1.4 Cylindrical and Spherical Coordinates

Recall: Polar coordinates (in $\mathbb{R}^2$)

\[ x = r \cos \theta \quad y = r \sin \theta \quad r^2 = x^2 + y^2 \quad \tan \theta = \frac{y}{x} \]

**Cylindrical Coordinates**

Definition: The cylindrical coordinates $(r, \theta, z)$ of a point $(x, y, z)$ are defined by

\[ x = r \cos \theta \quad y = r \sin \theta \quad z = z \]

Note that (once again)

\[ r^2 = x^2 + y^2 \quad \tan \theta = \frac{y}{x} \]

examples: cylindrical $\rightarrow$ rectangular
rectangular $\rightarrow$ cylindrical

Describe surfaces with 'cylindrical' equations: $r = k : \quad \theta = k : \quad z = r$

**Spherical Coordinates**

The length of a vector $x \hat{i} + y \hat{j} + z \hat{k}$ in $\mathbb{R}^3$ is $\rho = \sqrt{x^2 + y^2 + z^2}$, not a coordinate using cylindrical coordinates.

Definition: The spherical coordinates of a point $(x, y, z)$ in space are triples of form $(\rho, \theta, \phi)$, as defined by:

\[ x = r \cos \theta = (\rho \sin \phi) \cos \theta \quad y = r \sin \theta = (\rho \sin \phi) \sin \theta \quad z = \rho \cos \phi \]

where \[ \rho = \sqrt{x^2 + y^2 + z^2} \geq 0 \quad 0 \leq \theta < 2\pi \quad 0 \leq \phi \leq \pi \]

In this coordinate system: $\rho =$ distance from origin, $\theta =$ angle (on $xy$-plane) w/ the positive $x$-axis, and $\phi =$ angle with positive $z$-axis.

conversions: spherical $\rightarrow$ rectangular
rectangular $\rightarrow$ spherical

Describe surfaces with 'spherical' equations: $\rho = k$ (sphere) : $\phi = k$ (cone)