. The insulating effect of flexible exchange rates portrayed in Proposition 2 will be viewed by some as collateral damage.

These results can of course be traced to the assumptions that characterize the benchmark model, and to the simple theory of exchange rate determination that results. In particular, the results depend upon the log utility of consumption.

4. Policy coordination in the benchmark model

One might expect that there would be little need for policy coordination in a model that exhibits so little macroeconomic interdependence. And indeed, that is precisely what O&Rb and C&Pb found when the productivity shocks are perfectly correlated across sectors in each country. In this section, we illustrate their result, and then, letting productivity shocks vary across sectors, we provide counterexamples to the notion that the gains from coordination must necessarily be small in second generation models. Finally, we present a calibration of the benchmark model that suggests that the gains from coordination are as important as gains from reacting to the shocks in a sensible way at the national level (as in a Nash solution). But first, we must be more specific about the goals of monetary policy.

4.1. The goals of monetary policy

In second generation models, the utility of the representative household provides a natural measure of national welfare, and we will assume that central banks act benevolently. That said, second generation models exhibit two distortions that would tempt a central bank to create surprise inflations: (1) monopolistic competition makes households set prices too high (or work too little); and (2) the seigniorage tax makes households hold too little money.

It is common to eliminate the whole problem of seigniorage by simply ignoring the money term in the utility functions; the justification given is that seigniorage is a small component of government revenue (and therefore a small distortion) in OECD countries. It is also common to eliminate the possibility of monetary surprises by assuming that central banks are pre-committed to monetary policy rules.22 We will follow both of these practices.

To be precise, let $W = U - \log(M/P)$ and $W^* = U^* - \log(M^*/P^*)$. We assume central banks pre-commit to money supply rules that maximize $E[W]$ and $E[W^*]$. In a Nash solution, the home central bank chooses a rule for $M$ that maximizes $E[W]$ (conditional on the rule for $M^*$) and the foreign central bank chooses a rule for $M^*$ that maximizes $E[W^*]$ (conditional on the rule for $M$); in a Cooperative solution, the central banks coordinate on rules that maximize $E[W] + E[W^*]$.22

22 It is common in the closed economy literature to eliminate the distortions with fiscal subsidies. In a two-country setting, Benigno and Benigno (2003) show how to design production subsidies that make the monopolistic competition’s temptation to inflate balance the terms of trade’s temptation to deflate, letting (in their model) the Nash solution under discretion achieve the flexible price solution; there would still be a gain from coordination, since output is still inefficiently low in the Nash solution. We do not pursue these issues here.
Here again, assumptions that characterize the benchmark model have strong implications. In particular, Eqs. (1) and (12)\textsubscript{fixed} imply:

\[ E[W] = E[\log(C)] - 1/\mu \text{ and } E[W^*] = E[\log(C^*)] - 1/\mu. \]  

(15)

Central banks maximize the expected utility of consumption; the expected disutility of work is beyond their control.\textsuperscript{23}

Since we assume that central banks maximize \( E[\log(C)] \) and \( E[\log(C^*)] \), it seems natural to assume that the productivity shocks are log-normally distributed. In this case, the model yields exact solutions for all of the variables of interest, and of course it is natural to assume that the productivity shocks are log-normally distributed. In this case, the fixed imply log-normal distributions. In particular, Eqs. (1) and (12)\textsubscript{fixed} imply:

\[ E(c_N) = E(\hat{y}_N) - 1/2V[m-z_N], E(c_D) = E(\hat{y}_D) - 1/2V[m-z_D], E(c_E^*) = E(\hat{y}_E^*) - 1/2V[m^*-z_E^*] \]

\[ E(c_N^*) = E(\hat{y}_N^*) - 1/2V[m^*-z_{N^*}], E(c_D^*) = E(\hat{y}_D^*) - 1/2V[m^*-z_{D^*}], E(c_E) = E(\hat{y}_E) - 1/2V[m-z_E] \]  

(16)

Eqs. (15) and (16) enable us to derive exact solutions for national welfare:

\[ E[W] = E[\hat{W}] - (1/6)A \text{ and } E[W^*] = E[\hat{W}^*] - (1/6)A^*, \]  

(17)

where \( A = V[m-z_N]+V[m-z_D]+V[m^*-z_{N^*}]+V[m^*-z_{D^*}]+V[m-z_E] \).

