1. Let $\theta$ be an angle between $0$ and $2\pi$ such that $\tan \theta = -\frac{1}{4}$ and $\sin \theta < 0$.

   a. In what quadrant is $\theta$?
   b. Find the exact value of the following: you may rationalize denominators, but it is not necessary.

   \[
   \begin{array}{cccc}
   \cos \theta & \sin \theta & \sec \theta & \csc \theta \\
   \cot \theta & \cos(-\theta) & \sin(-\theta) & \\
   \end{array}
   \]

2. Two oceanfront homes are located 8 miles apart on a straight stretch of beach, each a distance of 1 mile from a paved path that parallels the ocean. Jane can jog 6 miles per hour on the paved path, but only 3 miles per hour in the sand on the beach. Jane jogs on a direct path on the sand that is at an angle of $30^\circ$ to the paved road, continues to jog on the paved road, and then jogs on the sand on a straight path at an angle of $45^\circ$ to the road (see the figure). Note that the angles and the distances in the figure are not to scale.

   a. Find the total distance that Jane jogs on this trip. Express your answer as an exact number.
   b. Find the total time of her trip. Recall that time equals distance divided by rate. Express your answer as an exact value written as a single fraction.

3. Given the function $f(x) = -3 \sin \left(2x + \frac{\pi}{2}\right) + 4$

   a. Find the amplitude of $f$.
   b. Find the period of $f$.
   c. Find the phase shift of $f$.
   d. Sketch the graph of $f$ over one period starting at the phase shift. Label on the graph the coordinates of the highest and lowest points on the graph, and the coordinates of the intersection points with the 'midline'.

4. Find the exact value of the of the following expressions:

   a. $\sin(405^\circ)$
   b. $\tan\left(\frac{5\pi}{6}\right)$
   c. $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$
   d. $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$
   e. $\tan\left(\cos^{-1}\left(\frac{7}{9}\right)\right)$
   f. $\cos^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right)$

5. Solve the equations on the interval $0 \leq \theta \leq 2\pi$:

   a. $\cos(2\theta) = -\frac{\sqrt{3}}{2}$
   b. $\sin^2 \theta - \cos^2 \theta = \sin \theta$

6. Establish the identity:

   a. $2 \sin^2 u - 1 = \frac{\tan u - \cot u}{\tan u + \cot u}$
   b. $\frac{\sec \theta + \tan \theta}{\cot \theta + \cos \theta} = \tan \theta \sec \theta$
1. a. Quadrant IV
   
   \[ \cos \theta = \frac{4}{\sqrt{17}} \quad \sin \theta = -\frac{1}{\sqrt{17}} \quad \cos(-\theta) = \frac{4}{\sqrt{17}} \]
   
   \[ \sec \theta = \frac{\sqrt{17}}{4} \quad \csc \theta = -\sqrt{17} \quad \cot \theta = -4 \quad \sin(-\theta) = \frac{1}{\sqrt{17}} \]

2. a. distance = \(9 + \sqrt{2} - \sqrt{3}\) miles \quad b. time = \(\frac{11+2\sqrt{2}-\sqrt{3}}{6}\) hours

3. a. Amplitude = 3 \quad b. period = \(\pi\) \quad c. phase shift = \(-\frac{\pi}{4}\)

4. a. \(\frac{\sqrt{2}}{2}\) \quad b. \(-\frac{1}{\sqrt{3}}\) \quad c. \(-\frac{\pi}{3}\)
   
   d. \(\frac{3\pi}{4}\) \quad e. \(\frac{4\sqrt{2}}{7}\) \quad f. \(\pi\)

5. a. \(\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}\) \quad b. \(\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\)

6. a. RHS: \[
   \frac{\tan u - \cot u}{\tan u + \cot u} = \frac{\sin u - \cos u}{\sin u + \cos u} \cdot \frac{\sin u \cdot \cos u}{\sin u \cdot \cos u} = \frac{\sin^2 u - \cos^2 u}{\sin^2 u + \cos^2 u} = \sin^2 u - \cos^2 u = \sin^2 u - (1 - \sin^2 u) = 2\sin^2 u - 1 = \text{LHS}
   \]

   b. LHS: \[
   \frac{\sec \theta + \tan \theta}{\cot \theta + \cos \theta} = \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta + \cos \theta} \cdot \frac{\sin \theta \cdot \cos \theta}{\sin \theta \cdot \cos \theta} = \frac{\sin \theta + \sin^2 \theta}{\cos^2 \theta + \cos^2 \theta} = \frac{\sin \theta}{\cos^2 \theta}
   \]

   RHS: \[
   \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = \frac{\sin \theta}{\cos^2 \theta}
   \]