3 CLASS NOTES: Mon. 3 Feb.

Note §4.7 is not on 1st midterm

4.7 FINANCIAL MODELS

SIMPLE INTEREST: \[ I = P \cdot r \cdot t \]

- \( I \) = interest charged/received
- \( P \) = amount (principal)
- \( r \) = interest rate (as a decimal)
- \( t \) = # time.

Ex: Borrow $800 at 6% annual interest for 2 years. What is interest?

\[ P = 800 \quad r = 0.06 \quad t = 2 \]

\[ I = P \cdot r \cdot t = 800 \cdot 0.06 \cdot 2 = 96 \]

Note total repayment = \( P + I = 896 \)
COMPOUND INTEREST: DEPOSIT $1000, IN AN ACCOUNT THAT PAYS 8% ANNUAL INTEREST. INTEREST WILL BE CREDITED QUARTERLY; KEEP IT IN BANK FOR 3 YRS.

SIMPLE INT: $1000 (.08) 3 = $240

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>START BAL</th>
<th>INTEREST</th>
<th>END BALANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>20</td>
<td>1020</td>
</tr>
<tr>
<td>2</td>
<td>1020</td>
<td>20.40</td>
<td>1040.40</td>
</tr>
<tr>
<td>3</td>
<td>1040.40</td>
<td>20.8080</td>
<td>1061.21</td>
</tr>
<tr>
<td>4</td>
<td>1061.21</td>
<td>21.2242</td>
<td>1082.43</td>
</tr>
</tbody>
</table>

A = 1000 (1.02)^n
After 3 yrs, \( A = 10000 \left( 1.02 \right)^{12} \)

\[ = 12682.4 \]

**Compound Interest Formula:**

\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]

- \( n \) = # periods/year
- \( A \) = Amount (at end)

**EXAMPLE:**

WANT $5000 IN 4 YEARS HOW MUCH SHOULD I DEPOSIT NOW IN AN ACCOUNT THAT PAY 6% ANNUAL INTEREST IF \( r \) INTEREST PAID MONTHLY.

\[ A = 5000 \quad r = .06 \quad n = 12 \quad t = 4 \text{ YEARS} \]

\[ (5000) = P \left( 1 + \frac{.06}{12} \right)^{12(4)} \]

\[ \frac{5000}{(1 + .005)^{48}} = P = \frac{A}{(1 + \frac{r}{n})^{nt}} \]
Must deposit $3,939.50

Deposit $500 in an account that pays 2.4% annual interest, compounded quarterly. How many years until I will have $1,000?

\[ P = 500 \quad r = \frac{2.4}{100} = 0.024 \]
\[ A = 1000 \quad n = 4 \quad 4t \]

\[
\frac{1000}{500} = (1 + \frac{0.024}{4})^{4t}
\]
\[
2 = 1.006^{4t}
\]
\[
\ln(2) = \ln(1.006)^{4t}
\]
\[
\ln(2) = 4t \ln(1.006)
\]

Exact: \[
(\frac{\ln(2)}{4 \ln(1.006)}) = t \approx 28.96 \text{ years} \]

29 years
$1$ deposited in an account that pays $100\%$ annual interest. Account held $1$ year.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

- $P = 1$
- $t = 1$
- $r = \frac{100}{100} = 1$

$$A = 1 \left(1+\frac{1}{n}\right)^n$$

$n=1$  
$$A = \left(1+\frac{1}{(4)^1}\right)^1 = 2 = 2$$

$n=2$  
$$A = \left(1+\frac{1}{(2)^2}\right)^2 = \left(1.5\right)^2 = 2.25$$

$n=4$  
$$A = \left(1+\frac{1}{(4)^4}\right)^4 = \left(1.25\right)^4 = 2.44$$

$n=10$  
$$A = \left(1+\frac{1}{(10)^{10}}\right)^{10} = (1.1)^{10} = 2.59$$

$n=100$  
$$A = \left(1+\frac{1}{(100)^{100}}\right)^{100} = (1.01)^{100} \approx 2.70$$

As $n$ increases, value of $A$ increases (slows down)

As $n \to \infty$, $A \to e$  

Natural base
CONTINUOUSLY COMPOUNDED INTEREST.

\[ A = P \left(1 + \frac{r}{n}\right)^n \rightarrow \text{as } n \to \infty \]

\[ A = Pe^t \]

\[ \text{ex} \quad P = \$400 \quad r = 0.05 \quad t = 3 \text{ yrs.} \]

\[ A = (400)e^{0.05 \cdot 3} = 400e^{0.15} = 464.73 \]

DEFN EFFECTIVE RATE (ANNUAL YIELD)
SAME (SIMPLE RATE) VALUE AS ACTUAL.

1000 @ 8% QUARTERLY (FOR 1 YR)

\[ 1000 \left(1 + \frac{0.08}{4}\right)^4 = 1082.43 \]

EFFECTIVE RATE 8.243%.

EFFECTIVE RATE: \[ r_e = \left(1 + \frac{r}{n}\right)^n - 1 \] (COMPounded \(n \) TiMEs)

\[ r_e = e^r - 1 \] (CONTINUOUSLY)
4.8 Exponential Growth/Decay Model

\[ A(t) = A_0 e^{kt} \]

(assume continuous growth/decay)

- If \( k > 0 \) amount grows
- If \( k < 0 \) amount decays.

\[ A(0) = A_0 \]

\[ A(t) = A_0 e^{kt} \]

\[ A(0) = A_0 e^{k(0)} = A_0 \]

\[ 1500 \text{ bacteria present at start} \]

\[ \frac{A(t)}{A_0} = e^{kt} \]

\[ \frac{2400}{1500} = e^{kt} \]

\[ e^{k(3)} = 1.6 \]

\[ 2400 = 1500e^{k(3)} \]

Think "word" problems
SOLVE FOR $k$: $3k$

Isolate $2400 = 1500e$

Exp'l Factor $\frac{2400}{1500} = e^{3k}$

$\ln\left(\frac{2400}{1500}\right) = \ln(e^{3k}) = 3k$

So $\ln\left(\frac{2400}{1500}\right) = k = \ln(\frac{24}{15})$

$\frac{8}{5}$

$k \approx 1.66667$

Bacteria is growing at a rate

$15.67\%$ per hour.