3.5 Graphs of Rat'l Fcn's (Wrap-Up)

\[ R(x) = \frac{2x^2 + 3x - 27}{x^2 - 9} = \frac{(2x+9)(x-3)}{(x+3)(x-3)} \]

Domain: \( \{ x \mid x \neq -3, 3 \} \)

\[ R(x) = \frac{2x+9}{x+3} \]

Intercepts: \( R(0) = \frac{2(0)+9}{0+3} = \frac{9}{3} = 3 \) \( (0, 3) \)

X-intercept: \( R(x) = \frac{2x+9}{x+3} = 0 \)

\( 2x + 9 = 0 \)

\( x = -\frac{9}{2} \)

Asymptotes: Vertical: \( x = -3 \)

Horizontal: \( R(x) = \frac{2x+9}{1x+3} \rightarrow y = 2 \)

\[ R(-4) = \frac{2(-4)+9}{(-4)+3} = \frac{1}{-1} = -1 \]

\[ R(-10) = \frac{2(-10)+9}{(-10)+3} = \frac{-1}{-7} \]

\[ \frac{-1}{-7} = \frac{1}{7} \]
3.6 POLYNOMIALS AND RATIONAL INEQUALITIES

Standard Form: \[ f(x) \leq 0 \]

Factor everything.

Find values of \( x \) for which all factors = 0

In each interval determined by values, use sample point to determine whether \( f(x) \) is positive or negative.
Example: 
\[
(x - 2)^2 \geq 0
\]

Zeros at \( x = 2, -5 \)

\[
\begin{align*}
\text{at } x = -6: & \quad f(-6) = (-6)^2 = 36 > 0 \\
\text{at } x = 0: & \quad f(0) = (-2)(5) > 0 \\
\text{at } x = 2: & \quad f(2) = (1)^2(8) = 8 > 0
\end{align*}
\]

\[ (-\infty, 2) \cup (2, \infty) \cup \{-5, 2\} = [-5, \infty) \]

Example: 
\[
2x^3 < 15x - x^2
\]

\[
\begin{align*}
2x^3 + x^2 - 15x & < 0 \\
x(2x^2 + x - 15) & < 0 \\
x(x - 5)(x + 3) & < 0
\end{align*}
\]

\[ a = 2, \ b = 1, \ c = -15 \]

\[
-1 \pm \sqrt{1^2 - 4(2)(-15)} \quad \frac{120}{8} \quad \frac{-1 \pm 11}{2}
\]

\[
-1 \pm \sqrt{121} \quad \frac{-1 \pm 11}{4} \quad \frac{-1 + 11}{4} = \frac{10}{4} = \frac{5}{2}, \quad \frac{-1 - 11}{4} = \frac{-12}{4} = -3
\]
SOLUTION

\[ (-\infty, -3) \cup (0, \frac{5}{2}) \]

**RATIONAL INEQUALITIES:**

**EX:** SOLVE: \( R(x) = \frac{4x^2 - 24x + 36}{x^2 - x - 12} \geq 0 \)

\[ \frac{4(x-3)(x-3)}{(x+3)(x-4)} \]

\[ R(0) = \frac{36}{-12} = -3 \]

\[ (-\infty, -3) \cup (4, \infty) \cup \{3\} \]
Solve: \[ \frac{(x-1)^2}{(x-4)^2} = 1 \]

\[ \frac{(x-1)^2}{(x-4)^2} - 1 = 0 \]

\[ \frac{(x-1)^2 - (x-4)^2}{(x-4)^2} = 0 \]

\[ \frac{6x-15}{(x-4)^2} = 0 \]

\[ 6x - 15 = 0 \]

\[ 6x = 15 \]

\[ x = \frac{15}{6} = \frac{5}{2} \]

Solution: \[ \left[ \frac{5}{2}, 4 \right) \cup (4, \infty) \]

Note: \[ \frac{(x-1)^2}{(x-4)^2} \leq 1 \quad (-\infty, \frac{5}{2}] \]
4.1 COMPOSITE FUNCTIONS

DEFN GIVEN f and g the composite function denoted \( f \circ g \) ("f composed with g") is defined by

\[
f \circ g(x) = f(g(x))
\]

f of g of x

EX

\( f(x) = x^2 \) \quad \text{and} \quad g(x) = 2x - 8.

FIND \( f \circ g(5) = f(g(5)) = f(2) = (2)^2 = 4 \)

\( 2(5)-8 \)

\( g \circ f(-2) = g(f(-2)) = g(4) = 2(4)-8 = 0 \)

\( (2)^3 \)

\( f \circ g(x) = f(g(x)) = f(2x-8) = (2x-8)^2 \)