Scope and the Grammar of Choice

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1 Introduction and Data

The essence of scope in natural language semantics can be characterized as follows: an expression e_1 takes scope over an expression e_2 iff the interpretation of the former affects the interpretation of the latter. Consider, for example, the sentence in (1) below, which is typical of the cases discussed in this paper in that it involves an indefinite and a universal (or, more generally, a non-existential) quantifier.

(1) Every^x student in my class read a^y paper about scope.

How can we tell whether the indefinite in (1) is in the scope of the universal or not? We can answer this question in two ways. From a dependence-based perspective, $\mathbf{Q}'y$ is in the scope of $\mathbf{Q}x$ if the values of the variable y (possibly) covary with the values of x. From an independence-based perspective, $\mathbf{Q}'y$ is outside the scope of $\mathbf{Q}x$ if y's value is fixed relative to the values of x.

This brings us to the first of our two central questions: should the scopal properties of ordinary, 'unmarked' indefinites be characterized in terms of *dependence* or in terms of *independence*? The difference between these two conceptualizations is that a dependence-based approach establishes which quantifier(s) $\mathbf{Q}'y$ is dependent on, while an independence-based approach establishes which quantifier(s) $\mathbf{Q}'y$ is independent of.

Logical semantics has taken both paths to the notion of scope: compare the standard, dependence-based semantics of first-order logic (FOL) – or the dependence-driven Skolemization procedure – with the independence-based semantics of Independence-Friendly Logic (IFL, Hintikka 1973, Sandu 1993, Hodges 1997, Väänänen 2007 among others). Natural language semantics has only taken the dependence-based path.

In this paper, we argue that there are advantages to following an independence-based approach to the scopal properties of natural language indefinites – and thus to importing into natural language semantics the main insight from independence-friendly logic.

The second central question we are concerned with is whether scope is a matter of syntax, semantics or both. And if the last answer is the correct one, what is the division of labor between syntax and semantics that best captures the scopal properties of natural language quantifiers in general and of indefinites in particular?

Once again, natural language semantics followed the lead of FOL and captured scope (more or less) syntactically. Scope effects emerge as a consequence of the different ways multiple quantifiers can be composed. At the heart of various accounts of the two quantifier scopings intuitively associated with sentence (1) above one finds the two FOL formulas in (2) and (3) below. Restrictor formulas are indicated by means of [ ] and nuclear scope formulas by means of ( ).

(2) $\forall x[\textsc{student}(x)]\ (\exists y[\textsc{paper}(y)]\ (\textsc{read}(x, y)))$

(3) $\exists y[\textsc{paper}(y)]\ (\forall x[\textsc{student}(x)]\ (\textsc{read}(x, y)))$

In this paper we depart from this tradition and partially separate semantic scope from syntactic scope.

1.1 The Free Upwards Scope of Indefinites

A well-known problem for the received view of scope in natural language semantics is that indefinites enjoy free upwards scope – disregarding not only clausal but also island boundaries. In contrast, the upward scope of universals is clause-bounded (see Farkas 1981, Fodor 1981, Abusch 1994 among others). This is exemplified in (4) and (5) below.

(4) John read a^x paper that every^y professor recommended.

(5) Every^x student read every^y paper that a^z professor recommended.

In sentence (4), the universal is in a relative clause restricting an indefinite – and the universal cannot scope over the indefinite. In sentence (5), the indefinite is in a relative clause restricting a universal which, in its turn, is in the syntactic scope of another universal. In this case, we have three possible readings, depending on the relation between the indefinite and each universal. That is, the indefinite can freely scope out of the relative clause. The three readings of sentence (5) are provided below. The IS reading in (7) is particularly important because it shows that exceptional scope cannot be analyzed away as a referential phenomenon (as argued in Farkas 1981 contra Fodor 1981).

(6) Narrowest Scope (NS): for every student x, for every paper y such that there is a professor z that recommended y, x read y.

(7) Intermediate Scope (IS): for every student x, there is a professor z such that, for every paper y that z recommended, x read y.

(8) Widest Scope (WS): there is a professor z such that, for every student x, for every paper y that z recommended, x read y.

The scopal freedom of indefinites is problematic for accounts in which semantic scope relations reduce to syntactic c-command relations. More generally, indefinites pose problems for any theory in which scope relations can only arise out of semantic composition because such theories need a special composition rule for indefinites that grants them their observed freedom. Whether this rule is embedded in a Cooper-storage account of scope (as in Abusch 1994) or it simply states that syntactically covert movement for scope is upwards free for indefinites (see Geurts 2000), there is no independent justification for the fact that such a rule can target only (in)definites, but not any *bona fide* quantifier.

