HW DUE NEXT FRIDAY:

SECTIONS START MONDAY, 9 APRIL.
(ALSO MS1 SECTIONS)

1.6 MATHEMATICAL MODELS

**EX (1.6.3a) EXPRESS DISTANCE FROM A POINT $P = (x, y)$ ON GRAPH OF $y = \sqrt{x}$ TO POINT $(1,0)$ AS A FUNCTION OF $x$.**

\[
\text{DIST} = \sqrt{(x-1)^2 + (y-0)^2}
\]

**DISTANCE FORMULA FROM**

$(x_1, y_1)$ AND $(x_2, y_2)$

\[
D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}
\]
\[ = \sqrt{(x-1)^2 + y^2} \]

\[ = \sqrt{(x-1)^2 + (\sqrt{x})^2} = \sqrt{(x-1)^2 + x} \]

\[ (1, 6, 82) \text{ Express area } A \text{ of rectangle inscribed in semicircle of radius 2 centered at origin as a function of } x. \]

\[ x^2 + y^2 = 4 \quad \rightarrow \quad y^2 = 4 - x^2 \]

\[ y = \sqrt{4 - x^2} \quad \text{for } (x, y) \]

\[ A = b \cdot h, \]

\[ = (2x)(y) \]

\[ A = (2x)\sqrt{4 - x^2} \quad \text{for } 0 \leq x \leq 2 \]

**CH 2 - LINEAR & QUADRATIC FUNCTIONS**

**2.1 LINEAR FUNCTIONS**

**DEF A LINEAR FCN IS OF FORM** \[ f(x) = mx + b \]

**GRAPH IS A LINE WITH SLOPE M ANY Y-INT AT (0, b) DOMAIN = ALL REALS = (-\infty, \infty) **
The average rate of change of a linear function is equal to the slope,

\[ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \]

Thus \( f(x) = mx + b \) is increasing over its domain if \( m > 0 \); decreasing if \( m < 0 \); constant if \( m = 0 \).

Given slope \( m \); point \( (x_0, y_0) \) on line

Point-slope form:

\[ m = \frac{\Delta y}{\Delta x} = \frac{y - y_0}{x - x_0} \]

\[ m(x - x_0) = y - y_0 \]
Given 2 points on line; first find slope: \( m = \frac{\Delta y}{\Delta x} \), then use point-slope form to find equation.

\[ \text{Line contains } (-4, 1) \text{ and } (2, 11) \]

\[ m = \frac{\Delta y}{\Delta x} = \frac{11 - 1}{2 - (-4)} = \frac{10}{6} = \frac{5}{3} \]

\[ y - 1 = \frac{5}{3} (x - (-4)) \]

\[ y - 1 = \frac{5}{3} (x + 4) = \frac{5}{3} x + \frac{20}{3} \]

\[ y = \frac{5}{3} x + \frac{23}{3} \]

\[ f(x) = \frac{5}{3} x + \frac{23}{3} \]

Example (201.52) Company buys a machine for $120,000. Depreciate value using straight line method over 10 years.
(a) Find function for \( V \) as a function of \( x \)

\[
M = \frac{\Delta y}{\Delta x} = \frac{0 - 120,000}{10 - 0} = \frac{-120,000}{10} = -12,000
\]

\[
V = -12,000x + 120,000
\]

(b) What is domain?

D = \([0, 10]\)

(c) Value after 6 years? \( V = f(x) \)

\[
x = 6 \quad V = -12,000(6) + 120,000 = -72,000
\]

\[
V = f(6) = \$48,000.
\]

(d) When will value be \( \$72,000 \)? \( V = 72,000 \)

\[
(72,000) = -12,000x + 120,000
\]

\[
-120,000 = -120,000
\]

\[
-48,000 = -12,000x
\]

\[
\frac{-48,000}{-12,000} = x \]

\[
x = 4
\]
\[-\frac{46000}{-12000} = x = +4\]

4 YEARS

2.3 QUADRATIC FUNCTIONS: THEIR ZEROS

DEF QUADRATIC FCN is of form

\[f(x) = ax^2 + bx + c\]

\[a, b, c\] REAL NUMBERS & TO

A QUADRATIC EQUATION IS AN EQN EQUIVALENT TO

\[ax^2 + bx + c = 0\] (STANDARD FORM)

NOTE \(f(x) = ax^2 + bx + c = 0\) FINDING ZEROS OR \(f\).

IF STANDARD FORM - FIND ZEROS BY

FACTORING OR USING QUADRATIC FORMULA.

\[\text{IF } ax^2 + bx + c = 0 \text{ THEN } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[b^2 - 4ac\] IS THE DISCRIMINANT

\[b^2 - 4ac > 0 \Leftrightarrow \text{ TWO REAL SOLUTIONS } \frac{-b}{2a}\]

\[b^2 - 4ac = 0 \Leftrightarrow \text{ ONE REAL SOLUTION, } x = \frac{-b}{2a}\]

\[b^2 - 4ac < 0 \Leftrightarrow \text{ NO REAL SOLUTIONS.}\]
\[
\text{Ex} \quad \text{Find zeros of } f(x) = 4x^2 - 17x - 15
\]

Set \( f(x) = 0 \):
\[
4x^2 - 17x - 15 = 0
\]

\( a = 4 \)
\( b = -17 \)
\( c = -15 \)

\[
\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{17 \pm \sqrt{(-17)^2 - 4(4)(-15)}}{2(4)} = \frac{17 \pm \sqrt{289 + 240}}{8} = \frac{17 \pm \sqrt{529}}{8} = \frac{17 \pm 23}{8}
\]

\[
\frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{17 + 23}{8} = \frac{40}{8} = 5
\]

\[
\frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{17 - 23}{8} = \frac{-6}{8} = \frac{-3}{4}
\]

Zeros are
\(-\frac{3}{4}, 5\)

\[\text{Ex} \quad \text{Solve: } (3x - 5)^2 = 8\]

\[
9x^2 - 30x + 25 = 8
\]

\[
9x^2 - 30x + 17 = 0
\]

\(3x - 5 = \pm \sqrt{8}\)

\(3x = 5 \pm \sqrt{8}\)

\(x = \frac{5 \pm \sqrt{8}}{3}\)

\(5 \pm 2\sqrt{2}, \frac{5 - 2\sqrt{2}}{3}, \frac{5 + 2\sqrt{2}}{3}\)
EQUATIONS "QUADRATIC IN TYPE"

BY A SUBSTITUTION, GET A QUAD. EQN.

EX: FIND ZEROS OR \( f(x) = \frac{2}{(2x+5)^2} - (2x+5) - 6 \)

SOLVE: \( (2x+5)^2 - (2x+5) - 6 = 0 \)

\[ y = 2x + 5 \]

\[ (y)^2 - (y) - 6 = 0 \]

\[ (y - 3)(y + 2) = 0 \]

\[ \downarrow \quad \downarrow \]

\[ y = 3 \quad y = -2 \]

\[ 2x + 5 = 3 \quad 2x + 5 = -2 \]

\[ -5 \quad -5 \quad -5 \quad -5 \]

\[ 2x = -2 \quad 2x = -7 \]

\[ x = -1 \quad x = -\frac{7}{2} \]