The best that the central banks can hope to do—individually or collectively—is to eliminate the variances and achieve the flexible price solution; as O&Rb have noted, the flexible price solution is constrained Pareto efficient in the benchmark model.\textsuperscript{26} And as we

\[ \text{Eq. (12)\textsubscript{fixed} imply } \log\{E[Y_j/Z_j]\} = \log\{y_j-z_j\} - 1/2[1/2V[y_j-z_j]] = \log(1/3) \text{ and } E(z_j) = \log(1/3)+E(z_j)-1/2V[y_j-z_j]. \]  

(2)

The unconstrained Pareto efficient solution would also eliminate the distortions created by monopolistic competition. (As noted earlier, this might be done with production subsidies.) Another rather amazing implication of the assumptions that characterize the benchmark model is that the monopolistic distortions do not interact with the stabilization problem (or the way that monetary policy reacts to shocks); they just make the expected work effort too small, and the work effort can be boosted with tax subsidies that are independent of the shocks.

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noted above, monetary policy can make consumption in a given sector mimic its flexible price solution by making the appropriate money supply (and therefore aggregate demand) move in proportion with the sectoral productivity shock. If the productivity shocks differ across sectors, then central banks face sectoral tradeoffs; these tradeoffs are implicit in the utility losses $A$ and $A^*$.

**4.2. Is there a role for policy coordination in the benchmark model?**

In Section 3, we showed that macroeconomic interdependence is rather limited in the benchmark model. Proposition 2 states that the supply side of the economy is completely insulated; the foreign central bank cannot, for example, help the home central bank stabilize it’s employment levels. However, the expected disutility of leisure is fixed in the benchmark model. Central banks maximize the expected utility of consumption, and the foreign money supply is the only policy instrument that affects home consumption of imports. So, despite the insulation of the supply side of the economy, policy coordination may actually play an important role in the benchmark model.

Nevertheless, both O&Rb and C&Pb found that there is no need for policy coordination.28 What accounts for their results? Both O&Rb and C&Pb assumed that productivity shocks are perfectly correlated across sectors within a given country; in effect, they have a single productivity shock in each country: $z^u_{N}=z^u_{D}=z^u_E$ and $z^*_u=z^*_N=z^*_D=z^*_E$. The central bank objectives reduce to minimizing:

$$A=V[m-z] + V[m-z] + V[m-z^*]$$

$$A^*=V[m^*-z^*] + V[m^*-z^*] + V[m-z]. \quad (18)$$

The Nash policy rules are obviously $m=z$ and $m^*=z^*$. Something fortuitous happens here. The home central bank, in doing what is best for its N and D sectors, just happens to do what is best for the foreign country’s consumption of imports. Each central bank moves its money supply in proportion with its domestic productivity shock, and this brings about the optimal flexible price solution in both countries. The Nash and Cooperative policy rules coincide!

From a modeling point of view, there is an obvious way to introduce a coordination problem in the setup we have already laid out: simply relax the assumption that the sectoral productivity shocks are perfectly correlated.29 This would certainly be motivated by empirical work on the Balassa–Samuelson hypothesis; for example, Canzoneri et al. (1999) show that the time series processes generating productivity shocks in the tradeable and non-tradeable goods sectors are quite different in OECD countries. We will explore this possibility in the next subsection.

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27 Note that this would eliminate the sectoral variance in Eq. (17).
28 Devereux and Engel (2003) found a similar result; in their model, there are only tradeable goods, but the assumption of unitary intertemporal elasticity of consumption is relaxed.
29 Tille (2002a) makes a similar point.
O&Rb introduced a coordination problem by generalizing the utility of consumption to a CRRA specification. O&Rb calculated three solutions to their model: (1) a “passive” solution in which central banks hold their money supplies constant, (2) the Nash solution, and (3) the Cooperative solution. They then argued that for reasonable parameterizations of their model, the gains from coordination are small. More specifically, let \( A_P, A_N, \) and \( A_C \) be the welfare losses (see Eq. (17)) in the passive solution, the Nash solution and the Cooperative solution. O&Rb showed that the relative gain ratio—\( R = (A_N - A_C)/(A_P - A_N) \)—is no bigger than 0.08 for values of the coefficient of relative risk aversion less than 4.\(^{30}\) In other words, the gains from coordination are second order compared to the gains from simply reacting to the shocks in a sensible way at the national level.