Choice-function / Skolem-function accounts of indefinites (see Reinhart 1997, Winter 1997, Kratzer 1998, Matthewson 1999 and Steedman 2007 among others) avoid such stipulative compositional rules by encapsulating the scopal freedom of indefinites into their lexical meaning. The main idea is that the core semantics of indefinites involves choosing a witness that satisfies the restrictor and nuclear scope of the indefinite – and the different ways in which a witness is chosen give the indefinite different semantic scopes. In particular, the choice of the witness can be dependent on the values of the variable introduced by another quantifier (which is exactly what Skolemization is in FOL).

For example, the account proposed in Steedman (2007) derives the two possible scopes of the indefinite *a^y paper about scope* in our initial sentence (1) by always interpreting the indefinite *in situ*, but letting it contribute a Skolem function $\mathbf{f}$ of variable arity. If $\mathbf{f}$'s arity is 0, then the function is a constant and the choice of the witness is not dependent on the universal quantifier *every^x student in my class*. This yields the reading under which the indefinite has wide scope, provided in (9) below. If $\mathbf{f}$'s arity is 1, the witness is chosen in a way that is dependent on the values of the universally-quantified variable x. This yields the reading under which the indefinite has narrow scope, provided in (10) below. For ease of comparison, we preserve the syntax in (2) and (3) above.

(9) $\forall x[\textsc{student}(x)]\ ([\textsc{paper}(\mathbf{f})]\ (\textsc{read}(x, \mathbf{f})))$

(10) $\forall x[\textsc{student}(x)]\ ([\textsc{paper}(\mathbf{f}(x))]\ (\textsc{read}(x, \mathbf{f}(x))))$

1.2 Syntactic Constraints on Exceptional Scope

We turn now to the second well-known problem for theories of scope that want to account for both indefinites and regular quantifiers, namely that syntax cannot be altogether disregarded, even in the case of indefinites. The syntactic constraint we want to capture, and which we dub the Binder Roof Constraint, is that an indefinite cannot scope over a quantifier that binds into its restrictor: in (11) below, *one*^{z} *of its*$_y$ *authors* can have only narrowest scope (see Abusch 1994, Chierchia 2001 and Schwarz 2001 among others).

(11) Every^x student read every^y paper that one^z of its$_y$ authors recommended.

While Steedman (2007) has a way of capturing this constraint, independence-friendly or choice-function approaches do not; see Chierchia (2001) and Schwarz (2001) for a discussion of this problem in the context of choice-function approaches. But the account in Steedman (2007) relies on a syntactic stipulation, and one would ideally want the constraint to follow from the general definition of the interpretation function that should require bound variables to always remain bound.

Thus, although indefinites exhibit upwards exceptional scope, configurational issues cannot be disregarded altogether. Given that choice / Skolem function approaches pack the scopal properties of indefinites into their lexical meanings, i.e., into the functions that they contribute, they need additional syntactic constraints to limit the freedom of interpretation that they incorrectly predict for indefinites. What we need is to account for the contrast between the scopal behavior of (in)definites and that of *bona fide* quantifiers, while still preserving a syntax-driven compositional interpretation procedure.

2 Outline of the Account

Following Steedman (2007) and Farkas (1997), we interpret indefinites *in situ*, thus partially divorcing scope from configurational matters. But we depart from previous linguistic accounts (including Steedman 2007, Farkas 1997 and choice-function approaches) in conceptualizing the scope of indefinites as an *independence*-based notion.

When we interpret an indefinite, we do not specify whether the witness it contributes is dependent on / covaries with other quantifiers. Instead, we specify whether the witness it contributes is independent of / fixed relative to other quantifiers. The main role of syntactic structure is to provide constraints on witness choice: if a quantifier $\mathbf{Q}x$ syntactically scopes over an indefinite $\mathbf{Q}'y$, it becomes possible for the values of y to be required to stay fixed relative to the values of x. If the possibility is taken advantage of, y will be independent of x.

Thus, the main proposal has two components: (*i*) just as in choice / Skolem function approaches, we take the essence of the semantics of indefinites to be choosing a witness; (*ii*) but, in contrast to choice / Skolem function approaches, we follow independence-friendly logic and take witness choice to be part of the interpretation procedure. That is, indefinites choose a witness *at some point in the evaluation* and require its non-variation from that point on. In contrast to Skolemization, which ensures non-variation by means of a higher-order functional variable, we ensure non-variation by directly constraining the values that the first-order variable contributed by the indefinite takes.

