4.8 Exponential Growth and Decay Models; Logistic Growth and Decay Models

**Exponential Growth:** \( A(t) = A_0 e^{kt} \), where \( A(t) \) is the population (or amount) at time \( t \), and \( k \) is a constant. If \( k > 0 \), the function represents **uninhibited growth**; if \( k < 0 \), the function represents **exponential decay**. Note that when \( t = 0 \), \( A(0) = A_0 \), which is therefore also a constant. The number \( k \) is called the **growth rate**.

**Example:** At the start of an experiment, there are 1500 bacteria present in a colony. Three hours later the number of bacteria has grown to 2400. Assume that the population size grows exponentially, so can be modeled by the equation \( N(t) = N_0 e^{kt} \), where \( k > 0 \).

a) Find the growth constant \( k \) and the growth formula \( N(t) \).
b) How many bacteria were there 2 hours after the experiment began?
c) When will the population size reach 4500?

**Radioactive decay:** \( A(t) = A_0 e^{kt} \), where \( k < 0 \). Note that \( k \) is the **decay rate**.

Definition: The **half-life** of a radioactive substance is the time required for half of a given sample to disintegrate. \( \text{half-life} = - \frac{ln 2}{k} \), so \( k = (- ln 2) \text{(half-life)} \)

**Example:** The half-life of radium-226 is 1620 years.

a) How much of a 2 gram sample remains after 100 years?
b) Find the time for 80% of the 2 gram sample to decay.

**Logistic Models:** Uninhibited growth may not be the best model for exponential growth. If there are limits to growth, a logistic model may be used. The population \( P \) after time \( t \) is given by \( P(t) = \frac{c}{1 + ae^{-bt}} \), where \( a, b, \) and \( c \) are constants with \( a > 0 \) and \( c > 0 \). The model is a growth model if \( b > 0 \) and is a decay model if \( b < 0 \). The number \( c \) is called the **carrying capacity** since the values for \( P(t) \) approach \( c \) as \( t \) approaches infinity. The number \( b \) is the **growth rate** (if \( b > 0 \)) or **decay rate** (if \( b < 0 \)). The properties of the logistic model are listed in the text.

**Example (4.8.26):** The logistic model \( P(t) = \frac{99.744}{1 + 3.014 e^{-0.799t}} \) represents the percentage of households using Microsoft Word since 1984.

a) What is the growth rate in the percentage of Microsoft Word users?
c) What was the percentage of Microsoft Word users in 1990?
d) During what year did the percentage of Microsoft Word users reach 90%?