O&Rb’s finding is reminiscent of earlier analyses of first generation models. Canzoneri and Edison (1990) used multipliers from the FRB’s Multi-country Model to simulate various game situations; they found \( R \)-ratios of about 0.01. Moreover, Canzoneri and Minford’s (1988) analysis of the theoretical structure of first generation models suggested that the gains from coordination in those models must necessarily be small. More recently, McKibbin (1997) surveyed the literature on first generation models, and in his words, “It appears from the evidence to date that the largest gains for the world economy are to be realized from some form of policy optimization at the individual economy level . . . Any additional gains from coordination of policies between economies have been found to be dwarfed by the potential gains from individual countries adjusting policies in a sensible manner.” McKibbin asserts that this conclusion has attained the status of a folk theorem in the literature on first generation models.

So, the interesting question here is whether the folk theorem is valid for second generation models as well. We use the benchmark model outlined in Section 2 to argue it is not.

4.3. Gains from coordination in the benchmark model with sector specific productivity shocks

As noted above, when productivity shocks are imperfectly correlated across sectors, central banks will not generally be able—individually or collectively—to achieve the optimal flexible price solution. To mimic the flexible price solution in a given sector, monetary policy has to make that sector’s demand move proportionately with its productivity shock. However, each country has three sectors and only one policy instrument; two money supplies cannot achieve six different levels of demand. This instrument insufficiency implies sectoral tradeoffs (characterized by the utility losses \( A \) and \( A^* \)) for the central banks, and it creates a potential role for policy coordination.

To analyze these sectoral tradeoffs, we need to make some more assumptions. We have already imposed a symmetric sectoral structure on the two countries. It seems natural to impose symmetry on the sectoral productivity processes as well; that is, \( z_N \) has the same

\(^{30}\) Benigno (2001) analyzed a more general intertemporal model and showed the gains from coordination can be significant when initial holdings of net foreign assets are not zero.
The lognormal distribution as $z_{N*}$, $z_D$ has the same distribution as $z_{D*}$, and $z_E$ has the same distribution as $z_{E*}$.

The Nash, Cooperative and Passive policies and utility losses are set out in Lemma 1:

**Lemma 1.** Gains from coordination in the benchmark model

*Nash policy rules:* $m=1/2z_N+1/2z_D$ and $m*=1/2z_{N*}+1/2z_{D*}$.

*Losses in the Nash solution:* $A_N=\frac{3}{4}\sigma^2_N+\frac{3}{4}\sigma^2_D+\sigma^2_E-1/2\sigma_{ND}-\sigma_{NE}-\sigma_{DE}$.

*Cooperative policy rules:* $m=1/3z_N+1/3z_D+1/3z_E$ and $m*=1/3z_{N*}+1/3z_{D*}+1/3z_{E*}$.

*Losses in the Cooperative solution:* $A_C=\frac{2}{3}(\sigma^2_N+\sigma^2_D+\sigma^2_E-\sigma_{ND}-\sigma_{NE}-\sigma_{DE})$.

*Passive policy rules:* $m=m*=0$.

*Losses in the Passive solution:* $A_P=A_P^*=\sigma^2_N+\sigma^2_D+\sigma^2_E$.

**Proof.** The calculations are straightforward; Appendix A provides the details.

In the Nash solution, the home central bank reacts to a weighted average of the productivity shocks its N and D sectors, with weights that reflect the sectors’ relative importance in the home consumers’ welfare; the home central bank does not care about foreign consumption of its exports. In the Cooperative Solution, the home central bank also reacts to the productivity shock in its export sector; weights reflect the sectors’ relative importance in home and foreign consumers’ welfare.