Consider, for example, the formula in (12) below. Suppose that the set of values for x satisfying the restrictor formula ϕ is $\{\alpha_1, \alpha_2\}$ and that the set of values for y satisfying the restrictor formula ϕ' is $\{\beta_1, \beta_2\}$, i.e., suppose that the universal $\forall x$ quantifies over the set $\{\alpha_1, \alpha_2\}$ and the universal $\forall y$ quantifies over the set $\{\beta_1, \beta_2\}$. The existential $\exists z$ chooses a witness satisfying its restrictor formula ϕ'' and its nuclear scope formula ψ. The witness choice can happen in three different ways, as shown in (13): it can be fixed relative to no variables (narrowest scope), it can be fixed relative to the variable y contributed by the lower universal (intermediate scope) or it can be fixed relative to both x and y (widest scope). Syntactically, the scope of the existential is the same, namely narrowest scope; semantically, the existential can have three possible scopes depending on how witnesses are chosen.

(12) $\forall x[\phi]\ (\forall y[\phi']\ (\exists z[\phi'']\ (\psi)))$

(13)

Narrowest scope (NS):	z is fixed relative to no variable, i.e., z (possibly) covaries with both x and y
Intermediate scope (IS):	z is fixed relative to y and (possibly) covaries with x
Widest scope (WS):	z is fixed relative to both x and y

The three semantic scopes are schematically depicted by the matrices in (14) below. In the NS case, the values of z are (possibly) different for any two different pairs of values for x and y. In the IS case, the values of z are (possibly) different for the two values of x, i.e., α_1 is associated with witness γ and α_2 is associated with witness γ'. But the witnesses are fixed relative to the two values of y, i.e., witness γ is associated with both β_1 and β_2 and so is witness γ'. In the WS case, the values of z are fixed relative to any combination of values for x and y: γ is associated with all four pairs of values $\langle\alpha_1, \beta_1\rangle$, $\langle\alpha_1, \beta_2\rangle$, $\langle\alpha_2, \beta_1\rangle$ and $\langle\alpha_2, \beta_2\rangle$.

(14)

NS

<table>
<tr><th>x</th><th>y</th><th>z</th></tr>
<tr><td>α_1</td><td>β_1</td><td>γ</td></tr>
<tr><td>α_1</td><td>β_2</td><td>γ'</td></tr>
<tr><td>α_2</td><td>β_1</td><td>γ''</td></tr>
<tr><td>α_2</td><td>β_2</td><td>γ'''</td></tr>
</table>

IS

<table>
<tr><th>x</th><th>y</th><th>z</th></tr>
<tr><td>α_1</td><td>β_1</td><td rowspan="2">γ</td></tr>
<tr><td>α_1</td><td>β_2</td></tr>
<tr><td>α_2</td><td>β_1</td><td rowspan="2">γ'</td></tr>
<tr><td>α_2</td><td>β_2</td></tr>
</table>

WS

<table>
<tr><th>x</th><th>y</th><th>z</th></tr>
<tr><td>α_1</td><td>β_1</td><td rowspan="4">γ</td></tr>
<tr><td>α_1</td><td>β_2</td></tr>
<tr><td>α_2</td><td>β_1</td></tr>
<tr><td>α_2</td><td>β_2</td></tr>
</table>

The main questions we need to answer are: (i) how should we formalize such matrices? and (ii) how should we formalize the non-variation requirement contributed by existentials relative to these matrices? The following section answers these questions and provides the formal account of the data introduced in section 1.

3 Scope in First-Order Logic with Choice (C-FOL)

Our account is couched in a slightly modified first-order language with restricted quantification. A compositional translation procedure from English into (a higher-order version of) this language can be defined in the usual Montagovian way, but we won't do this here.

While the syntax of the language is fairly standard, the semantics is not. The main formal novelty is that, in contrast to standard Tarskian semantics where evaluation indices are single assignments, indices of evaluation in the language we define have a more articulated structure: (i) following the versions of independence-friendly logic in Hodges (1997) and Väänänen (2007), we evaluate formulas relative to *sets* of assignments $G, G', \ldots$ instead of single assignments $g, g', \ldots$; (ii) in the spirit of the main insight in Steedman (2007), we evaluate a quantifier relative to the set of variables $\mathcal{V}$ introduced by the syntactically higher quantifiers, i.e., the indices of evaluation contain the set of variables $\mathcal{V} = \{x, y, \ldots\}$ introduced by all the previously interpreted quantifiers $\mathbf{Q}x, \mathbf{Q}'y, \ldots$.

A set of assignments G is represented as a matrix with assignments $g, g', \ldots$ as rows:

G	$\ldots$	x	y	z	$\ldots$
g	$\ldots$	α_1 $(= g(x))$	α_2 $(= g(y))$	α_3 $(= g(z))$	$\ldots$
g'	$\ldots$	β_1 $(= g'(x))$	β_2 $(= g'(y))$	β_3 $(= g'(z))$	$\ldots$
g''	$\ldots$	γ_1 $(= g''(x))$	γ_2 $(= g''(y))$	γ_3 $(= g''(z))$	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

or

$\ldots$	x	y	z	$\ldots$
$\ldots$	α_1	α_2	α_3	$\ldots$
$\ldots$	β_1	β_2	β_3	$\ldots$
$\ldots$	γ_1	γ_2	γ_3	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

To keep the formalism as simple as possible and as close as possible to the standard Tarskian semantics for FOL, we work with total assignments.

