5.2 Trigonometric Functions: Unit Circle Approach

The unit circle is the circle of radius 1 centered at the origin. Consider a vertical tangent line to this circle at the point \((1, 0)\). Consider a point on this line \(t\) units from the point \((1, 0)\). If this point is above the \(x\)-axis denote \(t\) as positive; if it is below the \(x\)-axis \(t\) is negative. Imagine that the line segment from \((1, 0)\) to the point \(t\) is wrapped around the unit circle. Denote the endpoint \(P = (x, y)\).

Definition: Let \(P = (x, y)\) denote the point on the unit circle that corresponds to \(t\). Then the six trigonometric functions of the real number \(t\) are defined as follows:

\[
\begin{align*}
\cos t &= x \\
\sec t &= \frac{1}{x} \quad (x \neq 0) \\
\sin t &= y \\
\csc t &= \frac{1}{y} \quad (y \neq 0) \\
\tan t &= \frac{y}{x} \quad (x \neq 0) \\
\cot t &= \frac{x}{y} \quad (y \neq 0)
\end{align*}
\]

Note that one can determine the six trig functions of a real number \(t\) just by knowing the point \((x, y)\).

example: Suppose the point \(\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)\) is the point that corresponds to \(t\). Find the six trigonometric functions of \(t\).

Now consider the ray from the origin through the point \(P\) corresponding to \(t\). Let \(\theta\) be the angle in standard position determined by this ray and the positive \(x\)-axis. We see that \(s = t = 1 \cdot \theta\), so \(t = \theta\). So we can define the six trigonometric functions of the angle \(\theta\):

\[
\begin{align*}
\sin \theta &= \sin t \\
\csc \theta &= \csc t \\
\cos \theta &= \cos t \\
\sec \theta &= \sec t \\
\tan \theta &= \tan t \\
\cot \theta &= \cot t
\end{align*}
\]

Note that we can use points on the unit circle to determine trig functions of angles (or real numbers) whose points lie on the \(x\)-axis or \(y\)-axis.

Special angles: triangles can help us determine the value of the trigonometric functions at \(\theta = \frac{\pi}{6} \left(30^\circ\right), \theta = \frac{\pi}{4} \left(45^\circ\right), \text{and } \theta = \frac{\pi}{3} \left(60^\circ\right)\).

Using symmetry to evaluate trig function values for integer multiples of the special angles.
5.2 Trigonometric Functions: Unit Circle Approach (continued)

Here is a summary of the trigonometric function values of some angles:

<table>
<thead>
<tr>
<th>θ (radians)</th>
<th>θ (degrees)</th>
<th>sin θ</th>
<th>cos θ</th>
<th>tan θ</th>
<th>csc θ</th>
<th>sec θ</th>
<th>cot θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>---</td>
<td>1</td>
<td>---</td>
</tr>
<tr>
<td>π/6</td>
<td>30</td>
<td>1/2</td>
<td>√3/2</td>
<td>1/√3</td>
<td>2</td>
<td>2√3</td>
<td>√3</td>
</tr>
<tr>
<td>π/4</td>
<td>45</td>
<td>√2/2</td>
<td>√2/2</td>
<td>1</td>
<td>√2</td>
<td>√2</td>
<td>1</td>
</tr>
<tr>
<td>π/3</td>
<td>60</td>
<td>√3/2</td>
<td>1/2</td>
<td>√3</td>
<td>2</td>
<td>2/√3</td>
<td>1/√3</td>
</tr>
<tr>
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<td>90</td>
<td>1</td>
<td>0</td>
<td>---</td>
<td>1</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
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<td>−1</td>
<td>0</td>
<td>---</td>
<td>−1</td>
<td>---</td>
</tr>
<tr>
<td>3π/2</td>
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<td>−1</td>
<td>0</td>
<td>---</td>
<td>−1</td>
<td>---</td>
<td>0</td>
</tr>
<tr>
<td>2π</td>
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<td>1</td>
<td>0</td>
<td>---</td>
<td>1</td>
<td>---</td>
</tr>
</tbody>
</table>

Note that trigonometric function values can be determined using a circle (still centered at the origin) of any radius r.

Application (5.2.129): While driving, Arletha observes the car in front of her with a viewing angle of 22°. If the car is 6 feet wide, how close is Arletha to the car in front of her.