There are several ways of measuring the welfare gains from Nash and Cooperative policies. One is the $R$-ratio—$R=(A_N-A_C)/(A_P-A_N)$—introduced in the last section. It measures the gain from coordination relative to the gain from simply reacting to the shocks in a sensible way at the national level. A potential problem with the $R$-ratio is that it may be the ratio of two small numbers; neither gain may be worth mentioning. Tille (2002b) suggests decomposing the $R$-ratio to look into this possibility. $A_P$ represents the welfare gap (or utility loss) created by nominal rigidities; if prices were flexible, $A_P$ would disappear. Tille suggests we measure the gains from Nash and Cooperative policies as the fraction of the welfare gap that these policies close: $G_N=\frac{(A_P-A_N)}{A_P}$ and $G_C=\frac{(A_N-A_C)}{A_P}$. The $R$-ratio is then $R=G_C/G_N$. A final measure is the consumption equivalents of Lucas (2003).

Proposition 3 illustrates the theoretical possibilities exhibited by the benchmark model.

**Proposition 3.** The gains from coordination are not necessarily small in the benchmark model.

**Case 1.** $z_N=z_D=z_E=z$ and $z_{N*}=z_{D*}=z_{E*}=z*$ gives $A_N=1$; $G_C=0$ and $R=0$.

**Case 2.** $\sigma^2_N=\sigma^2_D=\sigma^2_E$ and $\sigma_{ND}=\sigma_{NE}=\sigma_{DE}=0$ gives $A_N=1/6$; $G_C=1/6$ and $R=1$.

**Case 3.** $\sigma^2_N=\sigma^2_D=0$ and $\sigma^2_E>0$ gives $A_N=0$; $G_C=1/3$ and $R=\infty$.

**Proof.** The calculations are straightforward.

31 Of course, this approach does not address the question of whether $A_P$ itself might be small. This is currently a subject of some debate.
and there are no gains from coordination. In Case 2, sectoral productivity shocks are assumed to be independent and identically distributed. The Nash and the Cooperative policies yield identical (incremental) welfare gains. This example illustrates two important points. First, the gains from coordination can (in theory) be just as important as the gains from reacting to the shocks at the national level; that is, $R=1$. However, when sectoral shocks are imperfectly correlated, monetary policy is simply not very potent in the benchmark model: even in the Cooperative solution, monetary policy is only able to close $1/3$ of the welfare gap created by nominal rigidities. In Case 3, productivity processes are assumed to be non-stochastic in two of the three sectors; nominal rigidities only cause welfare losses in the export sectors, where productivity is not perfectly predictable. Here, Nash and Passive policies coincide, since central banks do not care about foreign consumption of their country’s exports in a Nash solution. In this (admittedly contrived) example, the $R$-ratio is actually infinite, even though the gain from moving to the Cooperative solution is rather modest ($G_C=1/3$).

Cases 2 and 3 provide theoretical counterexamples to the notion that the gains from coordination must necessarily be small in second generation models of policy coordination. If there were structural limitations to the gains from coordination in the first generation models, as suggested by Canzoneri and Minford (1988), they do not appear to be present in second generation models. Of course, McKibbin’s (1997) folk theorem may still apply to the new models: the gains from coordination may turn out to be small in empirical applications.

4.4. A calibration of the benchmark model

We calibrate our simple benchmark model using US data to get a “first pass” notion of the quantitative importance of coordination in the second generation models. Lemma 1 indicates that the gains from coordination depend on the covariance matrix of sectoral productivity shocks. We use US data from the OECD STAN database to compute the covariance matrix of sectoral productivity shocks. Classifying ISIC sectors into tradeables and non-tradeables is fairly standard, but dividing the tradeable ISIC sectors into tradeables that are sold domestically (our D sector) and tradeables that are exported (our E sector) is not. As Case 3 in Proposition 3 clearly indicates, some notion of the distributions of the productivity shocks in the D and E sectors is essential in measuring the importance of policy coordination. Therefore, we classify all two digit ISIC sectors that export more than two thirds of their value added as belonging to the E sector; the remaining tradeable ISIC sectors belong to the D sector.

As might be expected, the correlation between productivity shocks in our D and E sectors is an important parameter in determining the importance of policy coordination. Fig. 1 plots $G_N$ and $G_C$ against this correlation; Fig. 2 plots the $R$-ratio. $G_N$ and $G_C$ are

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32 We divide value added by employment in FTEs to compute average labor productivity. We take agriculture, hunting, forestry, and fishing (ISIC 01-05) and manufacturing (ISIC 15-37) to be tradeables, and construction (ISIC 45) and business services (ISIC 50-74) to be non-tradeables.