As we already indicated informally, having sets of assignments as indices of evaluation enables us to encode when a quantifier $\mathbf{Q}'y$ is independent of another quantifier $\mathbf{Q}''z$ by requiring the variable y to have a fixed value relative to the varying values of z. This condition is of the form given in (15) below.

(15) Fixed value condition (basic version): for all $g, g' \in G$, $g(y) = g'(y)$.

The fixed value condition for y leaves open the possibility that the values of z vary from assignment to assignment, i.e., that $g(z) \neq g'(z)$ for some $g, g' \in G$.

Having sets of assignments allows us to state directly in the semantics that y does not co-vary with z. We are therefore able to state that the quantifier $\mathbf{Q}'y$ is not in the *semantic* scope of $\mathbf{Q}''z$, though it may well be in its *syntactic* scope. Partially separating syntactic scope from semantic scope is a main feature of our proposal.

Assuming that the quantifier $\mathbf{Q}'y$ is not in the semantic scope of $\mathbf{Q}''z$, we still want to allow for a third quantifier $\mathbf{Q}x$ to take both syntactic and semantic scope over $\mathbf{Q}'y$. One such case is the intermediate scope (IS) configuration discussed in the previous section for the formula in (12) above. In such cases, we want the value of y to be fixed relative to the values of z, but we want to allow for the possibility that the values of y covary with the values of x. We do this by relativizing the fixed value condition to the values of x, as shown in (16) below. The IS matrix in (14) above satisfies precisely this kind of relativized fixed value condition.

(16) Fixed value condition (relativized version):
for all $g, g' \in G$, if $g(x) = g'(x)$, then $g(y) = g'(y)$.

Thus, working with sets of assignments instead of single assignments enables us to formulate non-variation / fixed-value conditions relativized to particular variables. But note that we need to keep track of the variables introduced by the syntactically higher quantifiers in order to be able to relativize such conditions to only some of these quantifiers.

This brings us to the second way in which we add structure to our indices of evaluation: they contain the set of variables $\mathcal{V} = \{x, y, \ldots\}$ introduced by the previous quantifiers. These are the variables an existential *could* covary with – but, in contrast to the standard Tarskian semantics, the existential *does not have* to covary with them.

Existentials have a choice. They can choose which ones of the quantifiers that take syntactic scope over them also take semantic scope over them. That is, when we interpret an indefinite, we choose a subset of variables $\mathcal{V}' \subseteq \mathcal{V} = \{x, y, \ldots\}$ containing the variables that the indefinite possibly covaries with, i.e., the variables that the indefinite is possibly dependent on. The complement set of variables $\mathcal{V} \setminus \mathcal{V}'$ contains the variables relative to which the indefinite does *not* vary, i.e., the variables that the indefinite is independent of. We dub the resulting first-order language Choice-FOL or C-FOL for short.

Thus, an indefinite in the syntactic scope of a quantifier $\mathbf{Q}$ binding a variable x is in its semantic scope iff $x \in \mathcal{V}'$. This makes the following two (correct) predictions: (*i*) an indefinite may be in the semantic scope of a quantifier $\mathbf{Q}x$ only if $\mathbf{Q}x$ has syntactic scope over the indefinite and (*ii*) an indefinite may in principle be outside the semantic scope of a quantifier $\mathbf{Q}x$ that takes syntactic scope over it.

3.1 Existentials in First-Order Logic with Choice (C-FOL)

A model $\mathfrak{M}$ for C-FOL has the same structure as the standard models for FOL, i.e., $\mathfrak{M}$ is a pair $\langle \mathfrak{D}, \mathfrak{I} \rangle$, where $\mathfrak{D}$ is the domain of individuals and $\mathfrak{I}$ the basic interpretation function. An $\mathfrak{M}$-assignment g for C-FOL is also defined just as in FOL: g is a total function from the set of variables $\mathcal{VAR}$ to $\mathfrak{D}$, i.e., $g \in \mathfrak{D}^{\mathcal{VAR}}$.

The essence of quantification in FOL is pointwise (i.e., variablewise) manipulation of variable assignments. We indicate this by means of the abbreviation $g'[x]g$, defined in (17) below. Informally, $g'[x]g$ abbreviates that the assignments g' and g differ at most with respect to the value they assign to x.

