2A PROPERTIES OF QUADRATIC FUNCTIONS

THE GRAPH OF A QUADRATIC FUNCTION,

\[ f(x) = ax^2 + bx + c \]

IS A PARABOLA; OPENS UP OR DOWN. TURNING POINT \( \uparrow \) \( \downarrow \)

IS THE VERTEX. VERTICAL LINE THROUGH VERTEX IS LINE OF SYMMETRY.

\[ f(x) = a(x-h)^2 + k \]

VERTEX: \( \left( -\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right) = (h, k) \)

BOTH FORMS: \( a > 0 \rightarrow \) PARABOLA OPENS UP,

\( a < 0 \rightarrow \) PARABOLA OPENS DOWN.
SKETCH GRAPH OF $f(x) = 2x^2 + 8x - 10$

$a = 2$ OPENs UP

VERTEX AT $x = \frac{-b}{2a} = \frac{-(8)}{2(2)} = -2$

$x = -2$

$f(-2) = 2(-2)^2 + 8(-2) - 10$

$= 2(4) - 16 - 10$

$= 8 - 26$

$= -18$

INTERCEPTS:

$y$-INT: $x = 0, f(0) = -10$

$x$-INTs: $f(x) = 0$

$2x^2 + 8x - 10 = 0$

$2(x^2 + 4x - 5) = 0$

$x - 1, x + 5 = 0$

$x = -5, 1$
COMPLETE THE SQUARE

\[ f(x) = 2(x+2)^2 - 18 \]

\[ 4+2=2 \]

\[ V @ (-2, -18) \]

WHEN = 0, @ VERTEX

\[ x = -2 \]

\[ f(x) = 0 = 2(x+2)^2 - 18 \]

\[ 18 \quad 18 \]

\[ \frac{18}{2} \quad \frac{18}{2} = (x+2)^2 \]

\[ \sqrt{\pm\sqrt{9}} = x+2 \quad \rightarrow \quad x+2 = \pm3 \]

\[ \checkmark \]

Y-INT \quad x = 0

\[ f(0) = 2((0)+2)^2 - 18 \]

\[ 8 - 18 = -10 \]

FINDING A QUADRATIC FUNCTION FORMULA,
GIVEN INFORMATION ABOUT PARABOLA,

\[ f(x) = ax^2 + bx + c, \quad \text{Find } a, b, c. \]

REQUIRES THREE POINTS ON PARABOLA
\[(x_1, y_1) \rightarrow f(x_1) = y_1 = a(x_1)^2 + b(x_1) + c\]

Better if one point is vertex \((h, k)\)

Use \[f(x) = a(x - h)^2 + k\]

One other point allows us to find \(a\).

Ex: Find eq'n of parabola w/ vertex \((2, 5)\) through \((6, -3)\)

\[f(x) = a(x - h)^2 + k\]

\[= a(x - 2)^2 + 5\]

\(x = 6; f(6) = -3 = a(6 - 2)^2 + 5\)

\[-3 = a(16) + 5\]

\[-8 = 16a\] \[\Rightarrow \frac{-8}{16} = a = \frac{-1}{2}\]

\[f(x) = -\frac{1}{2}(x - 2)^2 + 5\]
2.5 **INEQUALITIES INVOLVING QUAD. FCNS**

STANDARD INEQUALITY: \( f(x) = ax^2 + bx + c \)

To solve:

1) Find zeroes of \( f(x) = ax^2 + bx + c \); these \( x \)-values break number line into three intervals. In each interval, test a point for sign of \( f(x) \).

Example: \( x^2 + 2x - 15 \geq 0 \) \((-\infty, 5] \cup [3, \infty)\)

\( x^2 + 2x - 15 = 0 \)

\[
\begin{align*}
x^2 + 2x + 1 &= 15 + 1 \\
(x + 1)^2 &= 16 \\
x + 1 &= \pm 4
\end{align*}
\]

\( x + 1 = 4 \quad x = 3 \)

\( x + 1 = -4 \quad x = -5 \)

\( f(-10) = 100 - 20 - 15 > 0 \)

\( f(0) = -15 \)

\( f(4) = 16 + 8 - 15 = 9 > 0 \)
2) Sketch graph of \( f(x) = 2x^2 + 6x + c \). Outputs are values of \( f(x) \) find portion(s) of graph that are above/below x-axis.

\[
\text{Solve: } 3x^2 + 10 - x - 10 \quad \text{and} \quad 3x^2 + x - 10 < 0
\]

\[ f(x) \]

\( a = 3 \) opens up.

x-intercepts: set \( 3x^2 + x - 10 = 0 \)

\[
(3x-5)(x+2) = 0
\]

\[
3x-5 = 0 \quad 3x = 5 \quad x = \frac{5}{3}
\]

\[
x + 2 = 0 \quad x = -2
\]

\[
\left( -2, \frac{5}{3} \right)
\]
2.6 BUILDING QUADRATIC MODELS FROM VERBAL DESCRIPTIONS & DATA

USED IN OPTIMIZATION PROBLEMS. FIND MAX OR A MIN.

EX: RANCHER BUILDS A CORRAL NEXT TO A STRAIGHT RIVER. SHE HAS 300' FENCE MATERIAL; BUILD A RECTANGULAR PEN W/ 3 SIDES:
WHAT ARE DIMENSIONS OF CORRAL W/ LARGEST AREA?

A = LENGTH \cdot WIDTH

f(x) = x \cdot y = x \left( \frac{300-x}{2} \right)

= x \left( 150-\frac{1}{2}x \right) = 150x - \frac{1}{2}x^2

f(x) = -\frac{1}{2}x^2 + 150x

VERTEX AT \quad \frac{-b}{2a} = \frac{-150}{2(-\frac{1}{2})} = 150'
\[ x = \frac{-150}{-1} = 150 \]

\[ \text{LENGTH} = 150' \]

\[ \text{WIDTH} = 75' \]

\[ \text{MAX AREA} = 150 \times 75' = 11,250 \text{sq ft} \]