33 This measure classifies ISIC sectors 19, 29, 30, 31, 32, 33, and 35 as export sectors.
roughly equal when the shocks have a very low correlation. In this case, $R$ is near one, and the gains from coordination are roughly equal to the gains from reacting to the shocks in a sensible way at the national level.

Fig. 1 shows that $G_N$ rises, and $G_C$ falls, as the correlation between the D and E sector shocks is increased. The reason for this is clear. In a Nash solution, the home central bank reacts to its D sector shock, but it does not care about its E sector shock (since it does not care about foreign consumption). The greater is the correlation between the D and E sector shocks, the greater is the extent to which the home central bank’s reaction to its D shock will also take care of the E sector. Nash policies are more effective, and there is less need for policy coordination. On the other hand, when the correlation between the D and E sector shocks is negative, the home central bank’s reaction to the D sector shock makes matters worse (for the foreign country) in the E sector.

So, where does our data put us in Figs. 1 and 2? We estimate the correlation between D and E sector productivity shocks to be $-0.15$. This implies that $G_N=0.13$, $G_C=0.17$ and $R=1.3$. The gains from coordination are 30% more important than the gains from simply reacting to the shocks in a sensible way at the national level. This exercise suggests that McKibbin’s (1997) folk theorem may not carry over to the second generation coordination models, and that the gains from coordination might well be significant.

The gains we have calculated are cardinal; their units are hard to comprehend. Lucas (2003) suggests reporting gains as consumption equivalents: what fraction of permanent consumption would consumers be willing to offer to obtain the Nash solution instead of the passive solution, or the Cooperative solution instead of the Nash solution. In the

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34 We combine the three productivity series in a VAR and use the residual covariance matrix in our calculations. The results differ somewhat from Case 2 in Proposition 3 because the variance of productivity shocks in the non-tradeables sector is considerably smaller than the variance of productivity shocks in either of the tradeables sectors. $G_N$ and $G_C$ are equal when the correlation is about $-0.075$. 

Fig. 1. Gains from Nash and Cooperative policies.
exercise above, consumers would only offer 0.0026% of consumption for the Nash solution, and an additional 0.0034% for the Cooperative solution. By this metric, the welfare gains are small.

It must be recalled, however, that the benchmark model is highly stylistic. We made strategic modeling choices to achieve algebraic simplicity, and to allow closed form solutions; but as we have already noted, these modeling choices can also produce some collateral damage. Here we argue that they probably give an unrealistic view of the true welfare costs of nominal inertia. For example, our linear specification of the disutility of labor implies that the Frisch (constant marginal utility of wealth) elasticity of labor supply is infinite; empirical studies suggest that it is very low (between 0.05 and 0.35). Also, our log specification of the utility of consumption implies that the coefficient of risk aversion is equal to one; most model calibrations make it higher. Nominal inertia causes undesired employment and consumption fluctuations; lowering the Frisch elasticity (or equivalently, making the disutility of labor convex) would increase the disutility of these employment fluctuations, and increasing the coefficient of risk aversion would increase the disutility of these consumption fluctuations. Moreover, we have assumed that price stickiness is the only form of nominal inertia; adding wage stickiness would cause firms to make inefficient hiring decisions, increasing the cost of nominal inertia. Finally, the benchmark model is essentially static; adding staggered price and wage setting would increase the inefficiencies caused by nominal inertia. In the work of Canzoneri et al. (2004), we find that the welfare costs of nominal inertia are much higher in a one sector, closed economy model with capital formation, staggered wage and price setting, and more realistic values of the Frisch elasticity. It would be interesting to add these features to our multi-sector, two-country model, and verify that the relative welfare measures—$G_N=0.13$,  

Fig. 2. Relative gains from Coordinated policies.