(17) $g'[x]g :=$ for all variables $v \in \mathcal{VAR}$, if $v \neq x$, then $g'(v) = g(v)$

Given that we work with sets of variable assignments, we generalize this to a notion of pointwise manipulation of sets of assignments, abbreviated as $G'[x]G$ and defined in (18) below. This is just the cumulative-quantification style generalization of $g'[x]g$: any $g' \in G'$ has an $[x]$-predecessor $g \in G$ and any $g \in G$ has an $[x]$-successor $g' \in G'$.

(18) $$G'[x]G := \begin{cases} \text{for all } g' \in G', \text{ there is a } g \in G \text{ such that } g'[x]g \\ \text{for all } g \in G, \text{ there is a } g' \in G' \text{ such that } g'[x]g \end{cases}$$

We can now turn to the central piece of our account, i.e., the definition of the interpretation function $[\![\cdot]\!]^{\mathfrak{M}G,\mathcal{V}}$. As already indicated, the indices of evaluations for C-FOL consist of a set of assignments G and a set of variables $\mathcal{V}$; $\mathbb{T}$ and $\mathbb{F}$ stand for true and false respectively.

Atomic formulas are interpreted as shown in (19) below.

(19) Atomic formulas: $[\![R(x_1, \ldots, x_n)]\!]^{\mathfrak{M}G,\mathcal{V}} = \mathbb{T}$ iff
   a. $\{x_1, \ldots, x_n\} \subseteq \mathcal{V}$
   b. $G \neq \emptyset$
   c. $\langle g(x_1), \ldots, g(x_n) \rangle \in \mathfrak{I}(R)$, for all $g \in G$

The condition $\{x_1, \ldots, x_n\} \subseteq \mathcal{V}$ in (19a) bans free variables. We assume that deictic pronouns require the discourse-initial sequence of variables to be non-empty, much like the discourse-initial partial assignments in Discourse Representation Theory / File-Change Semantics are required to have a non-empty domain.

The condition in (19c) is the central one: the set of assignments G satisfies an atomic formula $R(x_1, \ldots, x_n)$ if each assignment $g \in G$ satisfies it. That is, we distribute over the set G and, in this way, relate the C-FOL notion of set-based satisfaction to the standard FOL notion of single-assignment-based satisfaction. The non-emptiness condition in (19b) rules out the case in which the distributive requirement in (19c) is vacuously satisfied.

The interpretation of conjunction is the expected one: we just pass the current index of evaluation down to each conjunct.

(20) Conjunction: $[\![\phi \wedge \psi]\!]^{\mathfrak{M},G,\mathcal{V}} = \mathbb{T}$ iff $[\![\phi]\!]^{\mathfrak{M},G,\mathcal{V}} = \mathbb{T}$ and $[\![\psi]\!]^{\mathfrak{M},G,\mathcal{V}} = \mathbb{T}$.

The interpretation of existential quantification is provided in (21) below. Existentials have a choice – and this choice is encoded by the superscript $\mathcal{V}'$. The set of variables $\mathcal{V}'$ is a subset of the current set of variables $\mathcal{V}$ contributed by the previous / higher quantifiers, and it is relative to these quantifiers that the witness choice is fixed or not.

(21) Existential quantification: $[\![\exists^{\mathcal{V}'}x[\phi]\ (\psi)]\!]^{\mathfrak{M},G,\mathcal{V}} = \mathbb{T}$ iff $\mathcal{V}' \subseteq \mathcal{V}$ and $[\![\psi]\!]^{\mathfrak{M},G',\mathcal{V}\cup\{x\}} = \mathbb{T}$, for some G' such that
a. $G'[x]G$
b. $[\![\phi]\!]^{\mathfrak{M},G',\mathcal{V}'\cup\{x\}} = \mathbb{T}$
c. $\begin{cases} \text{if } \mathcal{V}' = \emptyset : g(x) = g'(x), \text{ for all } g, g' \in G' \\ \text{if } \mathcal{V}' \neq \emptyset : g(x) = g'(x), \text{ for all } g, g' \in G' \text{ that are } \mathcal{V}'\text{-identical} \end{cases}$

(22) Two assignments g and g' are $\mathcal{V}'$-identical iff for all variables $\upsilon \in \mathcal{V}'$, $g(\upsilon) = g'(\upsilon)$.

The two conditions in (21c) formalize what it means to make one of the allowed choices. If $\mathcal{V}' = \emptyset$, the value of the existential is completely fixed and the indefinite has widest scope: we enforce the basic, unrelativized fixed value condition in (15) above. If $\mathcal{V}' \neq \emptyset$, we enforce a relativized version of the fixed value condition (like the one defined in (16)) – and we relativize this condition to the set of variables $\mathcal{V}'$.

