3 CLASS NOTES: WED. 5 FEB.

4.8 (cont'd)

EXP'L GROWTH/DECAY MODELS:

UNINHIBITED GROWTH

EXP'L DECAY.

A = AMOUNT OF QUANTITY.

A₀ = CONSTANT = AMOUNT AT t = 0

k = GROWTH/DECAY RATE

k > 0 k < 0

MODEL A = A₀ e^{kt}

ex (LAST TIME)

START W/ 1500 = A₀

t = 3, A = 2400

\[ k = \frac{\ln\left(\frac{2400}{1500}\right)}{3} \]

\[ \ln\left(\frac{2400}{1500}\right) = 3k \]
\[ A = 1500 e^{(\ln(24/15)/3) \cdot t} \approx 1500 e^{(1.567) t} \]

b) How many after 2 hours? \( t = 2 \) \[ A = 1500 e^{1.567 \cdot 2} \approx 2052 \]

c) Popn of 4500? \( A = 4500 \)

\[ \frac{4500}{1500} = e^{(\ln(24/15)/3) \cdot t} \]

\[ 3 = \ln(24/15) \cdot t \]

\[ t = \frac{3}{\ln(24/15)} = t \approx 7.0123 \ldots \]

Radioactive Decay: \( A = A_0 e^{kt} \) (k < 0)

**DFN**: **Half-Life**: Time required for half of a sample to disintegrate.
\[ A = N_0 e^{kt} \rightarrow \left( \frac{1}{2} N_0 \right) = N_0 e^{kt} \]
\[ \frac{1}{2} = e^{kt} \]
\[ \ln\left( \frac{1}{2} \right) = kt \]
\[ \ln\left( \frac{1}{2} \right) - \ln 2 = \ln\left( \frac{1}{2} \right) \]
\[ \ln\left( \frac{1}{2} \right) = \frac{t}{k} = -\frac{\ln 2}{k} \]

**Half-life of Radium-226 is 1,620 years.**

Start with 2 g, \( A_0 = 2 \)

\[ A = 2e^{kt} \]
\[ 1 = 2e^{k(1620)} \]
\[ \frac{1}{2} = e^{1620k} \rightarrow \ln\left( \frac{1}{2} \right) = 1620k \]
\[ \ln\left( \frac{1}{2} \right) = k = -\frac{1}{1620} \ln(2) \]
\[ k \approx -0.000428896 \ldots \]
\[ k \approx -0.00043 \]
a) After 100 yrs, $t = 100$,

$$A = 2e^{-\frac{\ln 2}{1620}}[100] = 1.916 \text{ gm}$$

b) 80% gone; 20% remains

20% of 4 = 0.4 gm.

Isotope

Exp. (0.4) = 2e^{(-\frac{\ln 2}{1620})t}

\[0.2 = \frac{4}{2} = e^{(-\frac{\ln 2}{1620})t}\]

\[\ln (0.2) = (-\frac{\ln 2}{1620})t\]

\[\frac{1620 \cdot \ln (0.2)}{-\ln 2} = t \approx 3761.5 \text{ yr.}\]

Logistic Mode (For Growth)

$y = c$

Carrying Capacity

Logistic Curve
P = P_{0} e^{k t} \\
\text{LOGISTIC MODEL} \\
P = P(t) = \frac{C}{1 + a e^{-b t}} \\
a, b, C \text{ constants, } \quad a > 0, \quad b > 0 \\
C = \text{CARrying CAPACITY} \\
b > 0 \rightarrow \text{GROWTH} \\
b < 0 \rightarrow \text{DECAY} \\
\text{ex (4.8.26)} \quad P(t) = \frac{99.744}{1 + 3.014 e^{-799 t}} \\
\% \text{ of MS Word users since 1984.} \\
t = \# \text{YRS since 1984.} \\
a) \text{ GROWTH RATE. } \quad 0.799 \\
b) \% \text{ of MS users in 1990?} \\
\text{ [Work done on next page after class]}
\[ P(t) = \frac{99.744}{1 + 3.014 e^{-0.799t}} \]

b) If \( t = 6 \), find \( P \)
\[
P(6) = \frac{99.744}{1 + 3.014 e^{-0.799(6)}} \approx 97.3266...  
\]

\[ P \approx 97.3\% \]

c) If \( P = 90\% \) find \( t \)

\[
(90) = \frac{99.744}{1 + 3.014 e^{-0.799t}}
\]

\[
90(1 + 3.014 e^{-0.799t}) = 99.744
\]

\[
90 + (90)3.014 e^{-0.799t} = 99.744
\]

\[
(90)3.014 e^{-0.799t} = 99.744 - 90
\]

\[
e^{-0.799t} = \frac{99.744 - 90}{(90)3.014}
\]

\[
-0.799t = \ln\left(\frac{9.744}{(90)3.014}\right)
\]

\[
t = \frac{\ln\left(\frac{9.744}{(90)3.014}\right)}{-0.799} \approx 4.16323...
\]
5.1 ANGLES AND THEIR MEASURES

ANGLE

RAY: \( \overrightarrow{BA} \)

RAY: \( \overrightarrow{BC} \)

\( \angle ABC \) or \( \angle CBA = \theta \)

\( \theta > 0 \)

\( \theta < 0 \)

MEASURE ANGLES USING DEGREES.

\( 360^\circ \rightarrow \) FULL CIRCLE

\( 180^\circ \)
Radian Measure

Circle of radius $r$

$\theta = \frac{\text{arc}}{\text{radius}}$

$\theta = \frac{S}{r}$ (cm) \quad \Rightarrow \quad S = r \theta$

1 radian = $\frac{S}{r} = \frac{r}{r} = 1$

Full circle $\theta = 2\pi r$

$\theta = \frac{2\pi r}{r} = 2\pi$
Note 2π radians = 360°

\[
\begin{align*}
\pi & = 180° \\
\pi/2 & = 90° \\
\pi/4 & = 45° \\
\pi/3 & = 60°
\end{align*}
\]

Midterm: 6 problems
20-25 responses.

4 problems: Polynomials / Rational Functions

 poly → find zeros, factor, sketch/identify graph.

 Rational Functions
  - Domain
  - Intercepts
  - Asymptotes (Hor, Vert, Oblique)
  - "holes"
  - Sketch/Identify graphs

Solve Ineq.
2 PROBLEMS.

- FUNCTIONS: COMPOSITION, FINDING INVERSES.
- EQUATIONS (EXP, LOGARITHMIC)