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35 See Canzoneri et al. (2004).
Before we conclude, it should be noted that Fig. 1 also carries a second, and less sanguine, message. The imperfect correlation of sectoral productivity shocks creates a need for policy coordination in the benchmark model, but it also implies that monetary policy is much less effective. When sectoral productivity shocks are perfectly correlated, monetary policy can achieve the flexible price solution. But in our calibration exercise, Nash monetary policies only close 13% of the welfare gap; and Cooperative monetary policies only close another 17%. Monetary policy—fully coordinated, and based on full information about the shocks—can only close 30% of the welfare gap caused by nominal inertia.

5. Conclusion

O&Ra and C&Pa developed a benchmark model that paved the way for a second generation of policy coordination models incorporating optimizing households, monopolistic competition and nominal inertia. O&Rb asked the question: “Are the stabilization gains from having separate currencies largely squandered in the absence of effective international monetary coordination?” Based on their analysis of an extension of the benchmark model, they concluded that: “...under plausible assumptions the answer is no.” We have argued to the contrary that policy coordination may be more important in second generation models than it ever was in first generation models.

Our argument was based in part on theoretical counterexamples to the notion that the gains from coordination must necessarily be small in the benchmark model. (This notion achieved the status of a folk theorem in the literature on first generation models.) We also provided a calibration of the benchmark model which suggested that policy coordination was at least as important (to national welfare) as reacting to the shocks in a sensible way at the national level. However, the benchmark model is probably not rich enough to give a serious answer to O&Rb’s question.

The need for policy coordination in the benchmark model arose because of asymmetries in the stochastic processes for sectoral productivity. Asymmetries in nominal inertia would imply an additional need for policy coordination. For example, some sectors may exhibit wage inertia rather than price inertia, and some sectors may exhibit more nominal inertia (of either type) than others. Future modeling of second generation models should be guided by empirical evidence on where the more important asymmetries lie. Which sectors show more wage stickiness? Which sectors show more price stickiness? How pronounced is the asymmetry? The Balassa–Samuelson literature provides evidence on productivity asymmetries across tradeables and non-tradeables, but is this the most important sectoral breakdown? Currently, we do not have answers to these empirical questions. This is what’s yet to come.

36 See Canzoneri et al. (2003b). The policy tradeoffs they describe in closed economies would also arise in a two-country model similar to our benchmark model, and they would introduce a role for policy coordination.
Appendix A. Proof of Lemma 1

The Nash, Cooperative and Passive policies and utility losses are set out in Lemma 1:

**Lemma 1. Gains from coordination in the benchmark model**

(i) Nash policy rules: \( m = 1/2 \, z_N + 1/2 \, z_D \) and \( m^* = 1/2 \, z_N^* + 1/2 \, z_D^* \).

(ii) Losses in the Nash solution: \( \Delta_N = \Delta_N^* = 3/4 \, \sigma_N^2 + 3/4 \sigma_D^2 + \sigma_E^2 - 1/2 \sigma_{ND} - \sigma_{NE} - \sigma_{DE} \).

(iii) Cooperative policy rules: \( m = 1/3 \, z_N + 1/3 \, z_D + 1/3 \, z_E \) and \( m^* = 1/3 \, z_N^* + 1/3 \, z_D^* + 1/3 \, z_E^* \).

(iv) Losses in the Cooperative solution: \( \Delta_C = \Delta_C^* = 2/3(\sigma_N^2 + \sigma_D^2 + \sigma_E^2 - \sigma_{ND} - \sigma_{NE} - \sigma_{DE}) \).

(v) Passive policy rules: \( m = m^* = 0 \).

(vi) Losses in the Passive solution: \( \Delta_P = \Delta_P^* = \sigma_N^2 + \sigma_D^2 + \sigma_E^2 \).

**Proof.** (i) Let \( m = \gamma z_N + (1 - \gamma) z_D \) and \( m^* = \gamma^* z_N + (1 - \gamma^*) z_D^* \). Choose \( \gamma \) to minimize

\[
\Delta_N = V[\gamma z_N + (1 - \gamma) z_D - z_N] + V[\gamma^* z_N + (1 - \gamma^*) z_D - z_D^*] + V[\gamma^* z_N^* + (1 - \gamma^*) z_D^* - z_E^*] = (1 - \gamma)^2 V[z_D - z_N] + \gamma^2 V[z_N - z_D] + V[\gamma^* z_N^* + (1 - \gamma^*) z_D^* - z_E^*].
\]

The first order condition is \(-2(1 - \gamma) V[z_D - z_N] + 2\gamma V[z_N - z_D] = 0\). This implies \( \gamma = 1/2 \).