Thus, an existential formula $\exists^{\mathcal{V}'}x[\phi]\ (\psi)$ is interpreted relative to a set of variables $\mathcal{V}$ introduced by the sequence of quantifiers that take syntactic scope over the existential. The superscript $\mathcal{V}'$ on the existential indicates that, among the $\mathcal{V}$-quantifiers, only the $\mathcal{V}'$-quantifiers also take semantic scope over the existential: $\mathcal{V}'$ indicates the *non-variation* of the existential with respect to the quantifiers that contributed the variables in the complement set $\mathcal{V} \setminus \mathcal{V}'$. Importantly, such superscripts can meaningfully occur only on (in)definites, i.e., on existentials, and not on universals or any other *bona fide* quantifiers, because these superscripts constrain the witness choice for the existential. Crucially, the semantics of *bona fide* quantifiers cannot be given in terms of single witnesses.

The formalization of scopal (in)dependence is different from the formalization of referential dependencies. Although both phenomena involve a form of variable coindexation in syntax, the associated semantic interpretation rules are fundamentally different. For scopal (in)dependence, the set of variables that the indefinite is indexed with provides the parameters relative to which we choose the *new* entity introduced by the indefinite. For referential dependencies, e.g., anaphoric or bound pronouns, the variable that the pronoun is indexed with provides the *old* entity which the pronoun refers back to.

3.2 Deriving Syntactic Constraints on Scope

At this point, we can almost derive the syntactic constraint on scope introduced in section 1. The last piece we need is the interpretation of universal quantification. The definition for universally quantified formulas $\forall x[\phi]\ (\psi)$ requires some care because existential quantifiers can occur both in the restrictor formula ϕ and in the nuclear scope formula ψ. To derive the syntactic constraints on scope, we only need a simpler, preliminary definition that disregards the possibility that existential quantifiers can occur in restrictor formulas. We revise this definition in the next subsection when we account for exceptional scope.

Universally quantified formulas are interpreted as shown in (23) below. The main idea is that the nuclear scope formula ψ of a universal quantifier $\forall x[\phi]$ is evaluated relative to the set of of all assignments that satisfy the restrictor formula ϕ. That is, we collect all assignments g' such that ϕ is true relative to the singleton set of assignments $\{g'\}$ and pass the set G' consisting of all these assignments to the nuclear scope formula ψ.

(23) Universal quantification (preliminary): $[\![\forall x[\phi]\ (\psi)]\!]^{\mathfrak{M},G,\mathcal{V}} = \mathbb{T}$ iff $[\![\psi]\!]^{\mathfrak{M},G',\mathcal{V}\cup\{x\}} = \mathbb{T}$, where G' is the maximal set of assignments that satisfies ϕ relative to x, G and $\mathcal{V}$.

(24) G' is the maximal set of assignments that satisfies ϕ relative to the variable x, the set of assignments G and the set of variables $\mathcal{V}$ iff
$G' = \bigcup_{g \in G} \left\{g' : g'[x]g \text{ and } [\![\phi]\!]^{\mathfrak{M},\{g'\},\mathcal{V}\cup\{x\}} = \mathbb{T}\right\}$.

For example, consider the sentence in (25) below and its C-FOL translations in (26) and (27). We assume that an indefinite is always interpreted *in situ* and is translated by existentials that can have any superscript that is licensed by the interpretation of the previous quantifiers in the sentence. Given that the indefinite *a*^{y} *paper* is evaluated after the universal quantifier *every*^{x} *student*, it is interpreted relative to the non-empty set of variables $\{x\}$, so there are two possible superscripts, namely $\emptyset$ and $\{x\}$.

(25) Every^x student read a^y paper.

(26) $\forall x[\textsc{student}(x)]\ (\exists^{\emptyset} y[\textsc{paper}(y)]\ (\textsc{read}(x,y)))$

(27) $\forall x[\textsc{student}(x)]\ (\exists^{\{x\}} y[\textsc{paper}(y)]\ (\textsc{read}(x,y)))$

The only difference between the C-FOL translations in (26) and (27) is the superscript on the existential quantifier. If the superscript is $\emptyset$, as in (26), the existential receives a wide-scope interpretation, while if the superscript is $\{x\}$, as in (27), the existential receives a narrow-scope interpretation.

Thus, in contrast to the two FOL formulas in (2) and (3) above, C-FOL does not capture the wide-scope vs. narrow-scope readings of sentence (25) by means of two syntactically different formulas. The indefinite always has narrow scope syntactically, but at the point when it is interpreted, we may choose to select a witness that is independent of the higher universal quantifier and, thus, effectively removing the indefinite from the semantic scope of the universal.

In more detail, assume that the C-FOL formulas in (26) and (27) are interpreted relative to an arbitrary set of assignments G and the empty set of variables $\emptyset$. This is what is required by the definition of truth for C-FOL, provided in (28) below.

