3 Class Notes: Wed. 9 May

Midterms will be returned in section.

Ch 5 - Trigonometric Functions

5.1 Angles & Their Measure

Angle: Formed by 2 rays (half-lines) with common endpoint (vertex)

Notation: \( \angle B \) or \( \angle ABC \)

\( \text{vertex in middle,} \)

\( \angle AEC \)

Notation: Use Greek letters - \( \alpha, \beta, \gamma, \theta \)

Angle in standard position (on a coordinate system) if initial side is on positive x-axis and rotation counter-clockwise
Is a positive angle.

Clockwise: negative angle.

Measures of angles

Degrees: Full circle is

360°

Degrees can be broken into decimals, or minutes (1° = 60 minutes = 60') and seconds (1' = 60 seconds = 60'')

Example:

42°4' = 42°24'

Ex

35°36'30"

= 35°36.5'

8°35.608°
**Definition:** Radian measure. Place vertex of an angle at center of a circle of radius $r$. Let $s$ be length of arc on circle formed by two rays. This **radian measure** is given by:

$$\theta = \frac{s}{r}$$

**Note:** $s = r\theta$

**Example:**

$r = 8 \text{ cm}; \quad s = 10 \text{ cm} \rightarrow \theta = \frac{s}{r} = \frac{10 \text{ cm}}{8 \text{ cm}} = \frac{5}{4} = 1.25 \text{ radians}$

**Example:**

$r = 6 \text{ in}; \quad \theta = 1.5 \text{ radians} \quad s = r \cdot \theta = (6 \text{ in})(1.5) = 9 \text{ inches}$

**Note:** Radian measure is independent of size of circle.

$$\theta = \frac{s_1}{r_1} = \frac{s_2}{r_2} = \frac{s_3}{r_3}$$
NOTE: FULL CIRCLE IS $360^\circ$

IF $\theta = 360^\circ$

$S = \text{circumference}$

$= 2\pi r$

SO $360^\circ = 2\pi$

ALSO  $180^\circ = \pi$

$90^\circ = \frac{360^\circ}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

WELL KNOWN ANGLES:

<table>
<thead>
<tr>
<th>$\theta$ (DEGREES)</th>
<th>$30^\circ$</th>
<th>$45^\circ$</th>
<th>$60^\circ$</th>
<th>$90^\circ$</th>
<th>$180^\circ$</th>
<th>$360^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ (RADIANS)</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\pi}{3}$</td>
<td>$\frac{\pi}{2}$</td>
<td>$\pi$</td>
<td>$2\pi$</td>
</tr>
</tbody>
</table>

BIGGER LIST IN TEXT
CONVERSIONS: USE FACT $180^\circ = \pi$ RAO

CONVERT

$40^\circ$ TO RADIANS:

$$40^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = \frac{40\pi}{180} \text{ rad} = \frac{2\pi}{9} \text{ rad.}$$

CONVERT $\frac{\pi}{5}$ TO DEGREES:

$$\frac{\pi}{5} \text{ rad} = \frac{180^\circ}{\pi} \cdot \frac{\pi}{5} = \frac{180^\circ}{5} = 36^\circ$$

NOTE: $1$ Radian

$1$ Radian

$\frac{180^\circ}{\pi}$

$= \frac{\pi}{180^\circ}$

$= 57.3^\circ$

AREA OF A SECTOR: DEPENDS ON ANGLE.

$\frac{\theta}{2\pi}$

$\frac{\text{AREA OF SECTOR}}{\text{AREA OF CIRCLE}} = \frac{\theta}{2\pi}$

So AREA OF SECTOR $= \frac{\theta}{2\pi} \left(\pi r^2\right)$
\[ A = \frac{1}{2} \theta r^2 \]

**Den travel around the circle: object moves on a circle at a constant speed**

\[ S = \text{distance traveled} \]
\[ t = \text{time of travel} \]

**Linear speed**

\[ v = \frac{S}{t} \quad \text{cm/ sec} \]

**Angular speed**

\[ \omega = \frac{\theta}{t} \quad \text{rad/ time} \]

**Omega**

**Example (5.1.101)**

Gondola on a ride. SANS at 13 REV/ min. If gondola is 25 ft from center, what linear speed is in miles per hour?

\[ \frac{25 \text{ ft}}{13 \text{ REV}} = \frac{2.17 \text{ RAD}}{1 \text{ HR}} \]
\[ \frac{60 \text{ min}}{1 \text{ mile}} \]

\[ \frac{25 \cdot 13 \cdot 2\pi \cdot 60}{5280} = \frac{39000\pi}{5280} \approx 23.2 \text{ MPH} \]
UNIT CIRCLE: \( r = 1 \)
\[ x^2 + y^2 = 1 \]

DRAW VERT. TANGENT AT \((1,0)\) FIND A "\(t\)" ON LINE

'WRAP' LINE AROUND CIRCLE

**DEF** \( P = (x,y) \) BE POINT ON UNIT CIRCLE
THAT CORRESPONDS TO \( t \) THEN SIX TRIGONOMETRIC
FUNCTIONS ARE:

- **SIN**E: \( \sin t = y \)  
- **CO**SINE: \( \cos t = x \)
- **TA**NGENT \( \tan t = \frac{y}{x} (x \neq 0) \)
- **COT**ANGENT \( \cot t = \frac{x}{y} (y \neq 0) \)
- **SEC**ANT \( \sec t = \frac{1}{x} (x \neq 0) \)
- **CSC**ANT \( \csc t = \frac{1}{y} (y \neq 0) \)

**NOTE:** KNOWING JUST ONE POINT DETERMINES ALL 6 TRIG FUNCTIONS