(ii) Let \( \Delta_N = 1/4 V[z_D - z_N] + 1/4 V[z_N - z_D] + V[1/2 z_N + 1/2 z_D - z_E] = 1/2(\sigma_N^2 + \sigma_D^2 - 2\sigma_{ND}) + 1/4(\sigma_N^2 + \sigma_D^2 + 2\sigma_{ND}) + \sigma_E^2 - \sigma_{NE} - \sigma_{DE}. \) With symmetry, this reduces to \( 3/4 \sigma_N^2 + 3/4 \sigma_D^2 + \sigma_E^2 - 1/2 \sigma_{ND} - \sigma_{NE} - \sigma_{DE}. \)

(iii) Let \( m = \gamma_N z_N + \gamma_D z_D + \gamma_E z_E \) and \( m^* = \gamma_N^* z_N + \gamma_D^* z_D + \gamma_E^* z_E \). Choose the \( \gamma_j \) to minimize \( 1/2(\Delta_C + \Delta_C^*) \), which, with symmetry, is

\[
V[(\gamma_N - 1) z_N + \gamma_D z_D + \gamma_E z_E] + V[(\gamma_N^* - 1) z_N + \gamma_D^* z_D + \gamma_E^* z_E] + V[(\gamma_N z_N + \gamma_D z_D + (\gamma_E - 1) z_E] = (\gamma_N - 1)^2 + 2\gamma_N^2 \sigma_N^2 + (\gamma_D - 1)^2 + 2\gamma_D^2 \sigma_D^2 + (\gamma_E - 1)^2 + 2\gamma_E^2 \sigma_E^2 + 2(\gamma_N (\gamma_N - 1) + (\gamma_D - 1) \gamma_N + \gamma_D \gamma_N) \sigma_{ND} + 2(\gamma_E (\gamma_N - 1) + (\gamma_E - 1) \gamma_N + \gamma_E \gamma_N) \sigma_{NE} + 2(\gamma_D (\gamma_E - 1) + (\gamma_D - 1) \gamma_E + \gamma_D \gamma_E) \sigma_{DE}.
\]
First order conditions with respect to:

\[ \gamma_N (3\gamma_N - 1)\sigma_N^2 + (3\gamma_D - 1)\sigma_{ND} + (3\gamma_E - 1)\sigma_{NE} = 0 \]

\[ \gamma_D (3\gamma_N - 1)\sigma_{ND} + (3\gamma_D - 1)\sigma_D^2 + (3\gamma_E - 1)\sigma_{DE} = 0 \]

\[ \gamma_E (3\gamma_N - 1)\sigma_{NE} + (3\gamma_D - 1)\sigma_{DE} + (3\gamma_E - 1)\sigma_E^2 = 0 \]

These can be rewritten as \( \Sigma[(3\gamma_N - 1), (3\gamma_D - 1), (3\gamma_E - 1)] = [0, 0, 0] \), where \( \Sigma \) is the covariance matrix of the sectoral productivity shocks. The first order conditions are satisfied only by \( \gamma_N = \gamma_D = \gamma_E = 1/3 \) when \( \Sigma \) has full rank.

(iv) Substituting \( \gamma_N = \gamma_D = \gamma_E = 1/3 \) into the expression for \( A_C \) above yields,

\[
A_C = (4/9 + 1/9 + 1/9)\sigma_N^2 + (4/9 + 1/9 + 1/9)\sigma_D^2 + (4/9 + 1/9 + 1/9)\sigma_E^2 \\
+ 2(-2/9 - 2/9 + 1/9)\sigma_{ND} + 2(-2/9 - 2/9 + 1/9)\sigma_{NE} \\
+ 2(-2/9 - 2/9 + 1/9)\sigma_{ND} + 2(-2/9 - 2/9 + 1/9)\sigma_{DE} \\
= 2/3(\sigma_N^2 + \sigma_D^2 + \sigma_E^2 - \sigma_{ND} - \sigma_{NE} - \sigma_{DE}).
\]

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