(28) Truth: a formula ϕ is true in model $\mathfrak{M}$ iff $[\![\phi]\!]^{\mathfrak{M},G,\emptyset} = \mathbb{T}$ for any set of assignments G, where $\emptyset$ is the empty set of variables.

Furthermore, assume that there are exactly three students in our model $\mathfrak{M}$, namely $\{stud_1, stud_2, stud_3\}$. Then, the interpretation of the formulas in (26) and (27) proceeds as shown in (29) below. First, the universal quantifier $\forall x[\textsc{student}(x)]$ introduces the set of all students relative to the variable x and relative to each assignment $g \in G$. Then, the existential $\exists^{\emptyset/\{x\}} y[\textsc{paper}(y)]$ introduces a paper and chooses whether it is the same for every student (if the superscript is $\emptyset$) or whether it is possibly different from student to student (if the superscript is $\{x\}$). Finally, we check that, for each assignment in the resulting set of assignments, the x-student in that assignment read the y-paper in that assignment.

(29)
$$\begin{array}{cccc} \dots & \dots & \dots & \dots \\ \hline \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{\dots} & \multicolumn{1}{c|}{\dots} & \multicolumn{1}{c|}{\dots} \\ \hline \end{array} \xRightarrow{\forall x[\text{STUDENT}(x)]} \begin{array}{cccc} \dots & x & \dots & \dots \\ \hline \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_1} & \multicolumn{1}{c|}{\dots} & \multicolumn{1}{c|}{\dots} \\ \hline \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_2} & \multicolumn{1}{c|}{\dots} & \multicolumn{1}{c|}{\dots} \\ \hline \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_3} & \multicolumn{1}{c|}{\dots} & \multicolumn{1}{c|}{\dots} \\ \hline \end{array}$$

$$\left\{ \begin{array}{l} \xRightarrow{\exists^{\emptyset} y[\text{PAPER}(y)]} \begin{array}{cccc} \dots & x & y & \dots \\ \hline \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_1} & \multicolumn{1}{c|}{} & \multicolumn{1}{c|}{\dots} \\ \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_2} & \multicolumn{1}{c|}{paper} & \multicolumn{1}{c|}{\dots} \\ \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_3} & \multicolumn{1}{c|}{} & \multicolumn{1}{c|}{\dots} \\ \hline \end{array} \xRightarrow{\text{READ}(x,y)} \begin{array}{l} stud_1 \text{ read } paper \\ stud_2 \text{ read } paper \\ stud_3 \text{ read } paper \end{array} \\ \xRightarrow{\exists^{\{x\}} y[\text{PAPER}(y)]} \begin{array}{cccc} \dots & x & y & \dots \\ \hline \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_1} & \multicolumn{1}{c|}{paper} & \multicolumn{1}{c|}{\dots} \\ \hline \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_2} & \multicolumn{1}{c|}{paper'} & \multicolumn{1}{c|}{\dots} \\ \hline \multicolumn{1}{|c|}{\dots} & \multicolumn{1}{c|}{stud_3} & \multicolumn{1}{c|}{paper''} & \multicolumn{1}{c|}{\dots} \\ \hline \end{array} \xRightarrow{\text{READ}(x,y)} \begin{array}{l} stud_1 \text{ read } paper \\ stud_2 \text{ read } paper' \\ stud_3 \text{ read } paper'' \end{array} \end{array} \right.$$

Both independence-friendly and choice-function based approaches predict that indefinites with bound variables in their restrictors are able to take exceptional scope over the binders of those variables, i.e., the unattested IS or WS readings for sentence (11) above are predicted to be available (see Chierchia 2001 for a discussion of this problem in the context of choice-function approaches).

The C-FOL account allows only the NS reading. By the interpretation clause for existentials in (21) above, the restrictor formula ϕ of an indefinite is interpreted only relative to the variables that the indefinite possibly depends on, i.e., the restrictor of an existential $\exists^{\mathcal{V}'}$ is interpreted relative to the set of variables $\mathcal{V}'$. This captures the Binder Roof Constraint because the semantic scope of the restrictor ϕ is always the same as the semantic scope of the existential $\exists^{\mathcal{V}'}$.

Making the indefinite *one*^{z} *of its*$_y$ *authors* in sentence (11) above independent from the universal *every*^{y} *paper* makes the variable y contributed by the pronoun *its*$_y$ a free variable, which is ruled out by the interpretation clause for atomic formulas in (19) above.

3.3 Deriving Exceptional Scope

We finally turn to the account of exceptional scope. The sentence in (5) above is translated as shown in (30) below. The three readings, i.e., the WS, IS and NS readings, are obtained by letting the superscript on the indefinite be $\emptyset$, $\{x\}$ and $\{x, y\}$, respectively.

