5.1 ANGLES & THEIR MEASURE (CONT'D)

STANDARD FORM:
- VERTEX 180°
- FULL CIRCLE IS 360°

CIRCLE OF RADIUS r
- (0,0)
- (r,0)
- (r,0)

\[ \theta = \frac{\text{ARC}}{\text{RADIUS}} = \frac{S}{r} \]

RADIAN MEASURE

FULL CIRCLE (360°)

\[ \theta \text{ (RADIANS)} = \frac{\text{CIRCUMFERENCE}}{\text{RADIUS}} = \frac{2\pi}{r} \]

So 360° = 2\pi RADIANS

180° = \pi

RIGHT ANGLE \rightarrow 90° = \pi/2

CONVERSIONS:
- DEGREES \rightarrow RADIANS
  - MULTIPLY BY \( \frac{\pi}{180°} \)
- RADIANS \rightarrow DEGREES
  - MULTIPLY BY \( \frac{180°}{\pi} \)
\[360^\circ = \frac{\pi \sqrt{5}}{2} \text{ radians.}\]

\[
\frac{360^\circ}{180^\circ} = \frac{3\pi}{180^\circ} = 2\pi \text{ radians}
\]

\[
\frac{3\pi}{8} \text{ radians} = \square 0^\circ
\]

\[
\frac{3\pi}{8} \cdot \frac{180^\circ}{\pi} = \frac{3\pi}{8} \cdot \frac{180^\circ}{\pi} = \frac{135^\circ}{2} = 67.5^\circ
\]

\[
(4 \text{ radians}) \quad \frac{180^\circ}{\pi} = \frac{4 \cdot 180^\circ}{\pi} = \left(\frac{720}{\pi}\right)^\circ
\]

**Note:** 1 radian.

\[
1 \cdot \frac{180^\circ}{\pi} = \left(\frac{180}{\pi}\right)^\circ \approx 57.3^\circ
\]
Area of sector is proportional to angle

\[ \frac{A_1}{A_2} = \frac{\theta_1}{\theta_2} \]

\( \theta_2 = \text{full circle} = 2\pi \)

\( A_2 = \text{whole area} = \pi r^2 \)

\[ \frac{A_1}{\pi r^2} = \frac{\theta_1}{2\pi} \]

\[ A_1 = \frac{\theta_1 (\pi r^2)}{2\pi} \]

\[ A = \frac{1}{2} r^2 \theta \]

Draw area of a sector formed by central angle \( \theta \)

\[ A = \frac{1}{2} r^2 \theta \]

\[ A = \frac{\pi r^2}{4} \]

\[ = \frac{1}{2} r^2 \left( \frac{\pi}{2} \right) = \frac{\pi r^2}{4} \]
**DEFINITION**
Object that moves around a circle of radius $r$ (at a constant speed), $s = \text{distance traveled (on circle)}$, $t = \text{time of travel}$.

Then, (linear) speed: \[ v = \frac{s}{t} \]

Angular speed: \[ \omega = \frac{\theta}{t} \]

**EX** Convert 100 revolutions per minute.

\[
\frac{100 \text{ revolutions}}{\text{minutes}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} = \frac{200 \pi \text{ radians}}{\text{minutes}}
\]

**EX** \((5.1.101)\)
Spins at rate of 13 revolutions per minute.
Gondola is 25 feet from center. What is linear speed of gondola (in miles/hour)?

\[
25 \text{ feet} \times \frac{13 \text{ revolutions}}{\text{minute}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \approx 23.2 \text{ mph}
\]

(After class)
5.2 TRIGONOMETRIC FUNCTIONS: UNIT CIRCLE APPROACH

RIGHT TRIANGLE APPROACH.

CONSIDER CIRCLE (RADIUS = 1) CENTERED AT (0,0).

\[ x^2 + y^2 = 1 \]

HERE \( \theta = t \)

DFN:

\[ \cos(t) = x \]
\[ \sin(t) = y \]
\[ \tan(t) = \frac{y}{x} \]
\[ \cot(t) = \frac{x}{y} \]
\[ \sec(t) = \frac{1}{x} \]
\[ \csc(t) = \frac{1}{y} \]

BY Suppose \( \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \) IS POINT THAT CORRESPONDS TO \( t \). FIND ALL 6
**TRIG FUNCTIONS**

\[ \cos(t) = \frac{\sqrt{3}}{2} \]

\[ \sin(t) = \frac{-1}{2} \]

\[ \tan(t) = \frac{-1/2}{\sqrt{3}/2} = \frac{-\sqrt{3}/2}{\sqrt{3}/2} = \frac{-\sqrt{3}}{3} \]

\[ \csc(t) = \frac{1}{\frac{-1}{2}} = -2 \]

\[ \cot(t) = \frac{\sqrt{3}/2}{-1/2} = \frac{-\sqrt{3}}{1} = -\sqrt{3} \]