4.4 Logarithmic Functions

Note that the function \( f(x) = a^x \) is one-to-one, so it an an inverse function. Recall the procedure for finding \( f^{-1}(x) \):

Definition: the **logarithmic function to the base** \( a \), where \( a > 0 \) and \( a \neq 1 \), is denoted by \( y = \log_a(x) \), represents the exponent to which \( a \) must be raised to yield \( x \).

Key relationship: \[ y = \log_a(x) \iff x = a^y \]

Note: The domain of the logarithmic function \( y = \log_a(x) \) is \( x > 0 \). The range is \( (-\infty, \infty) \).

examples (of key relationship):

examples: Find the exact value of  
\( a) \log_2(64) \quad b) \log_5\left(\frac{1}{25}\right) \)

The graph of the logarithmic function \( f(x) = \log_a(x) \) is the reflection of the graph of the exponential function \( y = a^x \) in the line \( y = x \).

Properties of the logarithmic function \( f(x) = \log_a(x) \)

1. The domain is set of all positive real numbers: \((0, \infty)\). The range is the set of all real numbers: \((-\infty, \infty)\).
2. The \( x \)-intercept is \((1, 0)\); there is no \( y \)-intercept.
3. The \( y \)-axis \((x = 0)\) is a vertical asymptote.
4. A logarithmic function is decreasing if \( 0 < a < 1 \) and increasing if \( a > 1 \).
5. The graph of \( f \) contains the points \((1, 0), (a, 1), \) and \( \left(\frac{1}{a}, -1\right) \).
6. The graph is smooth and continuous, with no corners or gaps.

Notation:  
the natural log of \( x = \log_e(x) = \ln(x) \)  
the common log of \( x = \log_{10}(x) = \log(x) \)

Solving equations with logarithms (using a calculator)

examples:  
\( a) \log_2(3x + 2) = 5 \quad b) \log_x(64) = 3 \)

examples:  
\( a) 10^{x+1} = 342 \quad b) e^{2t-3} = 200 \)