(30)
$$\begin{array}{l} \forall x[\text{STUDENT}(x)] \\ \qquad (\forall y[\text{PAPER}(y) \wedge \\ \qquad\qquad \exists^{\emptyset/\{x\}/\{x,y\}} z[\text{PROFESSOR}(z)]\ (\text{RECOMMEND}(z, y))] \\ \qquad\qquad\qquad (\text{READ}(x, y))) \end{array}$$

Consider now the IS reading more closely. The superscript $\{x\}$ on the existential quantifier $\exists^{\{x\}} z[\text{PROFESSOR}(z)]$ indicates that only the first universal quantifier $\forall x[\text{STUDENT}(x)]$ takes semantic scope over it. That is, for each student x, we choose a professor z and require x to have read every paper that z recommended. However, the definition of universal quantification in (23) above fails to capture this intuition. The reason is that, when we evaluate the restrictor formula of the universal quantifier $\forall y[\text{PAPER}(y) \wedge \exists^{\{x\}} z[\text{PROFESSOR}(z)]$, we do not have access to the entire previous set of assignments that stores all the x-students: we only examine one assignment in that set at a time. Hence, we vacuously satisfy the relativized fixed value condition contributed by the existential and fail to ensure that a single z-professor is chosen for each x-student.

Thus, to account for our intuitions about exceptional scope in sentences that have indefinites in the restrictor of universals, we need to slightly modify the way C-FOL defines the interpretation of universally quantified formulas. The final definition is provided in (31) below. This definition makes the same predictions as the old one with respect to sentences that do not have indefinites in the restrictor of universals: for example, the analysis of sentence (25) – or of sentence (11) – proceeds in the same way.

(31) Universal quantification (final version): $[\![\forall x[\phi]\ (\psi)]\!]^{\mathfrak{M},G,\mathcal{V}} = \mathbb{T}$ iff $[\![\psi]\!]^{\mathfrak{M},G',\mathcal{V}\cup\{x\}} = \mathbb{T}$, for some G' that is a maximal set of assignments relative to x, ϕ, G and $\mathcal{V}$.

(32) G' is a maximal set of assignments relative to a variable x, a formula ϕ, a set of assignments G and a set of variables $\mathcal{V}$ iff

a. $G'[x]G$ and $[\![\phi]\!]^{\mathfrak{M},G',\mathcal{V}\cup\{x\}} = \mathbb{T}$

b. there is no $G'' \neq G'$ such that $G' \subseteq G''$ and: $G''[x]G$ and $[\![\phi]\!]^{\mathfrak{M},G'',\mathcal{V}\cup\{x\}} = \mathbb{T}$

The main difference between (23) above and (31) is that, in the former, we obtained the maximal set of assignments G' in a distributive way, i.e., by evaluating the restrictor formula ϕ relative to each assignment $g' \in G'$, while in the latter, we obtain it in a *collective* way, i.e., by evaluating the restrictor formula ϕ relative to G' as a whole.

Steedman (2007) cannot capture exceptional scope in general – and the IS reading of sentence (5) in particular. The reason is that the underlying semantics of that system is, ultimately, the familiar distributive Tarskian semantics, so scopal constraints that need to collectively refer to sets of assignments cannot be formulated.

4 Conclusion

This paper has proposed a novel solution to the problem of scope posed by natural language indefinites that captures both the fact that, unlike other quantifiers, indefinites have free upwards scope and the fact that the scopal freedom of indefinites is nonetheless syntactically constrained. As in independence-friendly logic, the special scopal properties of indefinites are attributed to the fact that their semantics can be stated in terms of choosing a suitable witness at a certain point during semantic evaluation. This is in contrast to *bona fide* quantifiers, the semantics of which cannot be given in terms of witnesses because it necessarily involves relations between sets of entities.

The syntactic constraints on the interpretation of indefinites follow from the fact that witness choice arises as a natural consequence of the process of (syntax-based) compositional interpretation of sentences and it is not encapsulated into the lexical meaning of indefinites, as choice / Skolem function approaches would have it.

We therefore expect an unmarked, ordinary indefinite to have free upward scope, exactly as we find in English and other languages that have an unmarked indefinite article. One way an indefinite can be special is by having constraints on its scopal independence, i.e., by requiring it to covary with some other quantifier. The cross-linguistic typology of indefinites appears to support this picture. In particular, dependent indefinites (see Farkas 1997) use additional morphology on top of the ordinary indefinite morphology to mark scopal dependence – e.g., in Hungarian, the indefinite article is reduplicated, while in Romanian the particle *cîte* is added immediately before the indefinite article.

The question that arises now is how the tools introduced here can be used to capture the rich variety of cross-linguistically attested special indefinites.

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